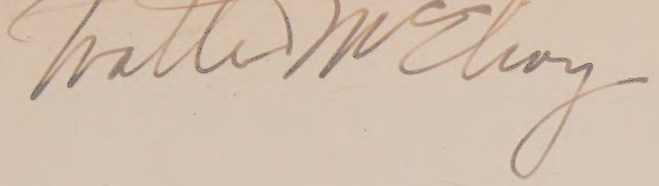

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HEAD OF DEPARTMENT OF APPLIED SCIENCE, WENTWORTH INSTITUTE, BOSTON, MASS.; FORMERLY INSTRUCTOR IN INDUSTRIAL ELECTRICITY, PRATT INSTITUTE

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PREFACE

The following text is designed primarily to meet the needs of young men who desire to follow an occupation connected with the electrical, or allied, industries. Such young men need a thorough grounding in the fundamental principles of electrical theory and measurement. They usually have neither the time nor the previous preparation for acquiring this knowledge through advanced or mathematical treatises. They need their information in a directly usable form. After acquiring the *words* of a law, they need, above all, to know what the law *means* in a practical sense. Thus they need extended drill in applying the same underlying principle to the many diversified problems of the electrical industries.

This text is based fundamentally upon a series of mimeographed notes which have been used for several years with both day and evening classes in Applied and Industrial Electricity at Pratt Institute. It may be justly claimed by the author, therefore, that his presentation, if not the usual one, or to some minds, in logical order, is at least *teachable;* that it secures results which are direct and which constitute a desirable preparation for more advanced study, as well as for practical work in the many lines of the electrical world.

In presenting each idea, it will be noted, such language is used as will make maximum use of the student's present knowledge or observation; a brief definition or discussion being followed immediately by a number of simple problems sufficient not only to enable the student to make the " sense of the matter " his own, in his own language and manner of thought, but also to cultivate in him some degree of alert-

ness and initiative in applying his knowledge. In addition, at the close of each chapter, a number of more advanced problems is given which have been in the process of collection and modification for a number of years.

The order followed, as has been suggested, is merely that which the author has found most practicable, and which will—by means of the amount of general information as to instruments, etc., placed early in the book—render the parallel laboratory course most efficient as a medium for teaching, illustration, and independent growth on the part of the student.

The author believes that the book has distinctively these four desirable qualities:

(1) It contains an adequate amount of information concerning electrical laws and practice in an immediately usable form.

(2) It applies the information contained to real things and not to abstract theory and conditions.

(3) It provides sufficient drill in concrete practical problems, which not only afford in themselves an additional amount of profitable information, but also develop in the student a capacity for applying what he has learned; the large number of diagrams aiding materially in providing exact and clean-cut conceptions of the conditions of the problems.

(4) It presents, out of the great mass of electrical phenomena, only those facts and principles which a technical student needs to know, and to know well. History and general theory, interesting as they are, have been omitted.

The text is confined to what is believed to be an adequate treatment of a *few* fundamental ideas rather than to a discursive treatment of *many*. Therefore it includes only ideas of the elementary principles of Direct and Alternating Current generation, distribution and utilization in light and power. This affords a sure foundation on which

to build more advanced courses, and yet presents a practical and well-rounded course for the man who is to receive no further electrical education.

It has been found that about fifteen weeks are needed for the student to satisfactorily cover the subject matter of the text, allotting five hours per week to recitations and six hours to laboratory work. But the book is also arranged so that by omitting the matter in smaller type and in Chapters VI, IX, and XI, a consecutive course can be given in twelve weeks, consisting of only three hours recitation and four hours laboratory work.

The previous preparation required is a knowledge of common school arithmetic and of the simplest algebra. Alternating current is presented without the use of even trigonometry, although the equations are given in each case for trignometric solution of problems. All the trigometry needed is explained on two pages of the Appendix. The elementary facts of Mechanics and of Heat may be taken up before studying the text, or may be supplied by the instructor from time to time as there is need.

The book then is especially adapted for use in the short, practical courses given in Trade, Industrial, and Technical High Schools and Apprenticeship courses, but also affords a substantial ground-work for the more advanced work in Electrical theory and practice in colleges and universities.

In conclusion, the author wishes to express his appreciation and thanks to Mr. Joseph M. Jameson, Head of the Department of Physics, and to Mr. Arthur L. Williston, formerly Director of the School of Science and Technology, Pratt Institute, for their assistance and constant encouragement in developing an Elementary Electrical course as here outlined. Grateful acknowledgment is also extended to my colleagues, Mr. John A. Randall and Mr. Warren L. Harlow for many valuable criticisms and suggestions.

W. H. Timbie.

Brooklyn, N. Y., November, 1910.

EDITOR'S NOTE

This text had already been in use for nearly two years when the "Wiley Technical Series" was conceived. Its very wide adoption and its general success as a text-book for an introductory course in electricity had thoroughly demonstrated its worth. When, therefore, the editor came to consider the texts on elementary electrical theory which would be needed for the Series, it was immediately recognized that in this volume there already existed a text which admirably met the requirements that had been laid down and upon which, as a whole, it would hardly be possible to improve. Mr. Timbie's consent to the incorporation of his "Elements of Electricity" as one of the "Wiley Technical Series" was therefore obtained.

With this brief word of explanation, the "Elements of Electricity" is presented in its new binding. It not only constitutes a concrete part of the material which the "Wiley Technical Series" is bringing out for use in applied courses, but also is relied upon to provide a fundamental part of the preparation upon which is based the subsequent texts on "Alternating Currents" and on "Direct and Alternating Current Machinery."

The Editor.

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Pages 526–540, Appendix, J. M. Jameson's "Elementary Practical Mechanics."

Page 544, Table of Equivalent Values, Kent's "Mechanical Engineers' Pocketbook."

Page 541, Table of *BH* Values, Caldwell's "Electrical Problems."

Page 332, Table of Electrochemical Equivalents, "Standard Handbook for Electrical Engineers."

Page 152, Table of Hysteresis Constants, Foster's "Electrical Engineer's Pocketbook."

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ELEMENTS OF ELECTRICITY

CHAPTER I

MAGNETS AND MAGNETISM

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In machines for the generation and use of electricity at the present time, some form or other of magnet is required. A thorough knowledge of magnets and the principles of magnetism is therefore necessary, in order to understand the construction and operation of generators, motors, and other electrical appliances. Accordingly, the subject will be presented only as it bears on electric machinery, and no attempt will be made to study it in its other aspects.

1. Magnets and Magnetism. It was known to the ancient Greeks that a "lodestone" or natural magnet possessed the power of attracting bits of iron to itself, and even could impart this attractive power to the iron pieces themselves, when they were rubbed on it.

This property of attracting iron and steel is called MAGNETISM, and a body possessing it is called a MAGNET.

Under the action of a sufficient magnetizing force, probably all materials possess this property to some degree at least, though iron and steel possess it in so much greater degree

than all other materials that they are generally referred to as the MAGNETIC substances, in distinction from all other materials. And even iron and steel differ in their magnetic qualities. Steel, on being magnetized, with proper treatment remains magnetized indefinitely, whereas iron, and especially soft iron, loses its magnetism as soon as the magnetizing force is removed. Thus PERMANENT MAGNETS are made of *hard steel*, and TEMPORARY MAGNETS of *soft iron* or *annealed steel*.

2. Poles. Certain parts, only, of a magnet possess this power of attracting iron. These parts are called the POLES, either NORTH or SOUTH, according to a distinction explained later. A line drawn across the magnet half way between the poles is called the EQUATOR.

3. Magnetic Lines. The nature of magnetism is that of a stream or CURRENT. This current of magnetism is represented by lines, called MAGNETIC LINES OF FORCE, which

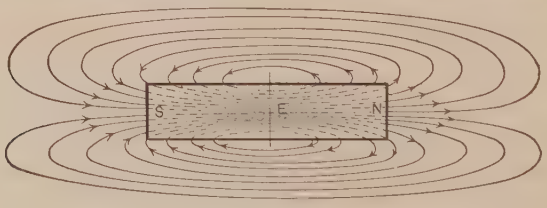

FIG. 1.—Bar magnet.

always flow out of the North pole, around into the South pole, and back through the magnet to the North pole again, thus forming a complete circuit. A NORTH POLE, then, may be defined as that portion of a magnet *out* of which the *lines of force flow*; a SOUTH POLE, that portion *into* which they flow. For convenience each pole is often considered to be concentrated at a point near the end. A bar magnet is represented as in Fig. 1. N = North pole; S = South pole; E = equator. Each magnetic line forms a *complete loop* or CIRCUIT, called a MAGNETIC CIRCUIT. The total number of lines threading the circuit makes up the magnetic FLUX

and is usually represented in formulas by the symbol ϕ. Thus when the expression, $\phi = 10{,}000$ is written, it means that the total number of lines of force passing through a given area is 10,000. These *lines of force* are as true force lines as the lines drawn to represent the tension in ropes, rods, etc., and can be combined into resultants and resolved into components as well as any other force lines.

The fact that the lines actually run through this magnet and form a complete loop, and do not merely start at the *N* pole and end at the *S* pole, is proved by breaking a magnet into several pieces. Each piece becomes a separate magnet with a *N* pole and *S* pole of its own. See Fig. 2.

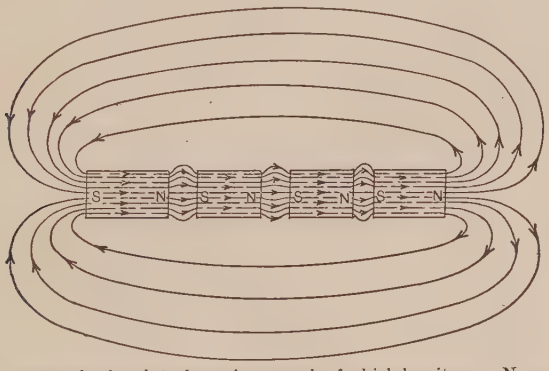

FIG. 2. A magnet broken into four pieces, each of which has its own N. and S. pole.

For convenience, the poles are sometimes *considered to be points* within the magnet, toward which all the lines seem to converge.

4. Magnetic Field. The *space* outside the magnet occupied by the magnetic lines is called the FIELD. The *number of lines per sq.cm.* (of surface at right angles to lines) is called the FIELD INTENSITY, or Field Strength.

A field intensity of 1 line per sq.cm. is called a GAUSS. Thus 10 gausses simply means 10 lines per sq.cm. The symbol for field intensity is *H*. To find the total flux ϕ in a given area, we have merely to multiply the field intensity *H* by the area *A*

We may express this relation by the equation

$$\phi = AH$$

which means

$$\phi(\text{lines}) = A(\text{sq.cms.}) \times H(\text{gausses or lines per sq.cm.}).$$

Example. What is the total flux flowing in a magnetic field of 80 sq.cms., if the average field strength is 2 kilogausses (i.e. 2000 gausses)?

$$\phi = AH$$

$$\phi = 80(\text{sq.cms.}) \times 2000(\text{gausses}) = 160{,}000 \text{ lines.}$$

Problem 1–1. The area of a pole face of a motor is 600 sq.cms. The average field intensity of magnetic field for this surface is 5000 gausses. What is the total number of magnetic lines coming out of the pole face?

Problem 2–1. What area must the pole face of a generator have, in order that the total magnetic flux may be 10,000,000 lines? H=8000 gausses.

Problem 3–1. What is the intensity at a point where 400 sq.cms. have a flux of 2,000,000 magnetic lines?

5. Distribution of Field Intensity. That part of a magnet will attract iron with the greatest force where the field intensity is greatest, that is, where the lines are flowing either out of or into the magnet in the greatest number per sq.cm. (The attraction is proportional to the square of the number of lines per sq.cm.)

The analogy to a stream of water is apparent; wherever the stream spreads out, the current is weak, and wherever it is contracted, the current is strong. Thus wherever the magnetic current spreads out and covers a large area, it is weak, and there are few lines per sq. cm., but wherever it is contracted into a small space, as near the magnet, there the current is strong and the number of lines per sq.cm. is great. Thus the intensity of the field is greatest near the poles and grows less as we go away from them. Fig. 3 illustrates this.

A represents a square centimeter. Notice that if this were moved nearer the magnet, a larger number of lines would pass

through it. If it were placed at one-half this distance, the number of lines passing through it would be not merely *twice* as great, but *four* times as great. At one-third the distance, the number of lines per sq.cm. becomes nine times as great, etc.

This may be stated as a general law as follows:

The intensity at any point in the field of an isolated magnetic pole varies inversely as the square of the distance of that point from the pole.

Or in an equation; $H \propto \frac{m}{d^2}$.

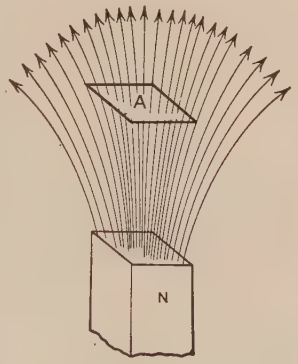

FIG. 3.—Intensity of magnetic field.

6. Nature of Magnetic Lines of Force. (1) Magnetic lines represent a *tension* along their length and tend to *shorten* as stretched rubber bands do.

(2) They also exert a lateral *crowding* effect on one another, tending to push one another sideways.

By means of these two facts we can explain many magnetic phenomena.

7. Attraction and Repulsion. When we place the *N* pole of one magnet near the *S* pole of another, the magnets attract one another. Fig. 4 shows this action. The lines

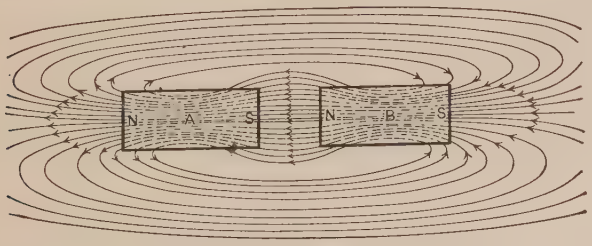

FIG. 4.—Unlike poles attract each other.

coming out of *B* enter the *S* pole of *A*, go through this, and emerging from the *N* pole of *A*, return to the *S* pole of *B*. The tension in the lines thus threading the two

magnets tends to pull the magnets together, as so many loops of rubber bands would.

If we place the north ends near each other as in Fig. 5, the lines coming out of the *N* poles turn one another aside

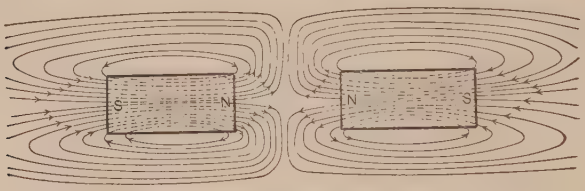

FIG. 5.—Like poles repel each other.

and their lateral crowding pressure tends to push the magnets apart.

The rule, then, is that *unlike poles attract*, while *like poles repel* one another.

If we place the magnets under a piece of glass, on which iron filings have been sprinkled, the filings will arrange themselves along the lines of force as shown in Figs. 6 and 7. This is, perhaps, the truest picture of an external magnetic field.

8. Permeability. Magnetic lines, as previously indicated, may be set up in any substance, though much more easily in some substances than in others.

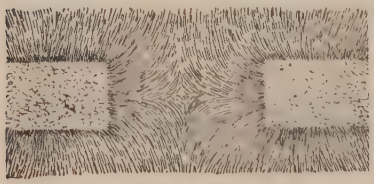

FIG. 6.—Shape of field between like poles shown by iron filings

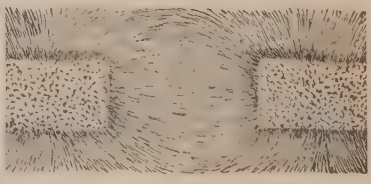

FIG. 7.—Shape of field between unlike poles shown by iron filings.

Since we can set up 1 line of force in 1 cu.cm. of *air* by 1 unit of magnetizing force, we say that air has a PERMEABILITY of 1. The same force might set up about 1500 lines in the same amount of wrought iron and 1000 lines in

steel. Therefore we say the permeability of wrought iron is 1500 and of steel is 1000, though it varies greatly in different specimens; and even the same specimen will vary as to its permeability, as explained in Chapter VI.

We may then define the permeability of a substance as the ratio of the number of magnetic lines set up in a unit volume of that substance, to the number set up in the same amount of air by the same magnetizing force.

9. Induction. The attraction of pieces of iron or steel to a magnet is a direct result of the high permeability of iron and steel. When a piece of soft iron, for instance, is placed in a magnetic field, its permeability being so much greater than that of the surrounding air, a larger number of magnetic lines is immediately set up in it than in the air. The action is as though the lines left the air and crowded into the iron. The piece of iron is then said to become a magnet by INDUCTION. This explains the action of iron filings when scattered in a magnetic field. Each particle of iron becomes a miniature magnet and acts like a compass needle.

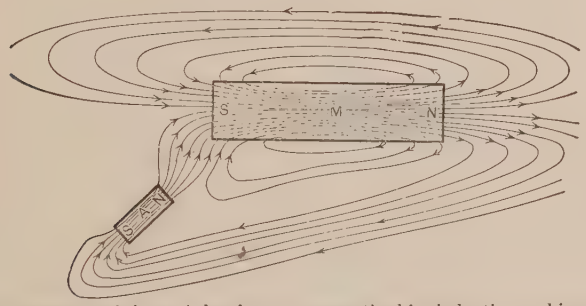

FIG. 8.—Piece of soft iron, *A*, has become magnetized by induction and is attracted to the magnet, *M*.

MAGNETIC INDUCTION is, then, the setting up in a magnetic substance of a greater number of lines than would be set up in air *by the same cause.*

Fig. 8 shows this process. *A* is a small piece of iron

which was not magnetized before it was placed in the magnetic field of the magnet *M*. The field is distorted by the lines apparently trying to leave the air and go through the iron, which now has a *N* and *S* pole of its own, since lines of force enter one end and leave the other. We have, then, two magnets with their unlike poles near each other, and they therefore attract one another. This explains why any piece of soft iron placed near a magnet is always attracted to it. The strength of the INDUCED MAGNET depends upon the permeability of the iron and the strength *H* of the magnetic field in which it happens to lie. The greater the field strength *H*, the stronger magnet it makes of a piece of iron. For this reason the field strength *H* is sometimes called the MAGNETIZING FORCE of the field. Since the field strength *H* is greater nearer the poles, so the magnetizing force on a piece of iron is also greater, the nearer the poles the piece of iron is placed. This explains why a piece of iron near a pole is attracted by so much greater force than when it is further away from the pole.

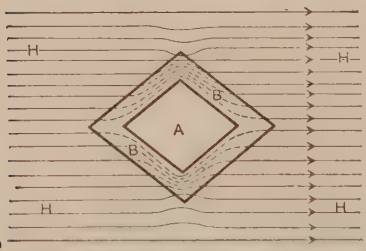

FIG. 9.—Piece of iron, *B*, acts as a screen and conducts lines around space *A*.

10. Magnetic Screens. This inductive property of iron is used also, whenever it is desired to cut the lines of force out of any part of a magnetic field. A shield of soft iron is put in the field as in Fig. 9. The iron screen *B*, since it has a much higher permeability than air, takes up nearly all the lines in the field *H* and conducts them around the space *A*. Instruments are sometimes thus shielded from the effects of a magnetic field. There is no *Magnetic Insulator*, that is, a material which will stop the lines, or one whose permeability is zero. So the above method of conducting as many of the lines as is possible around a

certain place, is resorted to, when it is desired to free it from magnetic lines.

11. Flux Density. The number of magnetic lines per sq.cm. in *any substance* is called the FLUX DENSITY in that substance.

The flux density in air is always H, which we have seen is also the magnetizing force. Note carefully these two uses for the letter H:

(1) Flux density in air.

(2) Magnetizing force.

The flux density in any material other than air is represented by the letter B, and under the influence of the same magnetizing force is as much greater than H, as the permeability of the material is greater than the permeability of air.

Thus, in Fig. 8, the piece of soft iron A was placed in the magnetic field of the magnet M. The flux density of the air at that place, before the iron A was placed in it, was H, but when the iron was placed in the field ,the flux density within the iron immediately became B which is much greater than H, because the permeability of the iron is greater than air.

The PERMEABILITY of a substance may thus be defined as the ratio of B to H, that is, the ratio of the flux density to the magnetizing force.

The Greek letter μ is used to express the value of the permeability. Thus we may write the equation

$$\mu = \frac{B}{H}$$

where μ = Permeability (*pure number*);
B = Flux density in *gausses*;
H = Magnetizing force, *gausses*.

Since in the air the magnetizing force is H and the flux density is also H, the equation for the permeability of air becomes,

$$\mu = \frac{H}{H} = 1.$$

Example. The piece of iron, A, Fig. 8, has a permeability of 900. It is placed at a point in the magnet's field where the intensity is 4 gausses. What is the flux density in the iron?

$$\mu = \frac{B}{H}$$

$$B = \mu H$$

$$= 900 \times 4 = 3600 \text{ gausses.}$$

Problem 4–1. How many lines of force per sq.cm. will be set up in a piece of steel by a magnetizing force of 18 gausses? The permeability of the steel is 1200.

Problem 5–1. It is desired to set up 400,000 magnetic lines of force in a piece of iron, the cross-section of which is 8 cms. $\times$ 10 cms. If the permeability of the iron is 750, what magnetizing force is required?

12. The Compass. If a magnet, which is free to turn on its axis, is placed in a magnetic field, it will take a position such that the lines of force *within it* run in the same direction as those in the magnetic field in which it is placed.

This is the rule for the action of the compass. A compass is merely a bar magnet suspended so that it turns freely. The earth is a large magnetic field with the lines running from south to north. Thus a compass takes a position parallel to these lines, or nearly south and north.

Of course, according to the definition we have given for north and south poles, the *North Magnetic Pole* is near the *South Geographical Pole* and vice versa. It causes no confusion to say that the compass points nearly to the North Pole.

The place to which the north end of a compass points is not the geographical North Pole of the earth, but a spot in Boothia, a peninsula in the northern part of Canada. At this spot, the earth's magnetic lines enter the earth, run through the earth, emerge near the geographical South Pole, and flow north along the earth's surface. This gives us on the earth's surface a practically uniform magnetic field flowing from south to north, called the Earth's Magnetic Field.

Practically, we are not so much interested in the North and South Pole as in the exact direction of the lines at different parts of the earth's surface; especially that part which is out of sight of land, and where the compass must be depended upon for direction. In order to map very completely the earth's magnetic field, The Carnegie Institute has had built at Brooklyn a special ship called the "Carnegie." This vessel is remarkable in that it is constructed almost entirely of non-magnetic materials. With the delicate instruments which can be used on the "Carnegie," an accurate magnetic survey of the important portions of the earth's surface is being made.

For a discussion of other magnetic phenomena, see any encyclopedia. Some special topics of interest are: History of the compass; determination of the *inclination* and *declination* of the magnetic needle; intensity of the earth's field; paramagnetic and diamag-

netic materials; magnetization of gases and fluids; effects of the medium in which the magnetic field lies.

All magnets which we use are in the earth's magnetic field. Thus the field about a magnet is really the resultant field of the earth's field and the magnet's field.

13. Unit Pole and Field Strength. It has already been stated that the intensity, or strength, at any point in a magnetic field is measured by the number of lines passing through a square centimeter of it.

Another way to measure the strength at a certain point in a magnetic field, is to measure the amount of force that it exerts on a *unit* magnetic pole placed at that point. When the field exerts a force of one *dyne* on a unit pole, the intensity or strength of that part of the field is said to be 1 line per sq.cm., or better, 1 *gauss*. The two methods of stating the field strength give the same numerical value, 1 gauss, 2 gausses, etc., which is represented by the symbol H.

A Unit Pole is a pole of such strength that, if placed a centimeter away in the air from a like pole, it will repel it with a force of *one dyne*. An isolated N pole is not a physical possibility, since every magnet must have a south pole for every north pole. We may, however, consider how such a pole would be acted upon when placed in a magnetic field.

If a unit north pole were placed near the north end of a bar magnet as in Fig. 1, it would be repelled by the north end of the magnet and would travel to the south end, not by the shortest path, but along any line of force on which it might happen to lie. Therefore a magnetic line of force represents the line along which a north pole is repelled or a south pole attracted.

When the field is strong enough to repel a unit north pole along a line with the force of one dyne, we say the field has a strength of one dyne, or of 1 gauss, i.e., 1 line per sq.cm.

We may, then, also say that a unit pole is a pole of such strength that when placed in a field of one gauss, it is acted upon by a force of one dyne.

14. Effect of a Magnetic Field upon Another Magnet. Since a magnet of unit pole strength is acted upon by a force of one dyne when placed in a unit field, it follows that if the pole strength is doubled, the action of the unit field on the magnet would be doubled. Or if we increased the field strength, its action on the magnet would be proportionally increased.

Thus we may compute the action of a magnet placed in the field of another magnet, by multiplying the pole strength of the magnet, by the strength of that part of the field in which it is placed. As an equation, this may be stated as follows:

$$F = mH.$$

where F = action of field upon magnet in *dynes;*
m = strength of magnet in *unit poles;*
H = strength of magnetic field in *gausses.*

Example. A magnet of 400 unit poles is placed in a magnetic field of 5 kilogausses. What force is exerted upon the magnet?

$F = mH.$
$F = 400\text{(unit poles)} \times 5000\text{ (gausses)}$
$= 2{,}000{,}000$ dynes
$= 2040$ grams $= 4.5$ lbs.

Problem 6–1. A magnet, when placed in a magnetic field of 200 gausses is attracted by a force of .3 lb. How many unit poles has the magnet?

Problem 7–1. If a magnet of 800 units pole strength were placed in a field of 1.4 kilogausses, what force would be exerted on the magnet?

15. Action of the Compass. Since every magnet has a north end and south end, there is always a double action upon it when placed in a magnetic field. The north end will tend to move along the lines of force in the direction of the field, and the south end, in the *opposite* direction. If the magnet is free to turn, as a compass is, it will turn until it lies along a line of force.

This explains why a compass always takes a position such that the lines of force within it are in the same direc-

tion as the field in which it lies. It always *tends to lie along a line of force,* and if placed otherwise, there will be a "force couple" acting on it to turn it into that position as in Fig. 10. Let a compass, pivoted at A, with pole strength m be placed in a position NS in a field whose direction is indicated by arrows HH. The north pole N would be repelled *along* the direction of H by a force mH, and the south pole S would be attracted in the *opposite* direction by a force $m'H$ equal to mH, and the compass would be turned around into the position N' S', parallel to force lines of field in which it lies.

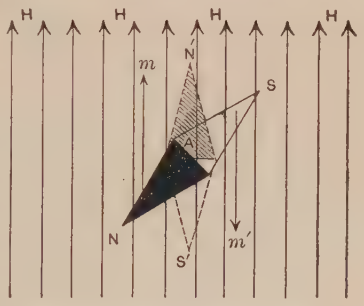

FIG. 10.—Action of magnetic field, H, on compass, A.

16. Mutual Action between Two Magnets. It has been found by experiment, that the force with which the poles of two magnets attract or repel each other, in the air, is equal to the product of their pole strengths, divided by the square of the distance between them, or;

$$F=\frac{mm'}{d^2};$$

where F = force in dynes;
m = strength in *unit poles* (of *one* pole);
m' = strength in *unit poles* (of *other* pole);
d = distance between poles in *cms.*

Example. If the N pole of a magnet of 10 units strength be placed 4 cms. from the N pole of another magnet of 24 units pole strength, with what force will they repel each other?

(Consider the magnets of sufficient length that the S poles do not affect the action of the N poles).

$$F=\frac{mm'}{d^2};$$
$$m=10;$$
$$m'=24;$$
$$d=4;$$
$$F=\frac{10\times 24}{4\times 4}=15 \text{ dynes.}$$

Problem 8–1. How far from a pole of 400 units pole strength must another pole of 3000 units pole strength be placed in order that the force between them may be .2 lb.?

Problem 9–1. Two poles, 2.4 cms. apart, attract each other with a force of 4940 dynes. One has a strength of 2430 unit poles. What strength has the other?

17. Relation between Unit Pole and Force Lines. There are, then, two ways of indicating the pole strength of a magnet; either:

(1) By the number of unit poles it contains, or,

(2) By the number of lines of force coming out of it.

It is necessary often to reduce one set of units to the other. This can be done as follows:

A unit pole may be thought of as a point which sends out one line of force to every sq.cm. of surface situated 1 cm. distant from the pole. The shape of a surface, everywhere 1 cm. distant from a point, is a sphere of 1 cm. radius.

$$\text{Area of a sphere} = 4\pi r^2$$

where r = radius of sphere;

In this case we have said $r = 1$ cm.

$$\therefore \qquad 4\pi r^2 = 4\pi \text{ sq.cms.}$$

Since there is one line to each sq.cm. of surface, there must be 4π lines to the complete surface.

Therefore each *unit pole* is conceived to have a total of 4π *lines* of force coming out of or going into it.

A magnet with the strength of 24 unit poles would therefore have $24 \times 4\pi$ or 302 lines flowing out of the N pole around into the S pole. At a distance of 1 cm. from the pole, however, there would be a field intensity of 24 gausses, or 24 lines per sq.cm.

18. Relation between Equations for Magnetic Force. We have then two equations for magnetic force F:

Force between two magnets,

$$F = \frac{mm'}{d^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Force on magnet in magnetic field;

$$F = mH \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

It is well to consider the relation between these two equations, (1st) *Mathematically*, and (2d) *Analytically*.

(1st) *Mathematically*. We have seen that the field strength H at 1 cm. from a unit pole is 1 gauss, or 1 line per sq.cm. The field strength at 1 cm. distance from pole (m) of 64 units pole strength would be 64 gausses, or 64 lines per sq.cm.

Also we have seen that the field strength H varies inversely as the square of the distance from the pole m'.

Thus we may write the equation

$$H = \frac{m'}{d^2},$$

where H = field strength in *gausses*;
m' = pole strength in *unit poles*;
d = distance in *centimeters*.

If now we substitute this value for H in Eq. (2), $F = m \times H$, we get,

$$F = m \times \frac{m'}{d^2} = \frac{mm'}{d^2}.$$

But this is the same as Eq. (1) for the force exerted by one magnet on another.

Thus we see that the force exerted by one magnet on another is merely the force exerted by its magnetic field on that magnet, and can readily be found by the general equation for the action of a magnetic field on a magnet placed in it, $F = mH$.

(2d) *Analytically.* Consider Fig. 11. Let m = pole of 64 units pole strength. There is, then, a field intensity of 64 gausses at a distance of 1 cm. from the pole; that is, 64 lines emerge from the pole for every sq.cm. of the surface on a globe 1 cm. from the pole. Thus the field intensity along the surface AA 1 cm. from the pole = 64 gausses. Along a surface BB, 2 cms. from the pole, the field intensity would *not* be one-half as great, but only one-quarter as great; that is, $\frac{64}{4}$, or 16 gausses. At 3 cms. distant, the field would become only $\frac{1}{9}$ as great, or 7.1 gausses. Thus while there are 64 lines per sq.cm. at 1 cm. distance, there are only 16 lines per sq.cm. at 2 cms. and 7.1 lines per sq.cm. at 3 cms. distance. This follows from the fact that the strength of the field about a single magnetic pole varies inversely as the square of the distance from the pole.

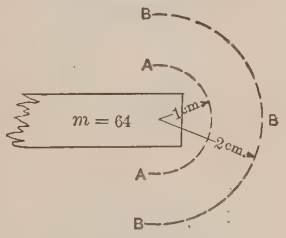

FIG. 11.

Suppose now a magnetic pole m' of 10 units pole strength be placed 1 cm. from m, of 64 units. It would be in a field of 64 gausses as explained above. The force exerted upon it would then be $H \times m = 64 \times 10 = 640$ dynes.

Also according to the equation (1) the force would be

$$\frac{mm'}{d^2} = \frac{64 \times 10}{1 \times 1} = 640 \text{ dynes.}$$

Suppose this magnet of 10 units pole strength be placed 2 cms. away from m. It would now be in a field of 16 gausses, as seen above, and the action would be $H \times m = 16 \times 10 = 160$ dynes.

Or by other method:

$$\frac{mm'}{d^2} = \frac{64 \times 10}{2 \times 2} = 160 \text{ dynes.}$$

Thus it is seen that the two equations $F = \frac{mm'}{d^2}$ and $F = Hm$ are identical and give the same result.

Therefore, if we know the number of *unit poles* a magnet contains and the *strength of the field* in which it lies, and we wish to know the force exerted on the magnet we have merely to use the equation

$$F = mH.$$

If, on the other hand, we know the number of *unit poles* two magnets contain and *their distance apart* we may use the equation:

$$F = \frac{mm'}{d^2},$$

Or we may find the field strength of one at the distance d by the equation $H = \frac{m'}{d^2}$ and consider the action of this field on the the other magnet by the equation $F = mH$.

Problem 10–1. In how strong a field is magnet of 400 units strength in problem 8–1, to fulfill conditions of the problem?

Problem 11–1. Two magnets, one of 800 units pole strength, the other of 500 units pole strength, are 3 cms. distant from each other. In how strong a magnetic field may each be considered to lie?

Problem 12–1. Compute in *three* ways the force exerted by the magnets of problem 11 on each other.

19. Ring Magnets. There are cases where a piece of iron may be strongly magnetized and still possess no poles, as in the ring, Fig. 12. Since the lines nowhere come out of the iron, they cannot produce any poles or external field. If, however, we break the ring, as in Fig. 13, we have the usual two poles, a north and a south, with an external field of great intensity, though small in area. The complete ring is used in some types of meters and also of transformers

where no external field is desired. The broken ring is the fundamental form of the magnetic circuit of generators and motors where an intense external field is desired in limited air space.

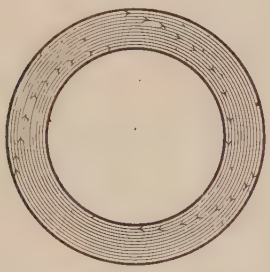

FIG. 12.—Closed iron magnetic circuit. No poles.

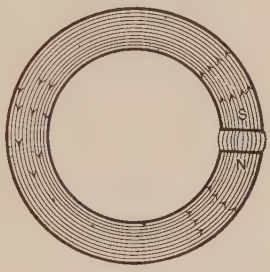

FIG. 13.—Nearly closed iron magnetic circuit.

20. Consequent Poles. Although two poles is the least number a magnet can have if it has any pole, it may possess any number greater than two. All those except the two end poles are called consequent poles.

N_1 and S_1 (Fig. 14), are consequent poles.

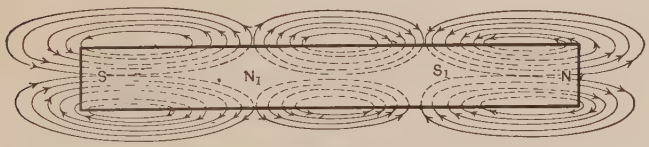

FIG. 14. Magnet with consequent poles, N_1 and S_1.

21. Permanent Magnets. Most of the electrical measuring instruments, such as ammeters, voltmeters, wattmeters, etc., contain a permanent magnet. The precision of the instrument depends upon the magnet remaining of the same strength from month to month.

We have seen that hard steel makes the strongest permanent magnet. For short magnets the steel should be "glass hard," that is, tempered by being heated to a red

heat and suddenly immersed in water, oil, or mercury. A magnet made of steel treated in this way will be very strong at first, but even *it* will gradually lose some of its strength, especially if handled roughly.

According to the "MAGNETIC MOLECULE" theory, it is easy to see that it would be no difficult matter to jar the molecules from their magnetic position.

22. Magnetic Molecules. Weber's theory of magnetism is perhaps the best one advanced to explain the different magnetic phenomena.

He supposes that all matter is made up of small molecules which are minute magnets. In iron and steel, these little magnets are strong, in all other materials they are weak. When a piece of material is not magnetized, these molecules lie in no regular position with regard to one another, as in Fig. 15. When the

FIG. 15.—Molecules of unmagnetized bar.

material is magnetized the molecules all lie with their *N* ends pointing the same way, as in Fig. 16.

A good illustration of this is to imagine a herd of cattle crowded into a pen. They will face in every direction. They thus fairly represent the molecules of an unmagnetized piece of iron. Now suppose someone comes to one end of the pen with some fodder. The cattle will all head toward that end of the pen. They then represent the position of the molecules in a magnetized piece of iron. The ease with which the molecules can be turned into this position is partially represented by the permeability of the material; as to whether or not a permanent magnet is formed depends on whether the molecules tend to retain this position

FIG. 16.—Molecules of magnetized bar.

of Fig. 16, or return to their original position of Fig. **15.** The molecules of hard steel tend to retain this magnetic position, whereas the molecules of soft iron tend to return to the original

position. Thus hard steel forms a PERMANENT magnet, and iron, a TEMPORARY one.

23. Aging of Magnets. It has been found that by every jar that a magnet receives, it loses some of its strength, though the amount lost by each successive jar becomes less and less. Thus all magnets which are to be of unvarying strength, are put through a process of "aging," which has the effect of settling their strength at a definite point. With reasonable care they will retain this strength indefinitely. By one method the steel is steamed for 30 hours or longer, then magnetized and steamed for 4 or 5 hours more. This treatment demagnetizes it more or less, of course, but the remaining magnetism is very nearly permanent, especially if the steel forms a nearly closed loop.

It has also been found that thin magnets are stronger in proportion to their weight than thick ones, so a laminated magnet, that is, one made of thin magnets bound together, is stronger than one made of a solid piece of steel.

Heat and rough treatment will ruin the best of magnets, hence the need of careful handling of electrical instruments.

The method of magnetizing iron and steel will be discussed in the next chapter.

SUMMARY OF CHAPTER I

MAGNETIC FLUX consists of lines of force which make a complete circuit. Symbol, ϕ.

THE INTENSITY at any point in this magnetic circuit, or field, is measured in gausses, which represent the force on a unit pole, or the number of lines per sq.cm. Symbol, H.

TOTAL FLUX, ϕ, equals average intensity, H, times the area, A, $\phi = AH$.

A NORTH POLE is the place where lines of force leave a magnet; a SOUTH POLE, where lines enter. The poles are often considered to be points. Like poles repel; unlike poles attract one another.

A UNIT North pole is one of such strength that if placed a centimeter distant from a like pole of equal strength, would repel it with force of one dyne.

4π LINES emerge from one unit N pole, and enter one unit S pole.

THE STRENGTH of a magnet may be stated in the number of unit poles it contains, symbol, m, or in the total flux coming out of the N pole ϕ. That is, $\phi = 4\pi m$.

FORCE exerted by one pole on another, $F = \frac{mm'}{d^2}$.

FORCE, by magnetic field on pole placed in it: $F = mH$.

FLUX DENSITY equals number of lines per sq.cm. Symbol, B.

In air, Flux Density equals field Intensity equals Magnetizing Force; that is, for air $B = H$.

PERMEABILITY is the relation of B to H in any material; equals the flux density for one unit of magnetizing force. Symbol μ.

PROBLEMS ON CHAPTER I

13–1. Plot the magnetic field outside and inside of a bar magnet.

14–1. Show the flux in the magnetic circuit of the two-pole dynamo with a drum armature, Fig. 17. Indicate the direction of the lines of force by arrow-heads.

15–1. Show the flux in the magnetic circuit of the dynamo with a ring armature, Fig. 18. Indicate direction of the lines as in Problem 14–1.

16–1. If one of two magnets which are 4 cms. apart has a pole strength of 5 units, what pole strength must the other have

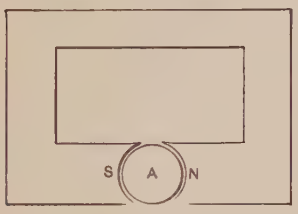

FIG. 17.

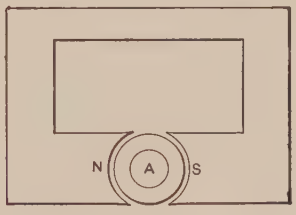

FIG. 18.

in order that there may be exerted between them a force of 89 dynes?

17–1. Draw the flux in the magnetic circuit of a four-pole motor, Fig. 19. Indicate the direction of the force lines by arrow-heads.

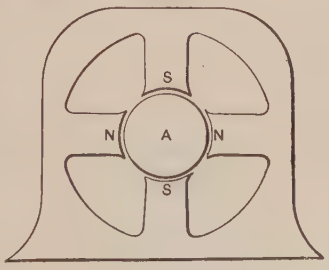

FIG. 19.

18–1. A bar of steel 100 cms. long, 4 cms. wide, 2 cms. thick is magnetized so that it has approximately 800 units pole strength per sq.cm. of cross-section.

(*a*) What is its pole strength, m?

(*b*) Compute total flux, ϕ, passing out of the N pole.

19–1. Calculate force in dynes with which the pole in Problem 18–1 would attract the S pole of a similar magnet of equal strength if placed 20 cms. from it.

20–1. One of the pole faces of a dynamo has an area of 20×30 cms., and 1,800,000 lines pass from the face into the smooth core of armature. What is the intensity of the magnetic field in the air gap between the pole face and the armature core?

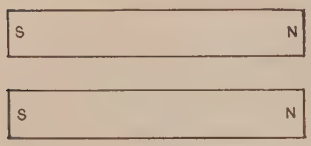

Fig. 20.

21–1. A magnet of 5 units pole strength is in a field of 16 gausses intensity. How much is the force on the pole?

22–1. Draw the magnetic field of two magnets placed side by side as in Fig. 20.

23–1. Two magnets are 5 cms. apart.
$m = 280$ unit poles.
$m' = 500$ unit poles.
(*a*) Find force between them.
(*b*) In how strong a field is m?
In how strong a field is m'?

24–1. A magnet of 120 units pole strength is in a field of 400 gausses.
(*a*) Force exerted on magnet?
(*b*) How far from another magnet of 1000 units would this first magnet have to be placed to be in a field equal to 400 gausses?
(*c*) What would be the force between the magnets in (*b*)?

25–1. A magnetic pole having a strength of 1800 lines is placed in a field of 2000 gausses. What is the action on the magnet?

26–1. A magnetic pole having 90,000 lines is placed in a magnetic field where there is an average of 1000 lines to every 50 sq.cms. What force (in pounds) is exerted on the magnet?

27–1. A magnet has a pole strength of 500 unit poles. How many lines emerge from the *N* pole?

28–1. What is the field intensity, 4 cms. from pole of magnet in Problem 27–1?

29–1. If a piece of iron, $\mu = 800$, were placed at point in field indicated in Problem 28–1, what would be the flux density in the iron? Assume magnetizing force to remain the same as in Problem 28–1.

30–1. If iron in Problem 29–1 has a cross-section area of 6

sq.cms. (at right angles to lines) how many unit poles are induced in it?

31–1. With what force will iron in Problem 29–1 be attracted to magnet?

32–1. The intensity at a certain point in the magnetic field of a magnet was 30 gausses. A piece of soft iron placed in that part of the field was magnetized to a flux density of 48,000 gausses. What was the permeability of the iron? Assume H to remain constant.

33–1. If the magnetic field of Problem 32 was due to a magnet of 4800 unit poles, how far from one of the poles is the point mentioned?

34–1. What would be the flux density in the iron of Problem 32 and 33 if it were moved 1 centimeter nearer the pole? Assume (μ) to remain constant.

35–1. A piece of wrought iron, the permeability of which is 1600, is placed in the magnetic field of a bar magnet of 2500 unit poles. It becomes magnetized with a flux density of 50,000 gausses. (*a*) What is the field strength at the point where the iron is placed? (*b*) How far is this point from the pole of the magnet?

36–1. The area of the pole face A of generator in Fig. 149 is 168 sq.in. (B) for the iron equals 12,000 gausses. How many lines emerge from the pole face?

CHAPTER II

ELECTROMAGNETS

Magnetic Field about a Straight Wire—Thumb Rule—Resultant of Circular and Parallel Fields—Moving Force in Voltmeter, Ammeter, Motor, etc.—Field about a Coil—Thumb Rule for Coil——Electromagnets—Magnetic Hoists—Telegraph—Generator and Motor Fields—Sucking Coils—Non-inductive Coils.

WHEREVER there is an electric current, there is also a MAGNETIC current. Another way of stating the same truth is: Electricity in *motion* always produces a magnetic field. This magnetic field is always at right angles to the electric current which accompanies it.

24. Field about a Straight Wire. When the wire carrying the electric current is straight, the magnetic field about the wire is circular, and a *N* pole placed near the wire would

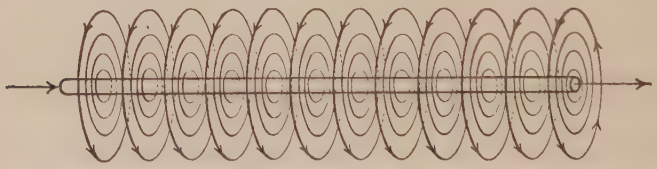

FIG. 21. Magnetic field about a straight wire carrying a current of electricity. Field is circular, not spiral.

whirl around and around the wire in a circle. If the current in the wire is reversed, the *N* pole would still whirl around the wire but in the opposite direction, showing that the field was the same shape but opposite in direction. Fig. 21 shows this circular field about a straight wire.

If now, we look along the wire in the direction in which the current is flowing, the magnetic field is whirling around the wire in the direction we would turn down a right-hand screw. Notice in particular that these whirls are *not spirals* but are *circles*. Fig. 22 shows a cross-section of the wire and magnetic field, and represents the way the field would appear if we looked at the end of the wire with the current going away from us. In Fig. 23 the current is reversed. Notice that the field is also reversed in direction.

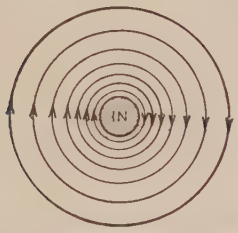

FIG. 22.—Field about a straight wire; end view.

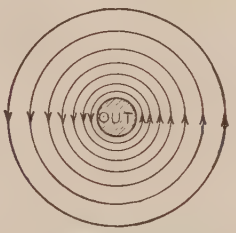

FIG. 23.—Field about a wire; end view; current reversed.

The best way for finding the direction of the magnetic field about a wire carrying an electric current is by the thumb rule.

25. Thumb Rule (for straight wire). If we *grasp the wire with our right* hand, so that the THUMB points in the direction of the *current*, the FINGERS will point in the direction of the magnetic *field*.

Similarly, if we know the direction of the magnetic field we can find the direction of the current. For if we place the fingers in the direction of the lines of force, the thumb will then point in the direction of the current.

Figs. 24 and 25 show this circular field about a wire carrying a current taken by means of iron filings.

26. Circular Field Combined with Parallel Field. It will be remembered that the field between two unlike poles as in Fig. 4 (which see) is nearly of uniform density.

If we now place a wire in this field perpendicular to the lines of force and send an electric current through the wire, this current sets up a circular field about the wire as in Fig. 25. But as this circular field is placed in the uniform field

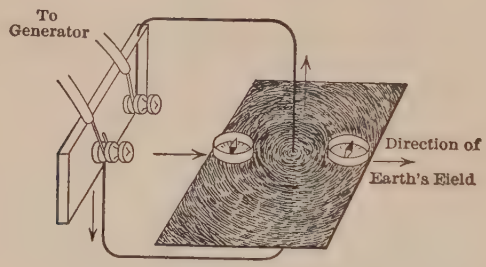

Fig. 24.—Field about a wire shown by iron filings.

of Fig. 4, the result is the combination field in Fig. 26. The lines are very much more crowded on the upper side of the

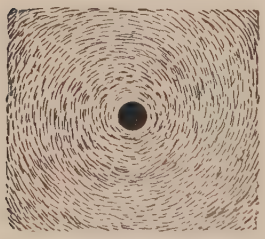

Fig. 25.—Field about a wire shown by iron filings.

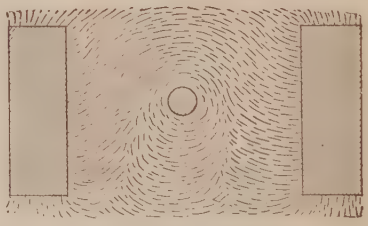

Fig. 26.—Combination of circular and parallel magnetic fields; shown by iron filings.

wire than on the lower side. The lateral pressure (crowding effect) of the lines on the upper side is therefore greater than the lateral pressure below the wire. Therefore the wire will be forced *down*.

This is the fundamental principle of electric motors, Weston voltmeters and ammeters, D'Arsonval galvanometers, etc. A large torque is obtained by having a very strong field with many wires in it, carrying heavy currents.

The sum of the pressures on all the wires can thus be made to amount to a very great force.

27. Explanation of Shape of Resultant Field. The cause of the lines being more crowded on one side of the wire is due to the circular shape of the field, which makes the direction of the force lines on one side of the wire to be exactly opposite to the direction of those lines on the other side. In Fig. 27, notice that the lines above the wire flow to the right, below to the left. When this wire is placed in uniform field flowing to the right, as in Fig. 28, the lines above

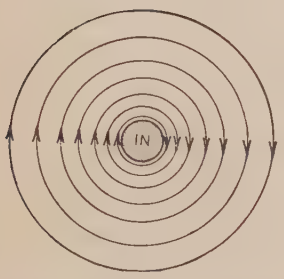

FIG. 27.—Field about a straight wire.

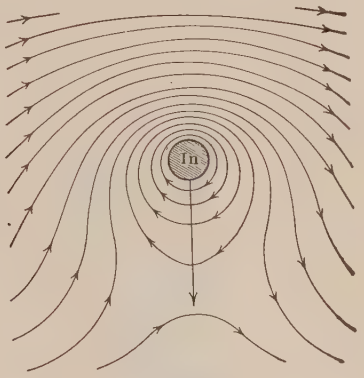

FIG. 28.—Wire carrying current placed in parallel magnetic field.

the wire, flowing to the right, join those of the field flowing to the right and thus strengthen the field above. Those below the wire, flowing to the left, neutralize some of those of the field flowing to the right, and weaken the field below the wire. Thus a strong field above the wire and a weak field below, resulting in a force urging the wire *down* as previously seen.

28. D. C. Voltmeters, Ammeters, etc. The Weston D. C. ammeters and voltmeters illustrate the practical application of this force existing between a parallel field

and a wire carrying a current. See Figs. 29 and 30, also Figs. 290 and 291.

A coil AB is carefully set on jeweled bearings between the poles of a permanent aged magnet N and S as described in Chapter I. Watchsprings W hold the coil in place, so that the pointer is held at zero on the scale, when no current is flowing through the coil. Suppose a current is led into the moving coil so that it goes *down* along the side B and up the side A. On a top view, Fig. 30, the current would go *in* at B and *out* at A. The clockwise circular field around B would strengthen the field of the permanent magnet

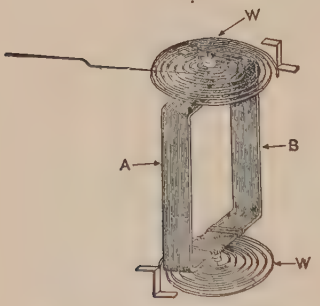

Fig. 29.—Moving coil of Weston D.C. meter.

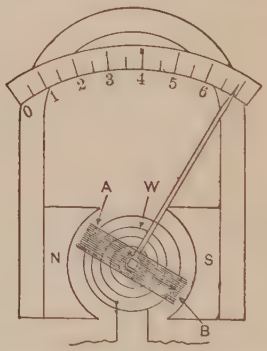

Fig. 30.—Construction of Weston D.C. meter.

NS, above the wires B and weaken it *below*. This would urge the side B downward. In the same way the counter-clockwise field about A would strengthen the magnet's field below A and weaken it above. Thus A would be urged *upward*.

These two actions would cause the coil to turn against the tension of the springs W. The stronger the current flowing through the coil the stronger the combination fields causing the coil to turn. The pointer would then indicate the current as it moved over a scale graduated in amperes. The instrument is then an ammeter. As the current in the

coil is proportional to the voltage across its terminals, the amount the coil turns must also be proportional to the voltage. The scale accordingly might be graduated to read volts. It would then be a voltmeter. These instruments are described fully in a later chapter, Figs. 280–9, show more complete details of the various types. For the similar action between the armature and field of a motor see Fig. 165 and description relating to it.

29. Field about a Coil Carrying a Current. If a wire is made into a loop as in Fig. 31, we find, by the THUMB rule, that the lines of force, which everywhere whirl around the wire, all *enter* the same face of the loop and all come out of the other face.

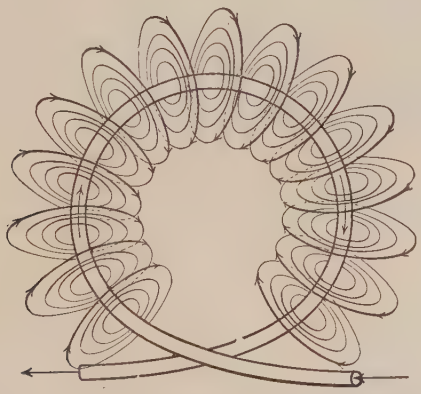

FIG. 31.—Field about a single loop carrying a current.

If we now place several loops together into a loose coil as in Fig. 32, most of the lines will thread the whole coil. If we make a close coil, practically *all* the lines will thread the whole coil, and return outside the coil to the other end.

The reason that practically no lines of force encircle the separate loops of a closely wound coil, but all thread the entire coil, is explained by referring to Fig. 33. This drawing represents an enlarged longitudinal section of the coil in Fig. 32. The current entering the ends of half loop at *A*, *B*, and *C*, comes out again

at D, E, and F. If the wire ends A and B were pushed nearer one another, the field on the right side of A would be in the opposite direction and would neutralize the field on the left side of B.

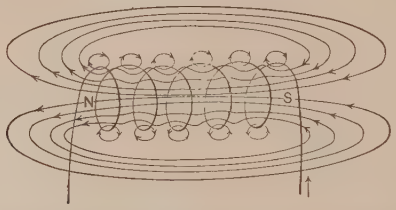

Fig. 32.—Field within and without a loose coil carrying a current.

The space between the wires A and B would thus be neutral or free of lines of force. The lines now would be compelled to continue on through the whole length of the coil, and would not slip into the spaces between the loops and encircle each wire with a separate field.

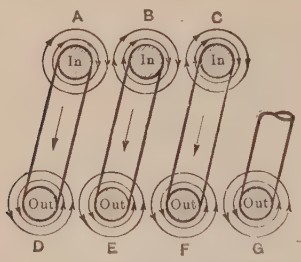

Fig. 33.—Longitudinal cross-section coil in Fig. 32.

We thus have the same shaped field as in and about a bar magnet; one end being a N pole, since all the lines come out of it, and the other end a S pole, since all the lines enter it. In order to find the polarity of such a coil, it is best to use a modified form of the Thumb Rule for a straight wire.

30. Thumb Rule (for coil). Grasp the coil with the right hand so that the Fingers point in the direction of the *current* in the coil, and the Thumb will point to the N pole.

N. B.—This is the thumb rule for the *straight wire reversed*.

31. Coil Equivalent to a Bar Magnet. A coil with a current flowing through it, is then the equivalent of a bar magnet with two poles. It obeys the laws of a bar magnet as stated in the first chapter. That is, unlike poles of two coils attract each other and like poles repel; if the coil is free to turn and is placed in a field, it will tend to take such

FIG. 34.—Browning electric lift magnet in action.

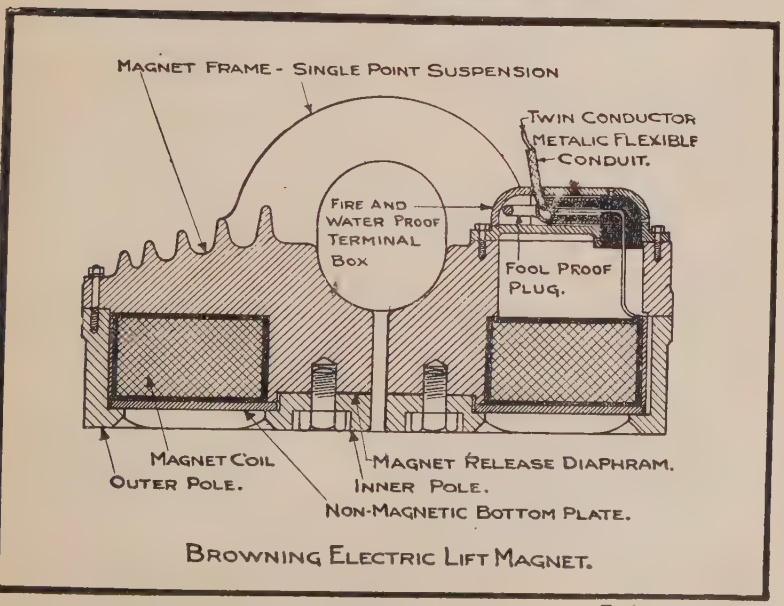

FIG. 35.

To face page 31

a position that the lines inside the coil are parallel to the lines in the field.

32. Electromagnets. Very strong magnets are made by inserting a piece of iron or steel, called a core, in the coil. The permeability of iron and steel is so much greater than air, as explained in Chapter I, that the same current in the same coil sets up thousands of times as many lines in the iron core as it would in the air alone.

33. Magnetic Hoists. The powerful poles of magnetic hoists are built in this way: Coils of wire are wound around a number of cores of soft iron. As long as there is no electric current going through the coils the iron is not a magnet. The face of the iron cores is brought in contact with pieces of iron or steel castings, etc., and the current turned on. The iron cores now become such strong magnets that each square inch of their ends will lift from 100 to 200 lbs. of iron.

To release the load of iron it is necessary to turn off the electric current, or reverse the direction of it.

Figs. 34 and 35 (Plate I), show the details of the Browning Lifting Magnet.

34. The Telegraph. The sounder of the telegraph system consists of two coils with soft iron cores which attract a small piece of soft iron to themselves, when a current is sent through them, and release it as the current is turned off, allowing it to be pulled away by the spring.

35. Generator and Motor Fields. But the most important use of electromagnets is in generators and motors, where they are used to create the intense magnetic fields necessary in the economical generation of electric power. In Fig. 36, which represents a 2-pole motor, the coils *AA* are wound on soft iron cores in such a way as to make one pole face a north pole and the other a south pole. The soft iron yoke *Y* joins the two cores together at one end, so that the magnetic circuit is composed entirely of iron except the space *G* where the armature goes. a large part of which

is also made of soft iron. The magnetic circuit is then a nearly closed iron circuit in the direction indicated.

Fig. 37 represents a 4-pole generator field and field coils.

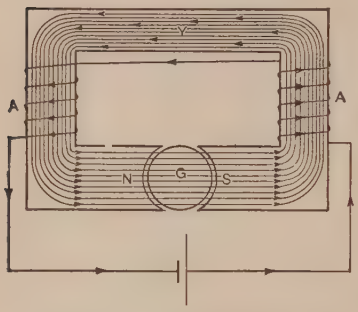

FIG. 36.—Magnetic field of 2-pole motor.

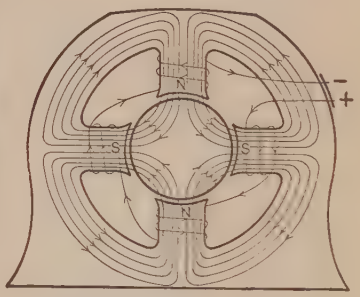

FIG. 37.—Magnetic field of a 4-pole motor.

36. "Sucking" Coils. Just as a bar magnet will attract a piece of iron, so a coil, since it has a field like a bar magnet, will attract a piece of iron. When the plunger of soft iron, *A*, in Fig. 38, is placed in the field of coil *B* it becomes magnetized as marked, the *N* pole being nearest the *S* pole of the coil. Thus it is attracted into the coil, being drawn up until the centers of coil and plunger coincide. The strongest attraction exists when the center of the iron plunger *nearly* coincides with the center of the coil.

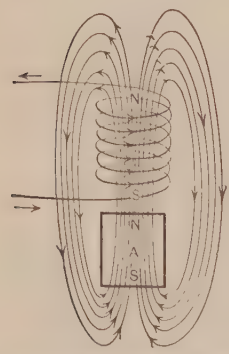

FIG. 38.—Sucking coil attracting iron plunger.

Extensive use is made commercially of this principle. It is the device used to regulate the arc of the arc lamp, Fig. 39. (For a full description, see Chapter XI.) Also to operate a circuit breaker, Fig. 40. In the latter of these the plunger is only sucked up, when the current in the coil becomes excessive. Then it automatically opens the switch, shutting off the power.

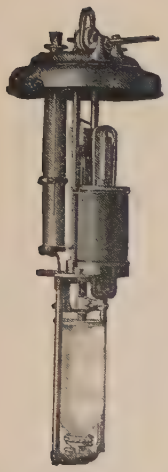

FIG. 39.—G. E. arc lamp. Lower coils suck up plunger of soft iron.

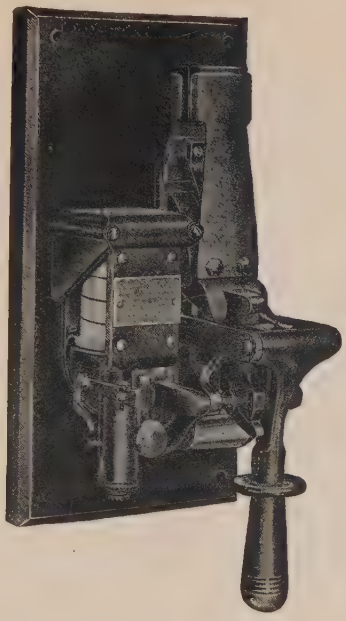

FIG. 40.—G. E. circuit breaker.

37. Non-Inductive Coils. Since the direction of the magnetic field about a wire depends on the direction of the electric current, it is seen that the fields of two currents flowing in opposite directions must oppose each other, and will neutralize each other if the wires are near enough together. Use is made of this fact whenever it is desired to have a current with a very weak magnetic field. Two wires are wound into a single coil so that the current in one wire flows in one direction, and the current in the other wire in the other direction, around the coil. Such a coil has no perceptible magnetic field, and is said to be NON-INDUCTIVELY wound. Resistance coils used in the measurement of resistance are wound in this way, as are almost all *resistance* coils in commercial ammeters and voltmeters. Fig. 116 shows such a coil taken from a Wheatstone bridge.

SUMMARY OF CHAPTER II

There is always a magnetic field about a wire carrying a current of electricity.

THUMB RULE FOR STRAIGHT WIRE. Grasp the wire with the right hand so that the thumb points in the direction of the current, the fingers then will point in the direction of the magnetic field around the wire. The magnetic field about a wire when combined with a uniform field, causes a crowding of the lines on one side of the wire, and a thinning out of the lines on the other side. The wire tends to move toward the thinner part of the field. This is the moving force of an electric motor.

An electric current flowing in a coil of wire makes an electromagnet of the coil, one face becoming a North pole and the other a South pole.

THUMB RULE FOR A COIL. Grasp the coil so that the Fingers point in the direction of current around coil, and Thumb points to the North pole. This is another form of the rule for straight wire.

A coil made thus and free to turn, is placed between the poles of a permanent magnet. On sending a current through it, the coil turns proportionately to the current. This is the principle of many Ammeters and Voltmeters.

The powerful magnets composing the magnetic field of generators and motors are made by coiling insulated wire around soft iron cores.

PROBLEMS ON CHAPTER II

1–2. Draw the magnetic field about the wire *A*, Fig. 41, when the current is flowing as indicated.

FIG. 41.

2–2. If *A*, Fig. 42, represents cross-section of wire with the current coming out, and the poles of the motor are as marked, draw resulting field between *N* and *S* pole. In what direction will wire *A* tend to move?

3–2. *A* and *B*, Fig. 43, represent the cross-section of a loop of wire on an armature in the magnetic field of a motor. If the current is as marked in *A* and *B* and poles of motor are as marked, in which direction will armature rotate?

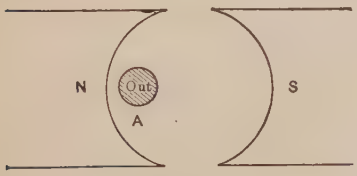

FIG. 42.

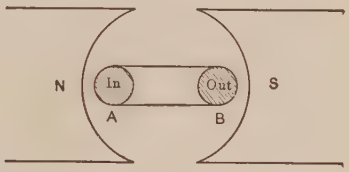

FIG. 43.

4–2. Draw magnetic internal and external field for iron core with electric current flowing around it as indicated in Fig. 44.

FIG. 44.

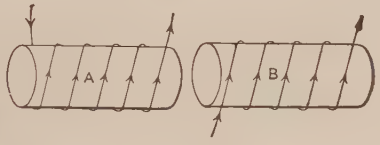

FIG. 45.

5–2. Draw field between coils *A* and *B*, in Fig. 45. State whether attraction or repulsion exists between them.

6–2. Show windings on cores *A* and *B*, with direction of electric current indicated and draw flux for magnetic circuit with direction of flux indicated in Fig. 46.

7–2. Draw 6-pole dynamo field, showing circuits and direction of flux, and electric current.

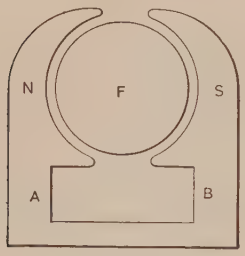

Fig. 46.

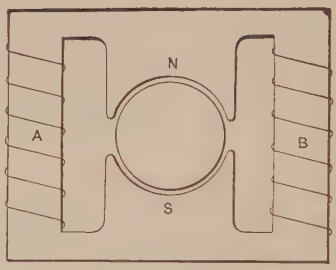

Fig. 47. Consequent pole motor frame.

8–2. Show direction of current in windings on *A* and *B* to produce Poles *N* and *S*, as marked in Fig. 47. Show magnetic flux as usual.

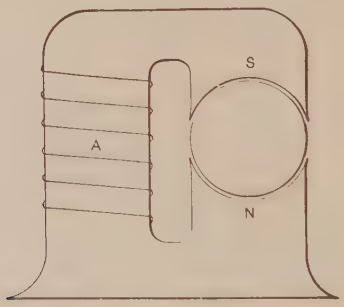

Fig. 48.

9–2. Show current in windings on *A* to produce *N* and *S* poles as marked in Fig. 48. Indicate magnetic flux as usual.

CHAPTER III

OHM'S LAW

The Electric Current—Units; Ampere (current); Volt (pressure); Ohm (resistance)—Analogy of Electrical to Hydraulic Units—Diagrams of Electric Circuits—Table of Convenient Symbols—Voltage, the Essential Factor—Potential—Electrical Potential—Brush Potential—Fall of Potential along a Uniform Wire—Ohm's Law—Applications of Ohm's Law—Series and Parallel Circuits—Voltage, Current, and Resistance Relations in Series and Parallel Circuits—Conductance.

38. The Electric Current. The *nature* of electricity has not yet been discovered. It is certain, however, that it is neither MATTER nor ENERGY in any form, and therefore cannot be a fluid. Yet the flow of electricity along a wire is very much like flow of water through a pipe. This analogy to the flow of water is helpful in explaining the flow of electricity, and will be used to show the action of an electric current.

39. Ampere (current). A current of water in a pipe is measured by the amount of water that flows through the pipe in a second: as, 1 gal. per sec., 8 gals. per sec., etc. In the same way, a current of electricity is measured by the amount of electricity that flows along a wire in a second as, 1 coulomb per sec., 8 coulombs per sec., etc. The coulomb of electricity is merely a quantity of electricity, just as a gallon is a quantity of water. Fortunately we have a special name for this *rate of flow* of 1 coulomb per sec., which is 1 AMPERE. This way of naming the *rate of flow* relieves us of all necessity of saying *per second* each time, as the *second* is part of the unit, the *ampere.* Thus 8 cou-

lombs per sec. is just 8 amperes; 25 coulombs per sec. is 25 amperes, etc.

Inasmuch as we are usually interested not in the *amount* of electricity alone, but in the *amount* that flows in a *second*, the coulomb is very rarely mentioned except as a base to define some other unit.

We have no general unit of rate of flow of *water*, so we always have to use some such cumbersome expression as gallons per sec. or cubic feet per sec.

40. Volt (pressure). The number of gals per sec. of water flowing through a pipe depends to a large extent on the PRESSURE under which it flows. This water pressure is measured in pounds per square foot. In the same way, the number of amperes (coulombs per sec.) of electricity flowing along a wire depends in part on the pressure under which the electricity flows. The electrical unit of PRESSURE is the VOLT. A *volt* means the same thing in speaking of a current of electricity that a *pound* pressure does in speaking of a current of water. Just as a higher pressure is required to force the same current of water through a small pipe than through a large pipe, so a higher electrical pressure is required to force the same current of electricity through a small wire than through a large wire. The *voltage* (pressure) between two points in an electric circuit is sometimes spoken of as the *difference* in *potential*, or the *drop* in *potential* or merely the "drop," between those two points. In the same way, the pressure between two points in a stream of water (as the top and bottom of a dam), is often spoken of as the *difference* in *level* or the *drop* in *level*, or merely the "drop," between those two points.

The distinction between AMPERES and VOLTS should now be plain. The *amperes* represent the amount of the current flowing through a circuit; the *volts* represent the pressure causing it to flow.

In the case of both water and electricity there may be a

great pressure and yet be no current. If the path of the water is blocked by a valve being turned the wrong way, there will be no current, yet there may be a high pressure. If the path of the electricity is blocked by a switch being thrown the wrong way, there will be no current (amperes), though the pressure (volts) may be high.

There is, therefore, *something* in addition to the pressure, that determines the amount of the current, both of water and electricity. This *something* is the *resistance* of the pipes in the case of water, and the *resistance* of the wires in electricity. The greater this resistance, the less the current under the same *pressure.*

41. Ohm (resistance). There is no general unit to measure the *resistance* which a pipe offers to the flow of water. The electrical unit of RESISTANCE is the OHM.

We say a wire has ONE OHM *resistance* when a pressure of ONE VOLT forces a current of ONE AMPERE through it. If the resistance were 2 ohms, or twice as great, the current would be only half as large: that is, one-half ampere would flow.

42. Comparison of Electrical and Hydraulic Units.

UNITS	ELECTRICITY	WATER
QUANTITY	COULOMB	GALLON
CURRENT	AMPERE 1 coulomb per second	GALLON per min.
PRESSURE	VOLT	POUNDS per sq.in.
RESISTANCE	OHM	

43. Diagrams of Electric Circuits. In order that the diagrams used later throughout the text may give no trouble, a few of the symbols in common use are indicated in this paragraph.

An electric current is always thought of as flowing from a higher to a lower level. We mark the higher level +, and the lower −, in order to denote in what direction the current is flowing. Sometimes arrow-heads are also put on the wire. The current always flows from the + to the −. A given point is then + to all points below its level and − to all points above its level. In Fig. 49 suppose the current to flow from *A* to *C*. If we are considering points *A* and *B*, the point *A* would be + and the point *B* −. But if we are considering *B* and *C*, *B* would then be + and *C* −.

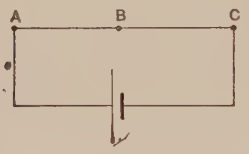

FIG. 49.—Diagram of electric circuit with battery cell.

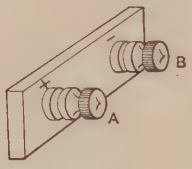

FIG. 50.—Terminals of electric circuit.

Or consider the terminals *A* and *B*, in Fig. 50. Since *A* is marked +, it means that the current will flow from *A* and down to *B* when the two points are connected by an electric circuit.

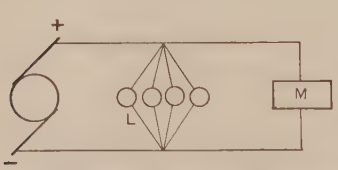

FIG. 51.—Diagram of generator delivering power to lamps and motor.

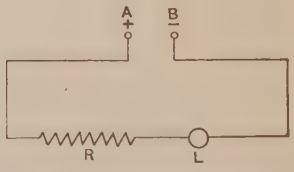

FIG. 52.—Diagram of electric circuit.

The terminals on all D.C. instruments are marked in this way, which is to indicate that the + terminal is always to be connected to the higher level of the circuit.

To represent a battery cell, we use the symbol |, the

longer line being the +, or the one from which the current flows *out* of the cell.

44. Table of Convenient Symbols.

Battery cell,

Generator, D.C.

Generator, A.C.

Motor or

Incandescent lamp,

Arc lamp,

Non-inductive resistance,

Inductive resistance

Switch—Single-throw,

Switch—Double-throw,

Galvanometer,

Voltmeter,

Ammeter,

Accordingly Fig. 51 represents a D. C. Generator lighting a bank of 4 incandescent lamps (L) and driving a motor M. The current is flowing from the generator along the top of the circuit to lamps and motor.

Sometimes merely the terminals to the source of the power are shown as A and B in Fig. 52.

A =power terminal; R =resistance.

B =power terminal; L =incandescent lamp.

The current comes from A, goes through R, then L and finally leaves at B.

45. Pressure, the Essential Factor. When we desire to have water piped to a house, the one thing above all others which we must have, in order to secure any flow at all, is PRESSURE. The water company agrees to furnish this water pressure. The flow of water into your house will then depend entirely upon your wishes. You have the

desired pressure always at hand and can allow it to force a great quantity of water through your pipe or only a small amount. You pay for the amount of water you allow this pressure to force through your pipes. If it had been necessary to maintain *twice* as great a pressure in order to force this same amount of water through your pipes, the company might charge you *twice* as much for the water used. In other words, they would charge you for the *work done* in pumping so much water into your house. At a given pressure, this amount of *work done* would be proportional to the amount of water which you allowed to flow into your house.

In the same way, if you desire to have electricity brought to your house, the one thing above all others that is necessary, in order to get a flow of electricity, is that an electric pressure be maintained. The electric company agrees to maintain this electric pressure. The flow of the current into your house depends entirely upon your wishes. The pressure is always at hand. You may allow it to cause a large current to flow to the house or only a small current. You pay for the amount of electricity you use, at that pressure. If the electric company maintained twice as much pressure and you used the same amount of electricity, you would have to pay twice as much for this quantity. In other words, they would charge you for the *work* done in forcing so much electricity into your house. At a given pressure, the amount of work done would be proportional to the amount of electricity which you allowed to flow into your house.

The starting point of all hydraulic and electric propositions is thus the PRESSURE. There can be no current either of water or electricity without pressure. The amount of work done in a stated time by a given current depends entirely upon PRESSURE. It is necessary, then, to study this feature of *pressure* a little more in detail.

46. Potential. In order to exert a pressure tending to

cause water to flow toward a given point, it is merely necessary to raise the water to a level *above* that point. That is, we give it potential energy with regard to that point, by the raising of it to a point which has a higher level or higher POTENTIAL than that point.' Thus if we wish to cause water to flow out of tap A (Fig. 53), we may connect it to a tank T. As long as the water in the tank is kept above the level ABB, the water will flow from T to A, because it is at a higher level or potential in T than in A. This *difference* in potential then, between two points is what causes a pressure between these two points. The greater this difference of potential the greater the pressure. When the *difference* of *potential* between the level of the water in T and A is only OB, the *pressure* is *less* than when the difference of potential is RB. The difference of potential is then a measure of the pressure. It is a matter of common experience that there is always a tendency for the water to flow toward the lower potential.

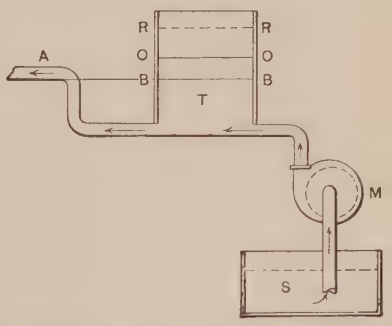

FIG. 53. – Water pressure at A depends upon water level in tank T.

47. Electrical Potential. Similarly, if we wish to cause a current of electricity to flow from one point to another, we have merely to raise the potential of the first point above that of the second. Then a pressure is set up proportional to their *difference* in potential. This difference of potential tends to send a current from the higher potential to the lower, as in the case of water.

Electricity always flows from a high potential point to a lower. A battery or generator may then be thought of, as being merely a pump which keeps one end of the line at a

higher potential than the other, thus setting up a pressure between these two points. Accordingly, the electric-power company runs two wires to your house and simply agrees to keep the difference in the potential between them up to a certain value; providing, of course, you do not allow so much electricity to flow that the generator is unable to pump fast enough to keep up this difference of potential. In such a case it would be just as if the water in Fig. 53 were allowed to flow out of A so fast that the pump M could not keep the level in the tank above the line ABB.

In referring to Fig. 49, we have said that the point A was at a higher level than B, and B higher than C. We may now

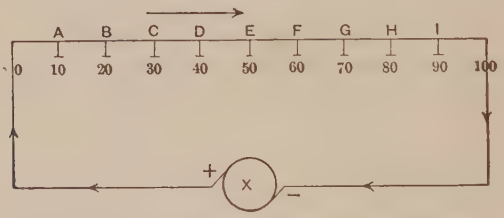

Fig. 54—Drop in potential along a uniform wire.

say that A is at a higher potential than B, and B than C. A current therefore flows from A to B and from B to C, due to these differences in potential. The battery acts as a pump to keep the left-hand side continually at a higher potential than the right-hand side. The difference of potential between A and B is sometimes spoken of as the *fall* of potential from A to B, or as the *drop* in potential from A to B. The same applies to B and C, or any other points. The difference in potential between two points in an electric circuit is called the *drop* in potential for that part of the circuit contained between those two points, and is the cause of any current flowing between these two points.

In Fig. 52 the terminal A is attached to the high-potential side of the generator, since it is marked + and B is attached to the low-potential side of the generator, since it is

marked −. The current therefore flows from A through R, then through L to B, and back to the low-potential side of the generator. The generator must continually raise electricity up to the high-potential side, to make up for that which flows away from that side to the low-potential side. If no electricity is allowed to flow away from A to B, then the generator has to raise no more electricity up to the potential of A. It merely has to keep the pressure up.

48. Fall of Potential along a Uniform Wire. In Fig. 54 a uniform wire $OABC \ldots$, 100 ft. long, has a current sent through it from O to H by means of a generator X. The point O must be a higher potential than H, since the current is flowing from O to H.

Let us mark this wire into 10-ft. lengths and letter the points A, B, C, etc., beginning 10 ft. from O. The point O will now be at a higher potential than A, A than B, B than C, etc. If we now measure by means of some instrument the difference in potential between any two of these points 10 ft. apart, we find the difference of potential is the same between any two. This shows that the drop of potential across equal distances along a uniform wire is the same.

The instrument used to measure drop or difference of electrical potential is called VOLTMETER. It indicates this potential difference in VOLTS.

If we now place the voltmeter across OB (*twice* the length of OA) and measure the drop between the points O and B, we find that the drop is twice that of OA. In the same way the drop across OC (three times the length of OA) is three times the drop across OA. This holds true for any multiple of OA, and proves that the drop across parts of a uniform wire carrying an electric current is proportional to the length of these parts. Now since the resistance of a wire is also proportional to its length, we may say:

The drop in potential across any part of an electric circuit

is proportional to the resistance of that part, if the same current is flowing through each part.

Example. The voltage across 6 ohms resistance is 20 volts. What will be the drop across 30 ohms, if the same current is flowing through each resistance?

Let V = drop across the 30 ohms.

Since the drops are proportional to the resistances, the current being the same,

$$\frac{V}{20}=\frac{30}{6}$$

$$V=100 \text{ volts.}$$

Problem 1–3. If the drop along 40 ft. of the line wire in a certain circuit is .002 volt, what is the drop per mile?

Problem 2–3. The drop across 20 ft. of wire is .06 volt. The drop across an unknown length is 14.4 volts. How great is unknown length?

Problem 3–3. Two resistances are connected in such a way that the same current flows through each. The resistance of one is .1 ohm, the other is unknown. The voltage across the known is .48 volt, across the unknown is 4.62 volts. What is the value of the unknown resistance?

49. Ohm's Law. The relation between *current* (amperes), *pressure* (volts), and *resistance* (ohms), is stated by the famous Ohm's law. This law is the beginning of all scientific knowledge of electricity and electric machinery.

The law is stated as follows:

The electric current along a conductor equals the pressure divided by the resistance.

$$\text{Current}=\frac{\text{Pressure}}{\text{Resistance}}.$$

In electric units:

$$\text{Amperes}=\frac{\text{Volts}}{\text{Ohms}}.$$

In symbols:

$$I=\frac{E}{R}.$$

Where I stands for *I*ntensity of current in amperes.
" E " " *E*lectromotive force in volts.
" R " " *R*esistance in ohms.

The student will save himself much work in the future if he will take the trouble to learn this law in its *three* forms.

1. $I=\frac{E}{R}$: Current equals voltage divided by resistance.

2. $E=IR$: Voltage equals current times resistance.

3. $R=\frac{E}{I}$: Resistance equals voltage divided by current.

Three applications of this law are given to illustrate these three ways of using the law. A more complete development of its use will follow.

Example 1. The pressure in an electric circuit is 50 volts, the resistance is 25 ohms. How many amperes flow in the line?

$$E=50 \text{ volts};$$

$$R=25 \text{ ohms};$$

$$I=\frac{E}{R}=\frac{50}{25}=2 \text{ amperes.}$$

Example 2. How many volts are required to force 3 amperes through 20 ohms?

$$I=3 \text{ amperes};$$

$$R=20 \text{ ohms};$$

$$E=IR=3\times 20=60 \text{ volts.}$$

Example 3. Through how many ohms can 110 volts force 5 amperes?

$$E=110 \text{ volts};$$

$$I=5 \text{ amperes};$$

$$R=\frac{E}{I}=\frac{110}{5}=22 \text{ ohms.}$$

Problem 4–3. An incandescent lamp uses .5 ampere on a 110-volt circuit. What is the resistance of lamp when burning?

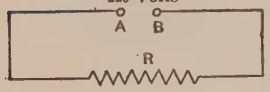

Fig. 55.

Problem 5–3. If $R=11$ ohms, Fig. 55, how many amperes are flowing in line?

Problem 6–3. What current is produced through a resistance of 5 ohms by an electromotive force of 10 volts?

Problem 7–3. Through what resistance will an electromotive force of 15 volts force a current of 3 amperes?

Problem 8–3. What voltage will produce a current of 4 amperes through a resistance of 4 ohms?

Problem 9–3. What electromotive force will produce a current of .03 ampere through a resistance of 1000 ohms?

Problem 10–3. Through what resistance will 121 volts produce 11 amperes?

Problem 11–3. What current is produced by 10 volts acting across .25 ohm?

Problem 12–3. A dynamo generates 500 volts; the resistance of the circuit including the dynamo is 20 ohms. What is the current?

Problem 13–3. An electric bell has a resistance of 400 ohms and will not ring with a current of less than .03 ampere. Neglecting battery and line resistance, what is the smallest electromotive force that will ring the bell?

50. Applications of Ohm's Law. Ohm's law can be applied to an electric circuit as a whole, or it can be applied to any part of it. It requires a great amount of care and practice to apply this simple law correctly in all cases. Accordingly, there is no part of electrical work where so many mistakes are made as in these various applications. Once the principle is firmly grasped, the student is able

at once to attack intelligently a wide range of electrical problems.

Many of the difficulties will be cleared up if one will keep in mind the *two* following statements of the law and will use them intelligently:

When applying the law to an *entire* circuit, state the law as follows:

(1) *The current in the Entire circuit equals the voltage across the Entire circuit divided by the resistance of the Entire circuit.*

Notice that the word ENTIRE applies to CURRENT, VOLTAGE, and RESISTANCE alike. Not to *one* only, or to *two* only, but to all *three* factors of the equation.

When applying the law to but a *part* of the circuit, state the law as follows:

(2) *The current in a certain Part of a circuit equals the voltage across that Same Part, divided by the resistance of that Same Part.*

Notice here again that the values for the *three* factors, CURRENT, VOLTAGE, and RESISTANCE, are all taken from the *same part* of the circuit. By far the greatest number of mistakes in applying Ohm's law comes from dividing the voltage across *one part* of a circuit by the resistance of *another part* and expecting to find the current in some part or other.

A few examples will make the correct use plain.

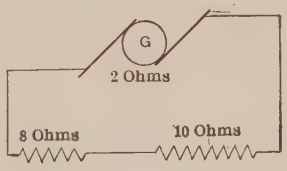

FIG. 56.

Example. The generator *G*, Fig. 56, has a resistance of 2 ohms and generates 150 volts pressure. What current flows through a circuit which has two resistances placed in series with the generator, if one resistance is 8 ohms and the other 10 ohms?

Solution. Since the pressure generated by the generator is the *voltage* of the *entire* system, the *resistance* of the *entire* system must be used in the equation;

$$\text{Current (in entire system)} = \frac{\text{voltage (across entire system)}}{\text{resistance (of entire system)}}.$$

Voltage (across entire system) = 150 volts;

Resistance (of entire system) = 2 + 8 + 10 = 20 ohms;

$$\text{Current (in entire system)} = \frac{150}{20} = 7.5 \text{ amperes.}$$

That is, $I = \frac{E}{R}$, applied to the *entire* circuit. Having found the current flowing in the circuit, it is now possible to compute the voltage necessary to force it through the different parts by applying Ohm's law to that part alone.

Suppose it is desired to find the voltage across the 8-ohm resistance alone.

Voltage (across 8 ohms) = current (in 8 ohms) × resistance (of 8 ohms).

Current (through 8 ohms) = 7.5 amperes.

Resistance (of 8 ohms) = 8 ohms.

Voltage (across 8 ohms) = 7.5 × 8 = 60 volts.

That is,

$E = IR$ applied to one part of current only.

Across the 10 ohms resistance, in the same way,

$$E = IR = 7.5 \times 10 = 75 \textit{ volts};$$

Across the interior of generator,

$$E = 7.5 \times 2 = 15 \textit{ volts}.$$

Total voltage = 60 + 75 + 15 = 150 volts, which checks with the value given as the voltage generated.

51. Brush Potential or Terminal Voltage. We may say then, that in the above example 75 volts of the 150 are used to force the current through the 10 ohms resistance; 60 volts are used to force it through the 8 ohms resistance, and the remaining 15 volts are used to force it through the generator itself.

The pressure used to force the current through the two resistances of 8 ohms and 10 ohms, would be the 75 volts, plus the 60 volts or 135 volts. Since this 135 volts is used entirely in forcing the current from one brush through the outside circuit and back to the other brush, it is called the BRUSH POTENTIAL or TERMINAL VOLTAGE of the generator for the given current. If a voltmeter were placed across the generator it would indicate the TERMINAL VOLTAGE, since that is all the voltage available for forcing a given current from brush to brush, through the external resistance. This BRUSH POTENTIAL of a generator must always be distinguished carefully from the total voltage generated by the generator, which is called the Electromotive Force of the generator. This is generally abbreviated to E.M.F.

Thus in above example,

the 150 volts equals the E.M.F.

" 135 " " " Brush Potential.

Example. Arc lamp A, Fig. 57, requires 5 amperes to operate it and has a resistance of 16 ohms. There is a resistance B of 4 ohms in series with A.

(a) What voltage is required to operate the lamp?

(b) What E.M.F. must a generator of 2 ohms internal resistance develop in order to run lamp and a resistance B of 4 ohms?

(c) What is voltage across B alone?

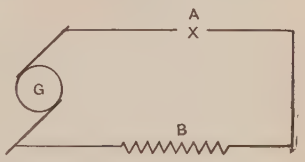

FIG. 57.

(d) What is the Brush Potential of the Generator?

Solution. (a) Voltage across lamp.

$I = 5$ amperes (current in *lamp*);

$R = 16$ ohms (resistance of *lamp*);

$E = IR = 5 \times 16 = 80$ volts (pressure across *lamp*).

(*b*) E.M.F. generated by generator G (that is, of course, voltage across *entire* circuit).

$I = 5$ amperes (current in *entire* circuit);

$R = 16 + 4 + 2 = 22$ ohms (resistance in *entire* circuit),

$E = IR = 5 \times 22 = 110$ volts (E.M.F. generated or volts across *entire* circuit).

(*c*) Voltage across B.

$I = 5$ amperes (current in B);

$R = 4$ ohms (resistance of B);

$E = IR = 5 \times 4 = 20$ volts (voltage across B).

(*d*) Brush potential. Brush potential is voltage across all that part of the circuit which is outside of the generator.

$I = 5$ amperes (current through circuit outside of generator);

$R = 16 + 4 = 20$ ohms (resistance outside of generator);

$E = IR = 20 \times 5 = 100$ volts (pressure outside of generator or BRUSH POTENTIAL.

Notice that in each case, no matter what the problem, the method is identical, great care being taken in using the equation to have the *current*, *resistance*, and *voltage* include exactly the same part of the circuit and no more.

52. Series and Parallel Circuits. There are *two* ways of connecting two or more pieces of electrical apparatus together.

(1) SERIES. When the pieces are connected in *tandem* they are said to be in SERIES. Lamps A and B, of Fig. 58, are in series.

(2) PARALLEL. When the pieces are connected so that the current is divided between them, they are said to be in PARALLEL with one another. MULTIPLE or SHUNT are other names for this combination. Lamps C and D of Fig. 59, are in *parallel* with each other.

The two combinations may exist in the same circuit, as in Fig. 60, where the parallel combination C and D is in series with the series combination A and B.

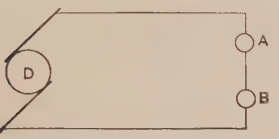

FIG. 58.—Series combination; A is in series with B.

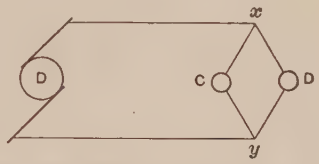

FIG. 59.—Parallel combination; C is in parallel with D.

Also as in Fig. 61, where the parallel combination C and D is parallel with the series combination A and B.

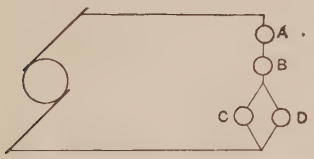

FIG. 60.—Series arrangement of parallel and series combinations.

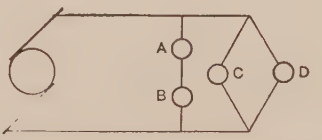

FIG. 61.—Parallel arrangement of series and parallel combinations.

53. Series—Resistance, Voltage, and Current Relations.

(1) RESISTANCE. The resistance of a series combination equals the sum of the resistance of the separate parts. So in Fig. 58, the resistance of the series combination of lamps A and B, equals the resistance of lamp A plus the resistance of lamp B.

(2) VOLTAGE. The voltage also, of a series combination equals the sum of the voltages across the separate resistances. Thus the voltage across the lamps A and B, Fig. 58, equals the voltage across A plus the voltage across B.

As has been stated earlier in this chapter, the voltage across any part of a series circuit is proportional to the resistance of that part. Thus the *voltage* across A would be 3 times the *voltage* across B, if the resistance of A were 3 times the resistance of B.

(3) CURRENT. The current in every part of a series circuit is the same. You cannot dam up an electric current. Therefore the current in lamp *A*, Fig. 58, must be the same as the current in lamp *B*, no matter what their respective resistances and voltages may be. The voltages and resistances always distribute themselves strictly in accordance with Ohm's law.

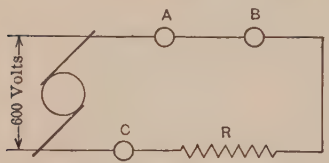

FIG. 62.—Series combination.

Example. Series combination (Fig 62.):

Resistance *A* equals 50 ohms.
" *B* " 150 "
" *C* " 75 "
" *R* " 25 "

Voltage across the combination equals 600 volts. Find:

(1) Combined resistance of *A*, *B*, *C*, *R*.
(2) Current through each.
(3) Voltage across each.

Solution.

(1) *Resistance* of combination:

$$50+150+75+25=300 \text{ ohms.}$$

(2) *Current* through each:

Apply Ohm's Law to the combination:

$$I=\frac{E}{R};$$

$$\text{Current (through combination)}=\frac{\text{Voltage (across combination)}}{\text{Resistance (of combination)}}.$$

$$I=\frac{600}{300}=2 \text{ amperes.}$$

There are, therefore, 2 amperes flowing through each part of the combination.

(3) *Voltage* across each.

Apply Ohm's Law to each part of the combination successively; to lamp A, for instance:

$$E = IR.$$

That is,

Voltage (across A) = *current* (through A) × resistance (of A.)

E $=2\times50$ $=100$ volts. (Voltage across A).

E' $=2\times150=300$ " (Voltage across B).

E'' $=2\times75$ $=150$ " (Voltage across C).

$E'''=2\times25$ $=50$ " (Voltage across R).

E (sum) $=600$ " (Voltage across the combination).

Problem 14–3. What brush potential must a generator produce to supply an electroplating current of 20 amperes through a circuit whose total resistance is .2 ohm?

Problem 15–3. A dynamo generates an electromotive force of 1150 volts and delivers a current of 20 amperes. The resistance of the circuit, including dynamo, is what?

Problem 16–3. (*a*) If the magneto-generator for ringing a telephone bell gives an electromotive force of 50 volts, what current will be transmitted through the circuit? The resistance of generator is 500 ohms, of line and bell is 125 ohms. (*b*) What will the brush potential be?

Problem 17–3. What E.M.F. will be required to force 2 amperes through a series circuit containing generator of $\frac{1}{2}$ an ohm, line wires of $1\frac{1}{2}$ ohms, and a lamp of 100 ohms resistance? (*b*) What will be the brush potential of generator?

Problem 18–3. Lamp, Fig. 63, has a resistance of 100 ohms, and line wires, 2 ohms each. (*a*) What is current in line? (*b*) What is voltage across the lamp? (*c*) How much voltage is used up in sending the current through the line wires?

Problem 19–3. A circuit consists of 8 ohms resistance in generator and 3 ohms in line wires, and has 30 lamps in series, each of 8 ohms. With a current of 4 amperes, what will be the drop in the generator; (2) in the line; (3) what the difference of potential across each lamp; (4) brush potential of generator?

Problem 20–3. (*a*) If each lamp, Fig. 64, of the combination takes .5 ampere, how many amperes must the generator deliver?

(*b*) Resistance of $A = 100$ ohms;
" " $B = 200$ "
" " $C = 150$ "
Combined resistance = ?

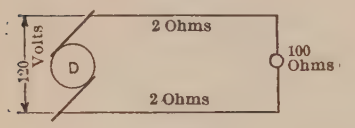

FIG. 63.

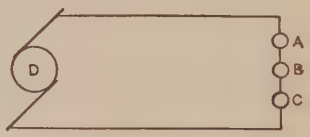

FIG. 64.

54. Parallel—Voltage, Current, and Resistance.

(1) *Voltage.* The voltage across each branch of a parallel circuit is the same as the voltage across the combination. In Fig. 59, the voltage across C is the same as the voltage across D, which is the same as the voltage across the combination, C and D, since they both lie between the same points, X and Y.

(2) *Current.* The current in a parallel combination equals the sum of the currents in the separate parts. As the current flowing from X to Y (Fig. 59), equals the current in C plus the current in D.

(3) *Resistance.* The resistance of a parallel combination equals the *reciprocal* of the sum of the *conductances* of the separate parts.

This method of finding the combined resistances of pieces in parallel, is a " last resort " method. The resistance can generally be found by a direct application of Ohm's law, but sometimes the above is more convenient.

The CONDUCTANCE is the reciprocal of resistance; that is,

$$\text{Conductance} = \frac{1}{\text{Resistance}}.$$

Thus if the resistance may be said to represent the *difficulty* with which an electric current is forced through a wire,

the CONDUCTANCE may be said to represent the *ease* with which the current can be forced through the wire. The name "Mho," the inverse of "Ohm," has been proposed as the unit of conductance. A wire of 1 ohm resistance has a conductance of 1 mho. A wire of 2 ohms resistance has a conductance of $\frac{1}{2}$ mho, etc.

$$\text{mho} = \frac{1}{\text{ohm}} \quad \text{or} \quad \text{ohm} = \frac{1}{\text{mho}}.$$

Since two resistances in parallel offer *less* RESISTANCE to the current than one alone, the CONDUCTANCE of the combination is *greater* than the conductance of one alone. In fact, conductance of pieces in parallel equals the sum of the separate conductances.

FIG. 65.—Parallel combination.

Consider Fig. 65.

Resistance of:

$L = 80$ ohms;
$M = 8$ "
$A = 16$ "

Conductance of:

$$L = \frac{1}{80} = .0125 \text{ mho};$$

$$M = \frac{1}{8} = .125 \text{ mho};$$

$$A = \frac{1}{16} = .0625 \text{ mho}.$$

The *conductance* of the combination therefore is:

$$\frac{1}{80} + \frac{1}{8} + \frac{1}{16} = \frac{16}{80} = \frac{1}{5} = .20 \text{ mho};$$

or

$$.0125 + .125 + .0625 = .20 \text{ mho}.$$

If the *conductance* of the combination is $\frac{1}{5}$ mho, the *resistance* of the combination must be $\frac{5}{1}$ or 5 ohms, since the resistance equals the reciprocal of the conductance.

Thus the rule for finding the resistance of a parallel combination is:

Add the *conductances* of the separate branches, which gives the *conductance* of the combination. Invert the *conductance* of the combination to get the *resistance* of the combination.

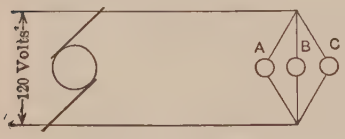

Fig. 66.

Example. In Fig. 66, (parallel combination.)

Resistance $A = 60$ ohms;
" $B = 40$ ohms;
" $C = 80$ ohms.

Voltage across combination $= 120$ volts.

Find:

(1) Voltage across each.
(2) Current through each.
(3) Current through combination.
(4) Resistance of combination.

(1) *Voltage* across *each* is 120 volts, because in parallel combinations, the voltage across *each* branch equals the voltage across the *combination*.

(2) Current through each.

Apply Ohm's Law to each branch successively.

$$I = \frac{E}{R};$$

$$\text{Current (through } A) = \frac{\text{Voltage (across } A)}{\text{Resistance (of } A)}$$

Current through A $\qquad I = \frac{120}{60} = 2$ amperes;

Current through B $\qquad I = \frac{120}{40} = 3$ amperes;

Current through C $\qquad I = \frac{120}{80} = 1.5$ amperes.

(3) *Current* through *combination* equals the sum of the currents in the separate branches:

$$2+3+1.5=6.5 \text{ amperes.}$$

(4) Resistance of combination:
Apply Ohm's Law to the combination:

$$\text{Resistance (of combination)} = \frac{\text{Voltage (across combination)}}{\text{Current (through combination)}}.$$

$$R=\frac{120}{6.5}=18.46 \text{ ohms.}$$

It may also be solved by the "inverse of the sum of the separate conductances," as follows:

$$\frac{1}{60}+\frac{1}{40}+\frac{1}{80}=\frac{13}{240} \text{ mho.} \quad \text{(Conductance of combination.)}$$

$$\text{Resistance (of combination)} = \frac{1}{\text{Conductance (of combination)}}.$$

Therefore

$$R=\frac{240}{13}=18.46 \text{ ohms.} \quad \text{(Checks with above ans.)}$$

Problem 21–3. If each lamp of the combination Fig. 67, takes .5 ampere, how many amperes must the generator deliver?

Problem 22–3. A divided circuit has two branches of 1 ohm and $\frac{1}{4}$ ohm respectively. What is the joint conductance of the two branches? What is the joint resistance?

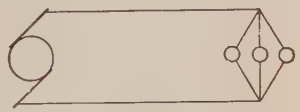

Fig. 67.

Problem 23–3. A circuit has three branches of 12, 4, and 6 ohms respectively. If 4 amperes flow in the circuit containing 6 ohms, what current will flow in each of the others?

Problem 24–3. What pressure will be required to force 10 amperes through a parallel combination consisting of 4 ohms, 5 ohms, and 8 ohms?

55. Simple Parallel Lighting Systems. The modern incandescent lamps are usually installed in parallel. The

resistance of all the lamps of any make is not the same. Nor is the voltage across all the lamps when installed the same. Still for convenience in calculating the "line drop," efficiency of transmission, etc., each lamp is assumed to take the same current. The error introduced by this assumption is usually too small to be taken into account.

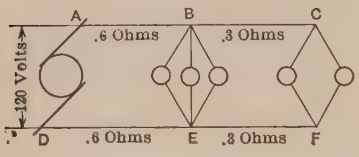

FIG. 68.

A general problem may be stated as follows:

A lighting circuit is arranged with two groups of lamps as in Fig. 68. One group has two lamps, the other, three. Each lamp takes 2 amperes. The generator has a terminal voltage of 120 volts.

Resistance of line wire AB and DE = .6 ohm each.
" " " " BC and EF = .3 ohm each.

Find:

(*a*) Volts lost in line wires.
(*b*) Voltage across each group of lamps.

First. Find the current distribution throughout circuit, assuming that each lamp takes 2 amperes.

Current through BC = 4 amperes
" " EF = 4 "
" " AB = 10 "
" " DE = 10 "

Second. By Ohm's law, solve for the "drop" in line wire.

Drop (in AB) = current (through AB) × resistance (of AB).
= .6 × 10 = 6 volts.
Drop (in DE) = current (through DE) × resistance (of DE).
= .6 × 10 = 6 volts.

Total volts used in line between generator and first group of lamps then equals,

$$6+6=12 \text{ volts.}$$

Generator voltage − voltage lost in lead wires = voltage at lamps (BE).

$$120-12=108 \text{ volts across } BE.$$

Volts (lost in BC) = current (through BC) × resistance (of BC);
$=4\times.3=1.2$;
Volts (lost in EF) $=4\times.3=1.2$;

Total lost between two groups of lamps $=1.2+1.2=2.4$ volts.
Voltage at first group − volts lost in connecting wires = voltage at second group.

$$108-2.4=105.6 \text{ volts across } CF.$$

Thus:

The voltage across first group = 108 volts.
" " " second " = 105.6 volts.

The above simple method by which this general problem was solved, can be used in solving any problem in Direct Current Light and Power distribution.

56. Standard Units. So far, only the fundamental ideas of volts, amperes and ohms have been set forth. It is essential, however, that the student know what the standard value of each is, and how it is obtained.

It is necessary to define independently but two of the three quantities. The third may be determined by the application of Ohm's law. It is customary at the present time to determine independently the *Ohm* and the *Ampere*, and then define the *Volt* as that pressure which will cause one *ampere* to flow through one *ohm*. Probably, a better method is to define independently the *Ohm* and the *Volt*,

and set the value of the Ampere as the current which one *volt* pressure will force through one *ohm* resistance. The reason why the second method is to be preferred is that the volt can be determined independently much more accurately than can the ampere.

THE STANDARD OHM is the resistance of a column of pure mercury 106.3 cms. long, of uniform cross-section, and weighing 14.4521 gms. at 0° C.

THE STANDARD VOLT is defined as $\frac{1}{1.0183}$ of the voltage of a Standard Weston cell. This latter definition has been adopted on account of the difficulty of maintaining and reproducing the standard conditions in solutions of silver nitrate.

THE STANDARD AMPERE is the rate of flow of a steady current which one STANDARD VOLT pressure forces through one STANDARD OHM resistance. The theory of these standard measurements is taken up more fully in Chapters V and XII.

SUMMARY OF CHAPTER III

Electricity is said to flow; thus we speak of an electric current.

UNITS

AMPERE. (Rate of flow). One coulomb per second, or current which will deposit in a given time a standard amount of silver from standard silver nitrate solution.

OHM. (Resistance to flow.) Resistance offered by standard column of mercury.

VOLT. (Pressure causing current to flow). Pressure to cause one ampere to flow through one ohm resistance.

POTENTIAL DIFFERENCE. The fundamental requisite for a flow of current in an electric circuit; can be called the electric difference in level between two points. The current flows from the higher to the lower level; potential difference is measured in volts.

OHM'S LAW.

$$\text{Amperes} = \frac{\text{volts}}{\text{ohms}}, \qquad I = \frac{E}{R}.$$

$$\text{Volts} = \text{amperes} \times \text{ohms}, \qquad E = IR.$$

$$\text{Ohms} = \frac{\text{volts}}{\text{amperes}}, \qquad R = \frac{E}{I}.$$

Ohm's law should be understood to mean always:

The current in a certain part of the circuit equals the voltage across that same part divided by the resistance of that same part.

SERIES CIRCUITS. Combined resistance of parts in series equals sum of the separate resistances.

Combined Voltage across parts in series equals sum of voltages across the separate resistances.

Combined Current. Current is same in each part.

PARALLEL CIRCUITS. Combined resistance of branches equals the reciprocal of the sum of the separate conductances.

Combined current equals the sum of the currents in the separate branches.

Combined Voltage. Voltage across each branch is the same as the voltage across the combination.

Conductance is the reciprocal of resistance.

PROBLEMS, CHAPTER III

25–3. If a car heater is supplied with a pressure of 550 volts from the trolley, how great must its resistance be that the current may not exceed 5 amperes ?

26–3. A 16-c.p. lamp requires .5 ampere, and its resistance at full candle-power is 200 ohms. What voltage must be impressed on its terminals?

27–3. Find drop across wire of 78.8 ohms resistance, if 200 amperes are forced through it.

28–3. Through M, Fig. 69, 5 amperes flow. Through M' 15 amperes. Total amperes in line equal what?

FIG. 69.

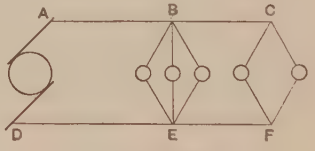

FIG. 70.

29–3. If each lamp, Fig. 70, takes .4 ampere, how much current flows in the following sections of the line: AB, BC, DE, and EF?

30–3. What pressure is needed for an incandescent lamp of 50 ohms resistance, through which flows a current of 1.04 amperes?

31–3. A parallel circuit has resistances in the several branches of 1, 2, 4, 5, and 10 ohms respectively. What is the conductance of the combination? What is the resistance?

32–3. Through each lamp, Fig. 71, 4 amperes flow.

Find current from A to B;
" " B to F;
" " B to C;
" " C to E.

33–3. A circuit has two branches, one of 2 ohms with a current of 6 amperes, the other of 15 ohms. What current flows in the second branch and what difference of potential is maintained between the terminals of the circuit?

34–3. Lamp *L*, Fig. 72, requires 2 amperes. Brush potential of generator is 220 volts:

(*a*) How many volts are used to send current through the line wires?

(*b*) What voltage is there across the lamp?

(*c*) What is the resistance of the lamp?

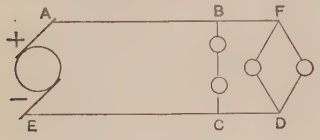

FIG. 71.

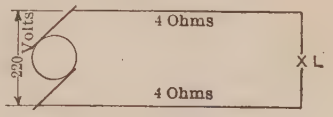

FIG. 72.

35–3. A 5000-ohm galvanometer has its terminals connected to two points in a circuit between which there is a difference of potential of .01 volt. What current flows through the galvanometer?

36–3. Three resistances of 20, 15, and 8 ohms are in series, with 200 volts applied across the outside. What will be the difference of potential across each resistance? Current through each?

37–3. There are seven arc lamps in series, Fig. 73, each requiring 5 amperes. If each has a resistance of 16 ohms, how many volts must dynamo supply to system?

FIG. 73.

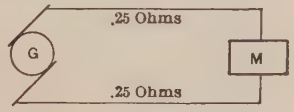

FIG. 74.

38–3. Motor requires 20 amperes at 110 volts, in Fig. 74. Line wires have .25 ohm each. What pressure must be supplied by generator?

39–3. A wire 40 ft. long has a drop of 2 volts across it. What will be the drop across 20 ft.?

40–3. It is desired to find the resistance of the motor in Fig. 75. The motor (*M*) is put in a circuit in series with a resistance (*R*) of .020 ohm. The drop across $R = .436$ volt; the drop across $M = .348$ volt. What is the resistance of the motor?

41–3. Each of the 8 arc lamps in Fig. 76 requires 6 amperes. The ordinary resistance of each lamp is 13 ohms. They are put 8 in series on an 800-volt line. How much *extra* resistance must be added to each lamp to cut the current down to the required 6 amperes?

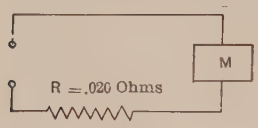

Fig. 75.

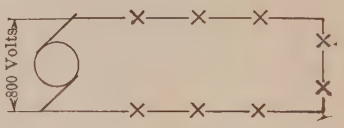

Fig. 76.

42–3. The average resistance of each lamp in Fig. 77 is 220 ohms. Each lamp takes .3 ampere.

(*a*) What is voltage across lamps?

(*b*) Voltage lost in line?

(*c*) What is brush potential of the generator?

Fig. 77.

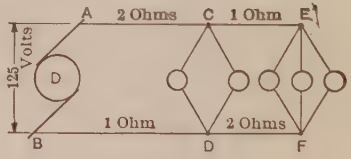

Fig. 78.

43–3. Each lamp in Fig. 78, takes .5 ampere.

(*a*) What is voltage across *CD* and *EF*?

(*b*) What is average resistance of each lamp across *CD*?

(*c*) Across *EF*?

(*d*) Volts lost in line?

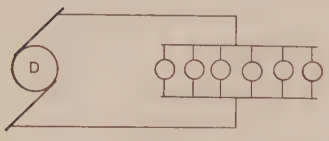

Fig. 79.

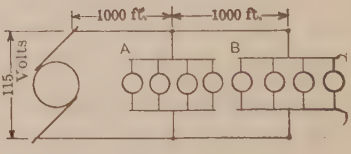

Fig. 80.

44–3. Generator *D*, Fig. 79, has an E.M.F. of 115 volts on open circuit; lamps have a resistance of 220 ohms each. Generator is able to supply .5 ampere to each lamp.

(*a*) What is brush potential when supplying this current?

(*b*) What is the internal resistance of dynamo? (Neglect feed-wire resistance.)

45–3. A storage battery has an E.M.F. of 2.20 volts on open circuit; the internal resistance is .002 ohm. What is the terminal voltage when it is delivering 10 amperes?

46–3. Each lamp in Fig. 80 is to take 1 ampere. There are available for feed wires two coils, one having a resistance of 2 ohms, the other 3 ohms; each coil contains 2000 ft.

(*a*) Should the 3-ohm lot be used between the dynamo and group *A*, or between group *A* and group *B*?

(*b*) What is the greatest voltage that can be obtained across *A* and across *B*, using these two coils as lead wires?

(*c*) What difference in the voltage across *A* would a poor use of the wire make?

47–3. Two resistances, 150 and 100 ohms respectively, are connected in parallel between two points *A* and *B*, across which the pressure is 110 volts.

(*a*) What current will flow in each circuit?

(*b*) If the 100-ohm circuit be reduced to 2 ohms, what current will flow in each?

(*c*) What is the resistance of the parallel combination in each case?

48–3. In Fig. 81,

Resistance of $A = 100$ ohms.
" $B = 120$ ohms.
" $C = 160$ ohms.

Find:

(*a*) Current through each resistance.
(*b*) Resistance of parallel combination (*A* and *B*).
(*c*) Combined resistance of system.
(*d*) Voltage across each resistance.

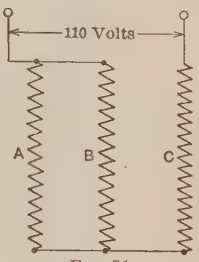

FIG. 81.

49–3. In Fig. 82, Voltage from *A* to $B = 40$ volts.
Current through resistance x is 2.5 amperes.
Resistance $y = 5$ ohms.
" $z = 4$ ohms.

Find:

Current through y. Resistance of x.
" z. Voltage from *B* to *C*.

50–3. A lighting circuit contains 65 arc lamps in series, each of 8 ohms resistance. If the resistance of the generator is 4 ohms and of the line is 8 ohms, what electromotive force must be main-

tained to supply a current of 3 amperes? What is the brush potential of the generator?

51–3. In Fig. 83, L has a resistance of 5.5 ohms, and requires a current of 20 amperes; M_2 requires 10 amperes.

(*a*) What voltage across L and across M_2?

(*b*) What is the combined resistance of AC and BF?

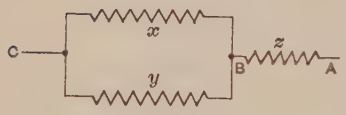

Fig. 82.

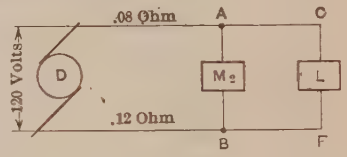

Fig. 83.

52–3. Each lamp, Fig. 84, has 220 ohms resistance, and is run on 110 volts.

Find:

(*a*) Total current through lamps.

(*b*) Volts lost in line.

(*c*) Brush potential of generator.

(*d*) E.M.F. of generator.

(*e*) Volts lost in generator.

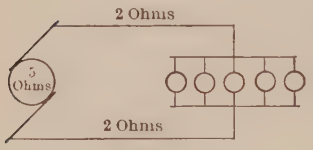

Fig. 84.

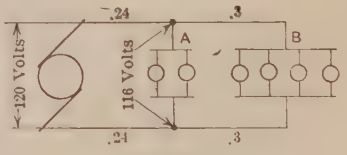

Fig. 85.

53–3. In Fig. 85, voltage across group $A = 116$ volts. Resistance of each lamp in group $A = 100$ ohms.

Find:

(*a*) Voltage across group B.

(*b*) Average resistance of each lamp in group B.

(*c*) Line drop between generator and group A.

(*d*) Line loss in volts between A and B.

(*e*) Current through each lamp in A and each lamp in B.

54–3. What would the values in Problem 53 become if the resistance from G to A were .3 ohm for each wire, and from A to B .24 ohm for each line wire?

55–3. A circuit consists of the following 4 parts joined in series. First part: 2 coils joined to each other in parallel, one having 4 ohms resistance, the other 7 ohms. Second part: a line of wire of 16 ohms resistance. Third part: 12 lamps in parallel, each having 220 ohms resistance. Fourth part: a line wire of 2 ohms resistance.

What voltage is required to send 5 amperes through this circuit?

56–3. In Fig. 86,

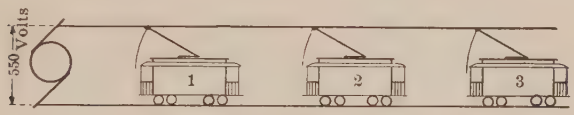

FIG. 86.

Car No. 1 is 1 mile from station and is taking 40 amperes.
" 2 " 3 miles " " 20 "
" 3 " 4 " " 25 "

Trolley has a resistance of .42 ohm per mile. Track resistance .03 ohm per mile.

Find:

(*a*) Voltage across each car.

(*b*) Total voltage loss in line.

57–3. What would the values in Problem 56 become if a feeder of .26 ohm per mile were run along the trolley, being tied to it every half mile?

CHAPTER IV

POWER MEASUREMENT

Use of Ammeter, Voltmeter, and Wattmeter—Electric Power; Watt; Kilowatt—Variations of Power Equation—Electric Energy; Kilowatt-hour; Watt-second or Joule—Electric Energy Converted to Heat Energy—Heat Equivalent of Electricity—Efficiency: of Electric Machines; of Electric Transmission; of Electric Lamps.

57. Ammeter. When we wish to know how many amperes are flowing through a *given* part of a circuit, we *insert an ammeter* in that part of the circuit. Since an ammeter reads only the current going through itself, we must actually cause the current we wish to measure to flow through the instrument. This means, we must *break the circuit* and insert the ammeter in such a way that the current flows through it. In other words, an *ammeter* is always placed in *series* in a circuit.

In Fig. 87, ammeter A measures the current through R because it is placed in series with R. Since the circuit

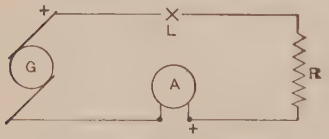

Fig. 87.—Ammeter A measures current in R and X.

Fig. 88.—Ammeter A measures current in M only.

is a simple series circuit, the ammeter also measures the current in arc L and generator G. Notice that the circuit is *broken* to insert ammeter.

In Fig. 88, the ammeter A measures the current flowing through the motor M, because it is in series with the motor.

It does not, however, measure the current in the lamp L, because it is not in series with the lamp. Neither does it measure the current through the generator G. The current through the generator must equal the current through motor M plus the current through lamp L, as explained in Chapter III.

Since an ammeter must always be of very low resistance in order not to cut down the current, it is necessary *always* to use a short-circuiting switch with the instrument. This protects it from injury in case there is too much current flowing through the line for the instrument to record.

In Fig. 89, with the short-circuiting switch in position S, nearly all the current flows through the switch-blade. If

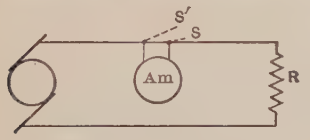

FIG. 89.—Ammeter with short circuiting switch.

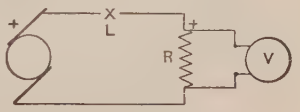

FIG. 90.

there is too much current in the line for the instrument to carry, there will be a noticeable reading on the instrument with switch S closed. If, however, the ammeter registers nothing, it will probably be safe to throw the switch to the position S', which causes all the current to flow through the ammeter, where it will be indicated.

Care must be taken to get the + side of the instrument on the + side of the line.

58. Voltmeter. Pressure between two points is measured by placing a voltmeter across these points. A voltmeter is not inserted in a circuit, but merely placed in shunt with the part of the circuit across which it is desired to find the voltage.

In Fig. 90, if it is desired to find the voltage across R, the voltmeter is connected as shown, to the two ends of R, without disturbing the former connections. Be sure to get + side of voltmeter on the + side of the line. A

voltmeter is of very high resistance, and therefore takes no appreciable amount of current from the line.

Just as an ammeter indicates the current flowing through *itself*, so a voltmeter reads the voltage across *itself*. Thus in Fig. 90, when we wish to measure the voltage across the resistance R, we place the voltmeter in parallel with R, so that the voltage across it will be the same as the voltage across R. Therefore when it indicates the voltage across itself, it also indicates the voltage across R.

59. Rule for Use of Ammeter and Voltmeter. Place AMMETER in *series*, always using a short-circuiting switch to prevent injury to the instrument.

Place VOLTMETER in *shunt*. Put the + side of each instrument on the + side of the line.

Fig. 91 shows the correct use of an ammeter and a voltmeter to measure the current and the voltage supplied to the motor M.

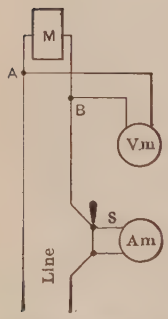

FIG. 91.—Ammeter and voltmeter connected to measure power used by M.

The short-circuiting switch S must be opened before the ammeter is read.

All the current that enters the motor must then flow through the ammeter and be indicated. The ammeter is of very low resistance (about .001 or .002 ohm) and does not cut down the flow of the current.

The voltmeter is of very high resistance (about 15,000 ohms), and does not allow any appreciable current to flow around the motor, yet enough goes through it to cause it to indicate the voltage across the terminal AB of the motor.

Suppose the voltage across the motor to be 110 volts, what would happen if an *ammeter* of .002 ohm resistance were by mistake placed across AB? (Remember—Ohm's law is always in operation.) For a detailed discussion of ammeters and voltmeters see Chapter XIV.

Work and Power Measurements

60. Power, Watt, Kilowatt. The flow of an electric current has been likened to the flow of water through a pipe. A current of water is measured by the number of gallons or pounds flowing per minute; a current of electricity, by the number of amperes, or coulombs per second. The power required to keep a current of water flowing is the product of the current in *pounds per minute* by head, or pressure, in *feet*. This gives the power in *foot-pounds per minute*. To reduce to horse-power, it is necessary merely to divide by 33,000, i.e., $\frac{\text{feet} \times (\text{lbs. per min.})}{33000} = \text{H.P.}$

In exactly the same way, the *power* required to keep a current of electricity flowing, is the product of the current in *amperes* by the pressure in *volts*. This gives the power in *watts*. To reduce to Kilowatts it is necessary merely to divide by 1000, i.e.

$$\frac{\text{volts} \times \text{amperes}}{1000} = \text{K.W.}$$

1 Kilowatt $= 1.34$ Horse-power or $1\frac{1}{3}$ H.P.
1 Horse-power $= 746$ watts or $\frac{3}{4}$ Kilowatt.

This method of computing electric power may be written in the form of an equation as

$$P = IE.$$

where P = power in *Watts*;
I = current in *Amperes*;
E = pressure in *Volts*.

The two following examples show the similarity in the methods of computing water power and electric power:

Example. What power is required to drive a pump which must raise water 100 ft. and supply 1500 gals. per hour? (1 gal. of water weighs 8.3 lbs.)

$$\begin{aligned}\text{Power} &= \text{pressure} \times \text{current};\\ &= \text{feet} \times \text{pounds per min.};\\ &= 100 \text{ ft.} \times \frac{1500 \times 8.3}{60} \text{ lbs. per min};\\ &= 20{,}800 \text{ ft. lb. per min.};\\ 20{,}800 \text{ ft.-lb. per min.} &= \frac{20{,}800}{33{,}000} \text{ H.P.} = .63 \text{ H.P.}\end{aligned}$$

About a 1 H.P. motor would then be required to drive the pump, when delivering the current.

Example. What power is required to drive a generator which must deliver 4 amperes at 120 volts?

$$\begin{aligned}\text{Power} &= \text{pressure} \times \text{current.}\\ &= \text{volts} \times \text{amperes.}\\ &= 120 \times 4 = 480 \text{ watts or } .480 \text{ K.W.}\\ 480 \text{ watts} &= \frac{480}{746} \text{ H.P.} = .64 \text{ H.P.}\end{aligned}$$

Thus a motor of somewhat less than a 1 H.P. would be required to drive the generator when delivering this current.

Note that these methods of computing the power are identical.

61. To Compute the Power in an Electric Circuit. If then we wish to know the power that is being consumed in *a certain part* of an electric circuit, we merely have to insert an ammeter to measure the current in *that part* of the circuit, and apply a voltmeter to measure the voltage across *that part* of the circuit, and multiply the Ammeter readings by the Voltmeter reading. This gives us our power directly in Watts. This is usually stated as an equation as follows:

$$\text{Watts} = \text{Volts} \times \text{Amperes.}$$

The same precautions must be observed in the use of this equation as in the use of Ohm's law. That is, the

Voltage and the Current must be measured for the *same part* of the circuit at the same time, and their product is the power consumed in *that part* of the circuit alone. The following example illustrates the use of the equation.

Example. A generator G, Fig. 92, is furnishing a current of 4 amperes to the line at a pressure of 120 volts. There is in the circuit a resistance R, which requires 5 volts to force the current through it, and a motor M, which requires 115 volts. How much power does the resistance R consume, and how much does the motor M consume?

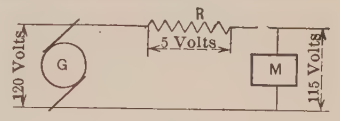

FIG. 92.

The resistance R consumes:

$$P = IE,$$

or

$$P(\text{watts in } R) = I(\text{current through } R) \times E(\text{voltage across } R).$$
$$P = 4(\text{amperes.}) \times 5(\text{volts}) = 20 \text{ watts.}$$

Motor M consumes:

$$P = IE,$$

or

$$P(\text{watts in } M) = I(\text{current through } M) \times E(\text{voltage across } M).$$
$$P = 4(\text{amperes}) \times 115 \text{ (volts)} = 460 \text{ watts.}$$

The total power consumed by R and $M = 460 + 20 = 480$ watts.

This checks with the total power delivered to the circuit as follows:

$$P = IE(\text{for circuit}),$$

or

$$P(\text{watts in circuit}) = I(\text{current in circuit}) \times E(\text{voltage across circuit}).$$
$$P = 4 \text{ (amperes)} \times 120 \text{ (volts)} = 480 \text{ watts.}$$

62. Variations of Power Equation. Just as Ohm's law has the three forms, (1) $I = \frac{E}{R}$, (2) $E = IR$, (3) $R = \frac{E}{I}$, so

this power equation $P=IE$ may have three forms, found as follows:

(1) $$P=IE;$$

but,

$$E=IR. \qquad \text{Ohm's law.}$$

Therefore,

$P=I(IR)$ or I^2R, substituting in (1).

(2) $$P=I^2R;$$

again,

$$I=\frac{E}{R}. \qquad \text{Ohm's law.}$$

Therefore,

$$P=\left(\frac{E}{R}\right)E=\frac{E^2}{R}, \text{ substituting in (1).}$$

(3) $$P=\frac{E^2}{R}.$$

It is well to learn this equation in its three forms:

$$P=IE=I^2R=\frac{E^2}{R}.$$

This will save a considerable amount of mathematical work.

When the volts and amperes are given, multiply direct to get the watts ($P=IE$); when the amperes and ohms are known, multiply ohms by square of amperes ($P=I^2R$); when volts and ohms are known, divide square of volts by ohms $\left(P=\frac{E^2}{R}\right)$.

The result is the same as though we used Ohm's law *first* to find the amperes and volts and then multiplied.

Example. Generator G, Fig. 93, furnishes .5 ampere to the line at 115 volts pressure. What power is consumed by (*a*) the 110-volt lamp L, and by (*b*) the 10 ohms resistance R?

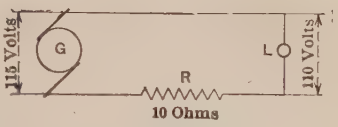

FIG. 93.

(*a*) For the lamp L:

$E=110$ volts;
$I=.5$ ampere;

(1) $P=EI=.5\times110=55$ watts.

Or find the resistance of the lamp.

$$R=\frac{E}{I}=\frac{110}{.5}=220 \text{ ohms.}$$

Then

$I=.5$ ampere;
$R=220$ ohms;

(2) $P=I^2R=.5\times.5\times220=55$ watts.

Or

$E=110$ volts;
$R=220$ ohms.

(3) $$P=\frac{E^2}{R}=\frac{110\times110}{220}=55 \text{ watts.}$$

The result of any of the three methods is the same, 55 watts. Therefore it is always best to use the easiest method for the data given. Since the voltage and the current in this case were given, it is best to use $P=EI$.

(*b*) For the resistance R.

$R=10$ ohms;
$I=.5$ ampere;
$P=I^2R=.5\times.5\times10=2.5$ watts.

There is no need of finding the voltage across R, and multiplying that by the current, though the result would be the same. Thus

115 volts − 110 volts = 5 volts across R.

$E=5$ volts;
$I=.5$ ampere;
$P=IE=.5\times5=2.5$ watts.

Problem 1–4. How many horse-power are required to hoist a load of 2000 lbs. 40 ft. per minute?

Problem 2–4. How many kilowatts are required to supply the power in Problem 1–4?

Problem 3–4. What power is being used by an incandescent lamp which takes .5 amperes at 110 volts?

Problem 4–4. What power does a car heater use if its resistance is 100 ohms and the pressure across it is 550 volts?

Problem 5–4. What power does an electric flatiron use on a 115-volt circuit if the resistance is 50 ohms?

Problem 6–4. How many watts are consumed in lead wires in Fig. 72?

Problem 7–4. A 110-volt arc lamp requires 6 amperes to operate properly. What power does such a lamp take?

Problem 8–4. How many watts are used by lamp *L*, in Fig 83? How many watts, by the line wires between *D* and M_2?

Problem 9–4. What power is consumed by each lamp in Group *A*, of Fig. 85?

Problem 10–4. The power being delivered by the generator in Fig. 86 is equivalent to how many horse-power?

63. Use of Wattmeter. Instead of using two separate instruments, an ammeter and a voltmeter, to measure the power consumed in a certain part of a circuit, we may use a single instrument called a WATTMETER. This instrument is a combination of an ammeter and a voltmeter, and thus has an ammeter part of *low resistance* to measure the current, and a voltmeter part of *high resistance* to take the voltage. The indicator shows the product of the volts times the amperes, that is, the *watts*.

A wattmeter, then, usually has *four* terminals, two for the ammeter leads and two for the voltmeter leads. The utmost care must always be exercised in using the proper terminals. An error in this respect will ruin a very expensive instrument.

Fig. 94, shows the correct method of connecting a wattmeter in the circuit to measure the power taken by the arc lamp *L*. The ammeter terminals *AA*, are put in *series* with the arc lamp *L* and the voltmeter terminals *VV* are connected in *parallel across* the arc lamp. For a detailed discussion of the wattmeter, see Chapter XIV.

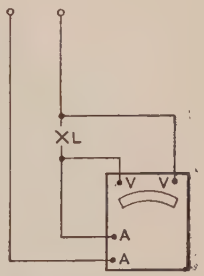

Fig. 94.—Wattmeter connected to measure power consumed by *X*.

64. Work — Commercial Units. In order to compute the amount of work done by a given engine, it is necessary to know the time it has been running, and the power it has been supplying, that is, its rate of doing work. If the power is measured in Horse-power and the time in Hours, the work done is measured in Horse-power-Hours and is the product of the *Horse-power* by the *Hours*.

Similarly, if the power is measured in Kilowatts and the time in Hours, the work done is measured in Kilowatt-Hours and is merely the product of the *Kilowatts* by the *Hours*.

The Horse-power-hour and the Kilowatt-hour are the commercial units of work.

$$1 \text{ H.P. hour} = .746 \text{ K.W. hour};$$
$$1 \text{ K.W. hour} = 1.34 \text{ H.P. hours}.$$

The use of these units is illustrated by the following examples:

Example. How much work is done in one day of 8 hours by a 150-K.W. generator running at full load?

$$150 \times 8 = 1200 \text{ K.W. hours};$$

or

$$1200 \times 1.34 = 1610 \text{ H.P. hours}.$$

Example. At 15 cts. per K.W. hour, what is the cost of burning 100 lamps for 8 hours if each lamp consumes 50 watts?

Power consumed in 100 lamps:

$$100 \times 50 = 5000 \text{ watts} = 5\text{K.W.};$$
$$8 \times 5 = 40 \text{ K.W. hours};$$
$$40 \times 15 \text{ cts.} = \$6.00.$$

65. Work: Small Units—Foot-Pound: Watt-Second or Joule. The commercial units of WORK, the H.P. hour and the K.W. hour, are too large to be used conveniently in some problems. Accordingly, use is made of two smaller units. The mechanical unit so used is the FOOT-POUND. The electrical unit is the WATT-SECOND, also called the JOULE.

1 H.P. hour = 1,980,000 ft.lbs.
1 K.W. hour = 3,600,000 watt-secs. or joules.
1 Joule = .74 ft.lb.

In the following pages, when dealing with small quantities of work the foot-pound and watt-second will be used; otherwise the commercial units the Horse-power-hour and the Kilowatt-hour will be employed.

Example. How much work is done when a 25-watt lamp is burned for 2 minutes?

$$25 \times 2 \times 60 = 3000 \text{ watt-secs. or joules;}$$

or

$$3000 \times .74 = 2220 \text{ ft.lbs.}$$

The following are the electrical units of WORK and POWER in general use:

Work: Watt-sec. = joule
K.W. hour;

Power: Watt
Kilowatt
Kilovolt-ampere (A. C. unit).

$$\text{WORK} = \text{POWER} \times \text{TIME}.$$
$$\text{POWER} = \frac{\text{WORK}}{\text{TIME}}.$$

The inter-relation among the different units can readily be seen by the following table:

Work Units: Watt-sec. = joule = volt-coulomb;
K.W. hour = 3,600,000 watt-secs.

Power Units: Watt = joule per sec. = volt-ampere = volt-coulomb per sec.
Kilowatt = 1000 watts.

$$\text{K.V.A. (A.C. unit)} = \frac{\text{Kilowatts}}{\text{Power factor}}.$$ (See Chapter XV and tables in Appendix.)

Problem 11–4. How much will it cost at 5 cts. per K.W. hour to run a 110-volt motor for 8 hours? Motor takes 40 amperes.

Problem 12–4. If work in Problem 11–4 were paid for at the rate of 4 cts. per H.P. hour, how much would it cost?

Problem 13–4. How much work is done when a 50-watt lamp is burned for 4 hours?

Problem 14–4. At 12 cts. per K.W. hour, what is the cost of running iron in Problem 5–4 for 6 hours?

Problem 15–4. What work is done in maintaining for 12 hours a current of 100 amperes in a wire of .8 ohm resistance?

Problem 16–4. A bill for electric energy was $16.40 for 120 hours. If the price was 10 cts. per K.W.-hour, what was the average power used?

Problem 17–4. When 100 incandescent lamps had been burned on a 110-volt circuit for 4 hours, a bill of $3.00 was presented, computed on the rate of 15 cts. per K.W. hour. What was the average current taken by each lamp?

Problem 18–4. At 5 cts. per K.W. hour, how much does it cost per year for transmission losses in Problem 52–3? Lamps are burned 2 hours each day.

Problem 19–4. What is the total energy lost in the line wires per month of 100 hours in Problem 43–3?

Problem 20–4. By how many foot-lbs. is 1,000,000 watt-secs. greater or less than 3 H.P. hours?

66. Electrical Energy and Heat Energy. An electric current may be used in one part of a circuit to produce mechanical motion, as in a motor; in another part of the circuit, to produce electrolytic action, as in an electroplat-

ing vat; in another part, to produce light, as in an electric lamp. In any part of the circuit where it is doing no one of these things, all the energy consumed goes into producing heat. It even produces heat in the portions of the circuit where it is also producing some other form of energy.

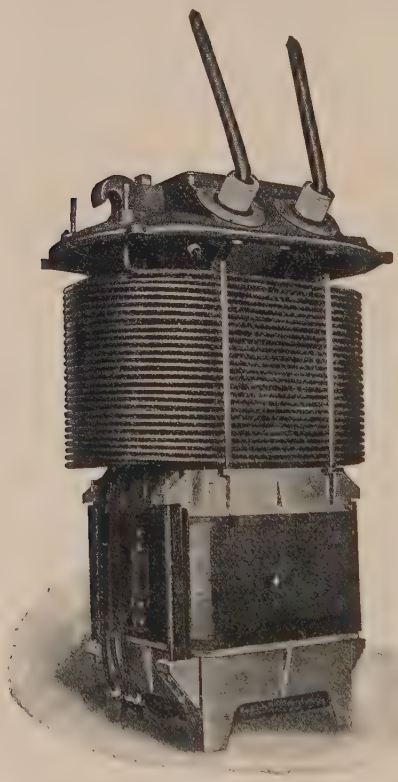

FIG. 95. Interior of transformer showing cooling coils.

A motor never gives out in mechanical energy all that it receives in the form of electrical energy. Some of the electrical energy is turned into heat energy. This heat is produced in overcoming the electrical *resistance*, just as heat is produced in a machine in overcoming mechanical *friction*.

It is safe to say that, in any part of an electric circuit where there is no transformation to other forms of energy, the whole of the electrical energy consumed is turned into heat.

Thus in the line wires in Fig. 96, all the electric energy consumed in forcing a current through them goes into heat.

The resistance of an electric circuit is similar to the friction of a machine. Just as the power used to overcome the resistance of the wire appears in the heat generated in the wire, so the power used to overcome the friction of a bearing appears in the heat generated in the bearing. The mechanical engineer strives to reduce the amount of power wasted in heat, by reducing the friction of the bearings. The electrical engineer may reduce the power wasted in heat, by reducing the resistance of the wire used to transmit a given current. In designing electric machinery, a great deal of care is taken to provide sufficient surface for radiation, so that the heat generated may not cause an excessive rise in temperature.

In Fig. 95 the interior of a transformer is shown. Notice the coil of pipes at the top. Water is circulated through these pipes in order to carry off the heat generated in the core and windings.

Example. The generator G in Fig. 96, maintains a pressure of 240 volts across the line. The line wires have a total resistance of 4 ohms. The motor M requires 5 amperes at 220 volts. How much electrical power is lost in heat in the line wire?

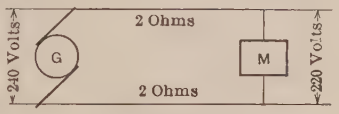

FIG. 96.

Solution. Since the line wire is producing no other form of power, all the electrical power consumed in it must be used in overcoming its resistance and must go into heating it. The power used in the line wire is found as follows:

$I = 5$ amperes;

$R = 4$ ohms.

$P = I^2R = 5 \times 5 \times 4 = 100$ watts or joules per sec.

Thus 100 joules are turned into heat each second. If this heat is not allowed to radiate, the wire would soon become hot enough to destroy the insulation and finally fuse the copper itself. Thus is seen the necessity for arranging wires in the coils of a transformer and in the field and armature coils of motors in such a way as to allow for the dissipating of the heat generated in overcoming the resistance of the coils.

Referring again to Fig. 96, it is seen that the motor consumes 220×5 or 1100 watts. The greater part of this energy is used in doing the mechanical work of turning the armature. But a considerable share of this energy even, is used in overcoming the electrical resistance of the motor.

Assuming the resistance of the motor is 2 ohms, the power consumed in overcoming its resistance, and thus in heating it would be:

$$P = I^2R;$$
$$= 5 \times 5 \times 2 = 50 \text{ watts.}$$

Of the total 1100 watts taken by the motor, not more than 1050 watts are turned into mechanical energy, 50 watts being used to overcome the electrical resistance of the motor. These 50 watts are called the I^2R Loss or sometimes the Copper Loss, since the electrical conductors are generally of copper.

67. Heat Equivalent of Electricity. If the heat generated by passing an electric current through a wire be measured by allowing it to raise the temperature of a known weight of water, it is found that one watt-sec. of electrical energy is the equivalent of .24 calorie. That is, one ampere flowing through an ohm resistance for 1 sec. will raise 1 gm. of water .24° C. This is generally stated in the form of an equation:

$$H = .24\, I^2Rt,$$

where H = Heat in calories;
I = Current in amperes;
R = Resistance in ohms;
t = time in seconds.

Since

$$I^2R = \text{watts},$$

then

$$I^2Rt = \text{watt-secs. or joules;}$$

$$\therefore \quad H = .24 \text{ watt-sec. or joule.}$$

The factor (.24) is called the HEAT EQUIVALENT of ELECTRICITY.

Example. Referring again to Fig. 96, we find that 100 watts were expended in heating the lead wires. The actual number of calories thus produced per sec. would be $100 \times .24 = 24$ calories per sec. This is a very high amount of energy to be lost in transmitting such a small quantity of electrical energy as the motor M is taking, and could easily be avoided by using line wire of less resistance.

Example. A bank of thirty-two c.p. incandescent lamps takes 10 amperes at 110 volts. If 88 per cent of the energy received by the lamps is given off in heat, how many calories are thus given off in 1 hour?

$10 \times 110 = 1100$ watts (delivered to lamps);
88% of $1100 = 968$ watts (given off in heat);
1 watt is equivalent to .24 cal. per sec.;
$968 \text{ watts} = 968 \times .24 = 232$ cal. per sec.
$232 \times 60 = 13{,}900$ cal. per min.
$13{,}900 \times 60 = 835{,}000$ cal. per hour.

Or, using the equation,

$$H = .24I^2Rt = .24 \text{ watt-sec.;}$$
$$= .24 \times (10 \times 110) \times 3600 \times (.88) = 835{,}000 \text{ cal.}$$

Problem 21–4. How much heat is generated per hour in an electric iron using 3.5 amperes at 110 volts?

Problem 22–4. If the price of the electric energy used in Problem 21–4 is 12 cts. per K.W. hour, what is the price per calorie?

Problem 23–4. How many calories per hour are given off by a car heater which takes 6 amperes at 550 volts?

Problem 24–4. How much heat is generated in 10 hours in the lead wires of Problem 18–3?

Problem 25–4. If only 14 per cent of the electric energy used in the lamp in Problem 18–3 goes into light, how many calories of heat does the lamp give off in burning 3 hours?

Problem 26–4. How long would a current of 5 amperes under a pressure of 40 volts have to run to supply enough heat to just melt 8 lbs. of ice? Latent heat of ice = 144 B.T.U.'s.

68. Efficiency. Since all pieces of electrical apparatus have resistance, there must always be some heat generated when a current is sent through them. Unless the apparatus is to be used for heating purposes, this energy is wasted, just as the energy is wasted which goes toward overcoming mechanical friction. It follows from this that no electrical machine is perfect. That is, it does not give out in useful energy, all the energy that is put into it. We can say, then, that no electrical machine has 100 per cent efficiency.

When we speak of the commercial efficiency of electrical machinery we mean the same that we do in speaking of the efficiency of any machine; the percentage that the useful *output* is of the *input*.

In many commercial tests, it is difficult to measure directly the input into a machine. But it may be *computed* from the output and the losses, since the input must equal the output plus the losses.

We then have the equation:

$$\text{Efficiency} = \frac{\text{output}}{\text{input}} = \frac{\text{output}}{\text{output} + \text{losses}}.$$

Example. A 3-H.P. motor requires 2.4 K.W. to drive it. What is its efficiency?

$$\text{Output} = 3.0 \text{ H.P.};$$
$$\text{Input} = 2.4 \times 1.34 = 3.2 \text{ H.P.};$$
$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{3.0}{3.2} = 93\%.$$

For a discussion of the efficiency of motors and generators see Chapter IX.

Problem 27–4. What efficiency has a 15-H.P. motor which requires 65 amperes at 220 volts?

Problem 28–4. A generator has an efficiency of 90 per cent. What current can it deliver at a pressure of 110 volts when it receives 10 H.P. from the driving engine?

Problem 29–4. What is the efficiency of a motor which does 428,000 ft.lbs. of work in 5 minutes, if the input is 2.5 K.W.?

69. Efficiency of Transmission. When energy is transmitted from one place to another, some energy is always used in the transmission line. In mechanical transmission of energy, there is the pulley and belt friction, etc., to be overcome. In electrical transmission, the resistance of the lead wires must be overcome. That method of transmission is most efficient which loses the smallest percentage of energy in the line. The efficiency of transmission may be defined as that percentage which the useful output of the line is of the total input.

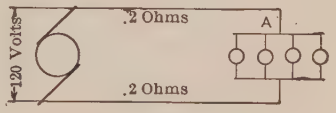

FIG. 97.

Example. The lamp bank *A*, Fig. 97, uses 12 amperes. What is the efficiency of the transmission; that is, what per cent of the total power delivered to the line is used by the lamps?

Total power $= (IE) = 120 \times 12 = 1440$ watts.
Watts lost in line $= (I^2R) = 12 \times 12 \times .4 = 57.6$ watts.
Watts left for lamps $= 1440 - 57.6 = 1382.4$ watts.

$$\text{Efficiency of transmission} = \frac{1382}{1440} = 96\%;$$

or

Volts lost in line $= (IR) = 12 \times .4 = 4.8$ volts.
Volts across lamps $= 120 - 4.8 = 115.2$ volts.
Watts delivered to lamps $= (IE) = 115.2 \times 12 = 1382$ watts.

$$\text{Efficiency} = \frac{1382}{1440} = 96\%.$$

Problem 30–4. What is the efficiency of transmission for a circuit which receives 4 K.W. from a generator at one end and delivers 3.8 K.W. to a motor at the other end?

Problem 31–4. What is the efficiency of transmission in Problem 42–3?

Problem 32–4. What is the efficiency of transmission in Problem 34–3?

Problem 33–4. What is the efficiency of transmission in Problem 38–3?

Problem 34–4. What is the efficiency of transmission in Problem 51–3?

Problem 35–4. What is the efficiency of transmission in Problem 43–3?

Problem 36–4. What is the efficiency of transmission in Problem 52–3?

Problem 37–4. What is the efficiency of transmission in Problem 53–3?

70. Efficiency of Lamps. Regarding a lamp as a *machine,* we may define the efficiency, as the ratio of the watts given out in light, to the total watts received at the lamp terminals. This is a very instructive way to study the performance of a lamp. The A.I.E.E. in 1907 defined the efficiency of an incandescent lamp as the candle-power produced by each watt received by the lamp. Thus a lamp taking .5 ampere at 110 volts, or 55 watts, and giving 14 c.p., would be rated according to the A.I.E.E. as $\frac{14}{55}$ or .25 c.p. per watt.

Neither of these methods, however, is in common use in giving the efficiency of any kind of electric light. The general practice is to state the number of watts required to produce 1 c.p. of light. The above 55-watt, 14 c.p. lamp would then be rated as a lamp of $\frac{55}{14}$ or 3.93 watts per c.p. Thus, the fewer the watts used per c.p., the more efficient the lamp. An arc lamp using .8 watt per c.p. is, therefore, much more efficient than an incandescent lamp using 3.93 watts per c.p. as above.

The problem of illumination is as much a matter of even distribution of the light as of the high efficiency of the lamp. The ideal lamp, of course, combines a high efficiency with an even distribution. For a fuller discussion of the subject of illumination, see Chapter XIII.

Problem 38–4. A carbon filament lamp of 16 c.p. takes 55 watts. What is the light efficiency of the lamp?

Problem 39–4. A tungsten lamp is rated as a 50-watt lamp. If the light efficiency is 1.5 watts per c.p., how many candle-power has it?

Problem 40–4. A certain 32 c.p. incandescent lamp gives out 13 watts in light and 89 watts in heat. (*a*) What is the efficiency of the lamp as a machine for a source of heat? (*b*) As a source of light? (*c*) What would be the commercial rating of the lamp?

SUMMARY OF CHAPTER IV

USE OF AMMETER. Measures current; has low resistance; always put in series with that part of the circuit in which the current is to be measured.

USE OF VOLTMETER. Measures voltage; has high resistance; connected in parallel across the points, between which the voltage is to be measured.

USE OF WATTMETER. Measures power; is a combination of ammeter and voltmeter. Ammeter side is connected in series with circuit, and voltmeter side across circuit. Great care must be used not to get ammeter side across the circuit.

COMPUTATION OF ELECTRICAL POWER.

$$\text{Watts (power)} = \text{amperes} \times \text{volts.}$$

This should be understood to mean that the Watts consumed in any part of a circuit equals the product of the Amperes flowing through that same part of the circuit times the Volts across that same part. Just as Ohm's law may be written in three ways, so the power equation may be written in three ways, thus:

$$P = IE = I^2R = \frac{E^2}{R}.$$

HEAT EQUIVALENT OF ELECTRICAL ENERGY.

$$1 \text{ watt-sec. (joule)} = .24 \text{ calorie.}$$

The total amount of heat developed by an electric current flowing for a given time through a resistance is computed from the following equation:

$$H = .24I^2Rt,$$

where H is measured in calories;
I " amperes;
R " ohms;
t " seconds.
.24 is the Electrical Heat Equivalent.

EFFICIENCY OF ELECTRIC MACHINES. The commercial efficiency of an electric machine, as of any machine, may be stated as the percentage of useful energy given out by the machine, in comparison with the energy put into the machine. The same is true of an electric transmission.

Electric lamps may be regarded as machines and their efficiency stated as above. The usual method, however, is to state the number of watts consumed per candle-power of light given out.

The efficiency of generators is generally most easily computed from the equation:

$$\text{Commerical efficiency} = \frac{\text{output}}{\text{output} + \text{losses}}.$$

PROBLEMS ON CHAPTER IV

41–4. The Weston D. C. Ammeters are designed to have 45 millivolts drop across them when they are giving their full scale reading.

(*a*) What must be the resistance of a 1 ampere ammeter?

(*b*) Of a 50 ampere ammeter?

(*c*) Of a 15 ampere ammeter?

(*d*) How much current would 15 ampere ammeter take if placed by mistake across 110 volts?

42–4. A Weston D.C. Voltmeter has approximately 100 ohms for each volt of the scale. What current does a 150-volt voltmeter take when placed across a 110 volt circuit?

43–4. Motor in Fig. 98 takes 20 amperes at 110 volts.

(*a*) What is the "brush potential" of the generator *G*?

(*b*) What is the efficiency of transmission?

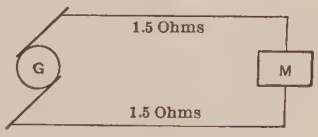

FIG. 98.

44–4. An engine supplies 160 H.P. to a generator delivering 200 amperes at 550 volts. What is the commercial efficiency of the generator?

45–4. A generator delivers a current of 90 amperes at a pressure of 110 volts. What power does it supply in kilowatts? How many H.P.?

46–4. The generator in Problem 45 has a shunt-wound field with a resistance of 200 ohms. How many watts are absorbed in the field?

47–4. The armature of generator in Problem 45 has a resistance of .02 ohm. What power is lost in the armature conductors?

48–4. An engine supplies 150 H.P. to a generator delivering 180 amperes at a pressure of 550 volts. What is the commercial efficiency of the generator?

49–4. Assuming an efficiency of 50 per cent for the whole arrangement, what current will be used by a 250-volt electric hoist when raising 2500 lbs. 200 ft. per minute?

50–4. Assuming that 70 per cent of the electrical input is usefully expended, what current will be required by a 500-volt railway motor in propelling a 60-ton car up a 2 per cent grade at a speed of 8 miles per hour?

51–4. Electrical energy for lighting costs 15 cts. per K.W. hour, what is the cost per month (30 days) of operating a 16 c.p. lamp, which takes 3.1 watts per candle, the lamp being in service 3 hours each day?

52–4. Electrical energy is supplied at 5 cts. per K.W. hour for driving a 10-H.P. motor. The efficiency of the motor at full load is 85 per cent. Find the cost of operating the motor for 100 hours.

53–4. A current of 0.5 ampere flowing through a lamp generates 180 calories of heat in 12 seconds. (*a*) What is the resistance of the lamp? (*b*) What power is expended in the lamp? Express in watts and in H.P.

54–4. A field coil of a generator contains 20 lb. of copper (specific heat 0.095); weight of insulation negligible. The resistance of the coil is 200 ohms. How fast does the temperature of the coil rise when a current of 2.1 ampere is flowing through the coil? Allow for no radiation.

55–4. How long would it take for the temperature of the coil in Problem 54–4 to rise 50° C., if no heat were given off from the coil in radiation?

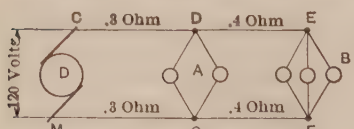

FIG. 99.

56–4. Each lamp, Fig. 99, takes 1 ampere.

Find:

(*a*) Voltage across group *A* and across *B*.

(*b*) Volts lost in *CD* and *GM* and in *DE* and *FG*.

(*c*) Watts delivered to each group.

(*d*) Watts lost in line.

(*e*) Efficiency of transmission.

57–4. Voltage across terminals of arc lamp in Fig. 100 is 110 volts. If arc takes 5 amperes at 85 volts and gives 800 c.p.

(*a*) What power is consumed by the series resistance R?

(*b*) What is efficiency of the lamp, as a machine? as a lamp?

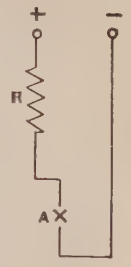

FIG. 100.

58–4. At 8 cts. per K.W. hour, how much will it cost, per week of 60 hours, to run a motor, having an average load of 40 H.P., and an average efficiency of 90 per cent?

59–4. What must be the H.P. of an engine to run a generator feeding 600 ½-ampere lamps at 110 volts? Five volts are lost in the line, the efficiency of the generator is 92 per cent, and the loss in the belt is 1 per cent.

60–4. At Group II, Fig. 101, there are four 55-watt 110-volt lamps. Group I consists of five 80-watt 115-volt lamps. Internal resistance of generator G is 4 ohms.

Find:

(1) Resistance of $AC+BD$.
(2) Volts lost between G and Group I.
(3) Brush potential of generator.
(4) E.M.F. of generator.
(5) Efficiency of transmission.
(6) Average resistance per lamp of Group I.
(7) Average resistance per lamp of Group II.

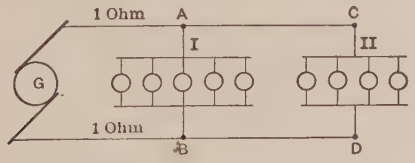

FIG. 101.

61–4. Generator, Fig. 102, delivers 5 K.W. to line. Motor uses 4.8 K.W.

Find:

(*a*) Watts lost in line.
(*b*) Voltage and current of motor.
(*c*) Brush potential of generator.

62–4. Assume internal resistance of generator in Problem 61 to be 4 ohms. The owner of the motor pays the owner of the generator 10 cts. per K.W. hour for energy. If it costs the owner of the generator 9 cts. per K.W. hour to generate energy, how much does he earn or lose in a month of 250 hours?

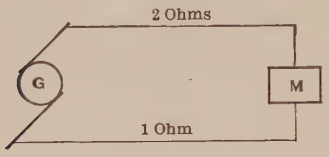

Fig. 102.

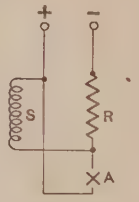

Fig. 103.

63–4. Resistance of shunt coil S in arc lamp in Fig. 103 is 440 ohms. Voltage across terminals = 55 volts. Arc A requires 7 amperes at 35 volts and gives 600 c.p.

Find:

(*a*) Resistance of R.

(*b*) Power consumed by lamp.

(*c*) Power consumed by resistances S and R.

(*d*) Efficiency of lamp,

(1) As a machine;

(2) As a lamp.

64–4. A wire having a resistance of 40 ohms is coiled in a vessel containing 300 gm. of oil of which the specific heat is 0.75. The vessel is copper and, together with copper wire, weighs 200 gm. (Specific heat = .095.) What will be the temperature rise after a current of 4.2 amperes has passed through the coil of wire for 5 minutes? Neglect radiation.

CHAPTER V

MEASUREMENT OF RESISTANCE

Circular Measure—Mil—Mil-foot—Resistivity—Temperature Coefficient of Resistance—Use of Wire Tables—Temperature measured by Change in Resistance—Methods of Measuring Resistance; Fall of Potential, Ammeter-voltmeter, Wheatstone Bridge, Voltmeter Method—Insulation Resistance by Galvanometer Deflection and Standard Megohm—Location of Faults.

71. Resistance—A Means of Current Control. We have seen in the previous work that the amount of current flowing in a given circuit depends upon the resistance of the circuit and the voltage across it. This is also true of any *Part* of a circuit. We can, therefore, control the amount of current either by controlling the voltage or the resistance, or both. The control of the resistance is generally the easiest, and, in many cases, the only available method. It is therefore necessary to become familiar with the common methods of computing and regulating the resistance of the several parts of a circuit.

For example, in line or "feed" wires of a system, there should be as little resistance to the flow of the current as is practicable. For this reason copper is generally used when much power is to be transmitted. The cost of copper is high, so we do not wish to use any more than is necessary. The wires then should be as small as is practicable. We must make the wire large enough to transmit the current, but small enough to avoid excessive cost of installation. Thus, it is necessary, even before it is put up, to be able to compute the resistance of a given length of copper wire of a certain cross-section area or weight.

72. Circular-Mil Measure. Inasmuch as most wire is drawn *round*, it is inconvenient always to retain the old method of measuring the cross-sectional area in *square inches*. The cross-section is circular, not square; so we use a circular measure, and not a square measure. In circular measure we make use of a *circular unit of area*, instead of a square unit of area. This circular unit of area is a CIRCULAR MIL; symbol, *C.M.* or cir. mil. It is the *area* of a circle whose diameter is 1 mil in length.

The term "mil" always means $\frac{1}{1000}$. As in our coinage 1 mill $=\frac{1}{1000}$ of a dollar, so in distance, 1 mil $=\frac{1}{1000}$ of 1 inch.

One CIRCULAR MIL, then, is the area of a *circle*, the diameter of which is *one mil* or *one-thousandth* of an inch. Just as a *square mil* is the area of a *square* whose sides are 1 mil or one-thousandth of an inch.

The area of a square whose sides measure 5 mils is 5×5 or 25 sq.mils. Similarly the area of a circle whose diameter is 5 mils is 5×5 or 25 circular mils. To find the number of *circular mils* in a *circle* of a given diameter, we have merely to square the number of mils in the diameter.

Examples. (1) Find area in circular mils of a circle of .015 inches in diameter.

.015 in. $=15$ mils.

Area of circle $=15\times15=225$ cir. mils. Ans.

(2) Find area of a circle of .346 in. dia.

.364 in. $=364.0$ mils.

Area of circle $=364\times364=132{,}500$ cir. mils. Ans.

The advantage of measuring circular sections in circular measure is too obvious to be mentioned.

Problem 1–5. Find the cross-section area in C.M. of a round wire .164 in. in diameter.

Problem 2–5. What is the circular mil area of a circle 2.18 in. in diameter?

Problem 3–5. Find the cross-section area of a round wire $\frac{1}{16}$ in. in diameter.

Problem 4–5. A round wire has a cross-section area of 500,000 C.M. What is its diameter in inches?

Problem 5–5. (*a*) Find the area of the circle in Problem 3 in square inches. (*b*) In square mils.

73. Computation of Resistance from Size, etc.; Mil-Foot. Just as in hydraulics the resistance of a large pipe is less than the resistance of a small one, so the electrical resistance of a large wire is less than the resistance of a small wire. Thus if the resistance of a wire of 1 cir.mil cross-section were 20 ohms, the resistance of a wire of 4 cir.mils would be not 4 times as great, but $\frac{1}{4}$ as great, or $\frac{1}{4}$ of 20 = 5 ohms.

Of course the resistance varies directly as the length; a wire 3 times as long as another of same cross-section would have 3 times the resistance. We take for our unit wire, 1 MIL-FOOT. A wire ONE FOOT long, and having a cross-section area of ONE CIRCULAR MIL is called a MIL-FOOT wire. In order, then, to find the resistance of a certain number of feet of wire of given cross-section, we merely have to multiply the resistance of a mil-foot by the length in feet and divide by the sectional area in circular mils. This is evident from the fact that the resistance of a wire *increases* as the length increases, but *decreases* as the section area increases.

This is generally expressed in the form of an equation:

$$R = \frac{Kl}{d^2};$$

where R = resistance of wire in ohms;
K = " 1 mil-ft. in ohms;
l = length in feet;
d = diameter in mils;

or

d^2 = section area in cir.-mils.

This resistance per mil-ft. (K) is sometimes called the Resistivity or Specific Resistance of a material, and varies with different materials. See Appendix.

Example. Assuming the resistance per mil-ft. of copper at 20° C. as 10.4 ohms, what is the resistance of 500 ft. of copper wire, .021 in. in diameter?

$$R = \frac{Kl}{d^2};$$

$$R = \frac{10.4 \times 500 \text{ ft.}}{(21 \times 21) \text{ cir. mils.}} = 11.8 \text{ ohms.}$$

Example. What size copper wire must be used to transmit an electric current 4 miles, if the line wire resistance is not to exceed 2 ohms?

$$R = \frac{Kl}{d^2};$$

$$2 = \frac{10.4 \times 4 \times 5280}{d^2} = \frac{219000}{d^2}.$$

or

$$d^2 = 109{,}500 \text{ cir.mil.}$$
$$d = 331 \text{ mils. diam.}$$
$$d = .331 \text{ in. diam.}$$

Note: Unless otherwise stated, the temperature of wires in all problems is to be taken as 20° C.

Problem 6–5. What is the resistance of a copper wire 4000 ft. long and $\frac{1}{4}$ in. in diameter?

Problem 7–5. What length of copper wire .064 in. in diameter will have a resistance of 8 ohms?

Problem 8–5. What diameter will a wire one mile long have, if the resistance is one ohm?

Problem 9–5. It is desired to transmit 400 amperes to a point 2500 ft. from the generator, with not more than 5 volts "line drop." What diameter copper wire must be used?

Problem 10–5. What current can be transmitted over a copper wire 2 miles long, $\frac{1}{2}$ in. in diameter with but 8 volts "line drop"?

Problem 11–5. What line drop is there in a 4-mile copper trolley wire carrying 200 amperes, if the wire is .325 in. in diameter?

74. Resistivity of Metals other than Copper. Although copper, on account of its low resistivity, is the metal most widely used for electrical conductors, aluminum and even galvanized iron are sometimes used. The resistivity of aluminum is 18.7 ohms per mil-ft. at 20° C., nearly twice that of copper. But its low specific gravity more than counterbalances this, so that for equal lengths and weights aluminum wire has less resistance than copper, and for this reason is coming into more general use.

The resistivity of iron and steel is about seven times that of copper. These materials, therefore, can be used only where a conductor of a large cross-section can be installed, as in the case of a third rail, or where very little current is to be transmitted, as in the case of the telegraph. For the resistivity of various pure metals and alloys, see Appendix.

Problem 12–5. What is the resistance of a mile of aluminum wire, .084 in. in diameter?

Problem 13–5. What diameter would a wire of aluminum have to be to fulfill conditions of Problem 8–5?

Problem 14–5. If a steel rail were used in Problem 9–5, what cross-section (sq.in.) would be necessary?

75. Temperature Coefficient of Resistance. It will be noticed that when the resistance of a mil-ft. of copper wire was given as 10.4 ohms, and of aluminum as 18.7 ohms, that the metal was assumed to be at a temperature of 20° C. The reason for stating the temperature is that the resistance of all pure metals *increases* as the temperature *rises*. The amount that the resistance increases per degree

rise of temperature for each ohm of the resistance at the standard temperature is called the TEMPERATURE COEFFICIENT of RESISTANCE. For all pure metals this coefficient is nearly the same, lying between the values .003 and .006, and depends upon what is taken as the standard or initial temperature.

Taking the resistance at 0° C. as the standard, copper increases .0042 ohm per degree for each ohm at 0°. Thus if the resistance of a copper wire is 80 ohms at 0° C. it will be increased by $80 \times .00420$ or .336 ohm for each degree the temperature rises above 0° C. At 50° C. the resistance would have increased $50 \times .336$ or 16.8 ohms. The resistance then at 50° C. would be the resistance at 0° C. plus the increase or, $80 + 16.8 = 96.8$ ohms. Other metals increase at a slightly different rate. See Appendix.

The resistance of a mil-ft. of commercial copper at 0° C. is variously given, because a very small trace of any impurity raises the resistance perceptibly. According to A.I.E.E. standard tables, the resistance per mil-ft. at 0° C. = 9.55 ohms. According to G. E. tables, the resistance per mil-ft. at 0° C. = 9.60 ohms.

In order to find the resistance per mil-ft. at any other temperature, it is merely necessary to find the increase in resistance due to temperature rise and add this increase to the resistance at 0° C. Thus the resistance per mil-ft. at 25° C. $= 9.6 + (9.6 \times .0042 \times 25) = 10.6$ ohms.

This effect of temperature on resistance of copper may be expressed in the form of an equation:

$$R = R_0(1 + .0042T);$$

where R = Resistance at temperature above or below 0° C.

R_0 = " " 0° C.;

T = Temperature of wire in degrees Centigrade.

Note: When using the temperature coefficient .0042 it is always necessary to use the Resistance at 0° C. as standard. The G. E. Co. has established as a secondary standard the resistance at 25° C.

The coefficient then becomes .00381. And the equation for effect of temperature on resistance becomes:

$$R = R_{25}(1 \pm .00381T);$$

where R = Resistance at temperature above or below 25°;
R_{25} = " " 25° C.;
T = Number of degrees wire is above or below 25° C.

The coefficient .00381 must be used because the standard resistance at 25° C. would be larger than at 0° C. and thus the increase per degree *for each ohm* at 25° C. would be smaller than *for each ohm* at 0° C., though of course the increase for the total resistance would be the same.

Example. The resistance of a copper wire is 2.48 ohms at 25° C. What will the resistance of this wire become at 40° C.?

$$\begin{aligned} R_{40} &= R_{25}(1 + .00381T) \\ &= 2.48(1 + .00381 \times 15) \\ &= 2.48 \times 1.0572 \\ &= 2.62 \text{ ohms.} \end{aligned}$$

Problem 15–5. The resistance of a copper wire is 4.90 ohms at 0° C. What is it at 20° C.?

Problem 16–5. The resistance of the field coils of a generator is 220 ohms at 25° C. When the coils become heated to 75° C. what will the resistance be?

Problem 17–5. What will the resistance of a coil of copper wire become at 7° C. if the resistance is 200 ohms at 25° C.?

Problem 18–5. What will the resistance of a coil of copper wire become at 7° C., if the resistance is 200 ohms at 0° C.?

Problem 19–5. A coil has a resistance of 12.5 ohms at 0° C. How high will the temperature be when the resistance reaches 14.5 ohms?

76. Coefficients for Various Initial Temperatures. It is possible to regard any temperature as the standard, providing the coefficient for that *standard* is used. There is thus a different coefficient for each initial temperature that is used as a standard. These coefficients are given by the A.I.E.E. in the following tables:

TEMPERATURE COEFFICIENTS FOR COPPER

T_i = initial or standard temperature,
a = coefficient, or change in resistance per degree C. for each ohm at temperature T_i.

T_i	a	T_i	a	T_i	a
0	.00420	17	.00392	34	.00368
1	.00418	18	.00391	35	.00366
2	.00417	19	.00389	36	.00365
3	.00415	20	.00388	37	.00364
4	.00413	21	.00386	38	.00362
5	.00411	22	.00385	39	.00361
6	.00410	23	.00383	40	.00360
7	.00408	24	.00382	41	.00358
8	.00406	25	.00381	42	.00357
9	.00405	26	.00379	43	.00356
10	.00403	27	.00377	44	.00355
11	.00402	28	.00376	45	.00353
12	.00400	29	.00374	46	.00352
13	.00398	30	.00373	47	.00351
14	.00397	31	.00372	48	.00350
15	.00395	32	.00370	49	.00348
16	.00394	33	.00369	50	.00347

By means of this table we may always use the same simple equation,

$$R_t = R_i(1 \pm aT);$$

where R_i = resistance at initial temperature;
R_t = " " second "
T = *change* of temperature;
a = coefficient at initial temperature.

Example. The resistance of a coil of copper wire at 19° C. is 240 ohms. What will be the resistance of the same coil at 42° C.?

$$R_t = R_i(1 + aT);$$
$$T = 42° - 19° = 23° \text{ C.}$$

From table, "a" at 19° = .00389;

$$R_t = 240(1 + .00389 \times 23)$$
$$= 240(1 + .0895)$$
$$= 261 \text{ ohms.}$$

Resistance at 42° C. = 261 ohms.

Problem 20–5. The resistance of a field coil is 130 ohms at 20° C. What will it be at 180° C.?

Problem 21–5. What will be the resistance of a copper wire at 10° C. if the resistance at 45° C. is 2.08 ohms?

77. Temperature Measured by Resistance. Electrical machines are generally sold under a guarantee that the wire in the coils, etc., will not rise more than a given number of degrees, when running under specified load for a specified time.

By measuring the resistance of the coils when at room temperature and then again at close of run, and applying the equation for temperature effect, the temperature rise can easily be found.

Example. The primary coils of a transformer have a resistance of 5.48 ohms at 25° C. After a run of 2 hours, the resistance has risen to 6.32 ohms. What is temperature rise of the coil?

The equation

$$R_t = R_{25}(1 + .00381T)$$

can be transformed to

$$T = \frac{R_t - R_{25}}{.00381 R_{25}};$$

when $R_t = 6.32 =$ resistance at high temperature;
$R_{25} = 5.48 =$ " " 25° C.;
$T = (?)$

$$T = \frac{6.32 - 5.48}{.00381 \times 5.48};$$

$$T = 40.3.$$

Temperature rise therefore is 40.3° C.

Problem 22–5. The cold (20° C.) resistance of an armature was 2.18 ohms. The hot resistance was 2.56 ohms. What was the temperature rise?

Problem 23–5. An electric soldering iron has a resistance composed of iron wire of 80 ohms when cold (20° C.). When hot the resistance rises to 150 ohms. What temperature is reached, assuming a constant temperature coefficient?

78. Temperature Coefficient of Alloys, etc. It has been stated that the temperature coefficient of resistance for all pure metals is nearly the same, that is, somewhere about .004. Alloys, though of a much higher resistance per mil-ft. have much lower coefficients, some having " O " and even negative coefficients at certain temperatures.

" Manganin," an alloy consisting of copper, nickel, and iron-manganese, for instance, has a resistance per mil-ft. of from 250 to 450 ohms according to the proportions of the different metals used, and a temperature coefficient so low as to be practically negligible.

Certain substances, notably carbon, porcelain, glass, etc., decrease in resistance very rapidly when heated. The cold resistance of a carbon lamp filament is about twice as great as the " hot " resistance. The porcelain " glower " of a Nernst lamp when cold is a good insulator, but when heated to incandescence it becomes a conductor. The filaments of the new tungsten lamps are pure metal and accordingly have a positive coefficient, which is about .0051.

79. Copper-Wire Tables. Tables have been prepared by wire manufacturers and by the A.I.E.E. which give the resistance of 1000 ft. or of a mile of copper wire of different standard sizes, and several temperatures. The sizes are designated by gauge numbers, diameter in mils, and section area in cir. mils, etc. There are several standard wire gauges, B. & S. (Brown & Sharpe) in general use in America, B. W. G. (Birmingham Wire Gauge) in general use in Great Britain. See Appendix for B. & S. table.

By means of these tables it is very easy to find the resistance of any length of wire of a given section area, etc.

Example. What copper wire (B. & S. gauge) should be used to transmit electric power 2 miles (out and back); resistance not to exceed 2.7 ohms; temperature to be assumed, 20° C.?

$$2 \text{ miles} = 2 \times 5280 = 10{,}560 \text{ ft.}$$

$$2.7 \text{ ohms for 2 miles} = \frac{2.7}{10.56} \text{ ohms per thousand ft.}$$

$$= .256 \text{ ohms per thousand ft.}$$

From wire table:

No. 5 = .3128 ohm per thousand ft.;
No. 4 = .2480 " " " "

No. 4 must be used in order not to exceed limit of .256 ohm per thousand ft.

Example. Suppose line in above Example is to work at a temperature of 45° C., what number wire will be required?

The resistance is not given in the tables at 45° C., but at 20° C. We must then find out what resistance a wire will have at 20° C., which has 2.7 ohms at 45° C.

According to the equation,

$$R_{20} = R_{45}(1 - aT).$$

From above table:

a (for 45°) = .00353;
$T = 45° - 20° = 25°$;
$R_{20} = 2.7\ (1 - .00353 \times 25)$;
$= 2.7(1 - .0883)$
$= 2.46$ ohms for the two miles;

or

.233 ohm per thousand ft.

The problem then becomes: What size wire has .233 ohm per thousand ft. at 20° C.?

From table:

No. 4 has .248 ohm per thousand ft.
No. 3 " .1967 " " " "

No. 3 must be used in order not to exceed limit of .233 ohm per thousand ft.

Problem 24–5. What size wire (B. & S.) is that mentioned in Problem 8–5?

Problem 25–5. What size is trolley wire in Problem 11–5?

Problem 26–5. What size wire must be used to transmit 40 amperes from generator to lamps, a distance of 600 ft. with 2 volts drop in line? Line to work at 30° C.

80. Standard Ohm. Measurement of Resistance. The standard ohm has already been defined as the resistance

of a column of pure mercury at 0° C., 106.3 cms. long, of uniform cross-section and weighing 14.4521 gms.

This piece of apparatus is difficult to construct and to operate, so several secondary standards have been adopted, such as a copper coil in oil. The value of an unknown resistance can be found by comparing it with one of these secondary standards, by means of the following methods:

Fall of Potential

There are needed for this method: (1) a *known* or *standard* resistance R, (2) a voltmeter (V), and (3) a source of power. See Fig. 104.

The known resistance (R) and unknown (R_x) are connected to the power in series, so that there will be the same current flowing through each. The voltmeter is placed to read the voltage (v_1) across R, and then to read the voltage (v_2) across R_x. Since the same current is flowing in each resistance, the voltage across each must be in the same ratio as the two resistances. That is, if the voltage v_2 is twice the voltage v_1, then the resistance R_x must be twice R, for we have seen that it requires twice the voltage to force the same current through twice the resistance. We place the coils in series in order to be sure that there is the same current in each. We have seen in Chapter III that this relation may be expressed as an equation:

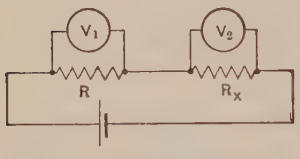

Fig. 104. Method of measuring resistance by voltmeter and standard resistance.

$$\frac{R_x}{R} = \frac{V_2}{V_1}.$$

Thus,

$$R_x = \frac{V_2 R}{V_1}.$$

Since R, v_2, and v_1 are known quantities, the value of the unknown resistance (R_x) may be found.

The advantages of this method are:

(1) The voltmeter need not be accurately calibrated, providing the deflections are proportional to the voltage.

(2) It is an accurate method of measuring *low* resistances if the voltmeter has a high resistance, or is replaced by a galvanometer. This shunts very little current around the series resistance.

(3) By means of a potentiometer in place of the voltmeter, no current is shunted around the resistance and thus very low resistances can be measured accurately.

81. Voltmeter—Ammeter. An unknown resistance (R) may be placed in series with an ammeter (A) and the voltage across it be found by means of a voltmeter (V), Fig. 105. Then $R=\frac{E}{I}$ (Ohm's law). This requires that both ammeter and voltmeter be accurately calibrated. The voltmeter must also be of high resistance. If there is not much current in the circuit, the voltmeter is generally connected around *both R and A*. The ammeter would then be a *millammeter*. If a millammeter were connected as in diagram, it would read the sum of the currents going through both R and V, which would be appreciably larger than that through R alone. On the other hand, if the voltmeter were put across R and A, the fall across A would be too small to be indicated and thus only the voltage across R would be read. See Chapter XIV.

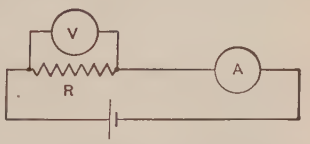

FIG. 105.—Method of measuring resistance by voltmeter and ammeter.

This method has the advantage of making use of the common instruments, ammeter and voltmeter, and is used wherever extreme accuracy is not desired.

82. Wheatstone Bridge. By far the most important apparatus for accurately measuring resistance is the well-known Wheatstone bridge.

It consists fundamentally of a *loop* of four resistances, R, R_1, R_2, R_3, one of which is unknown. Fig. 106. An

electric current, generally from a battery, is sent into the loop at B, and immediately divides into two parts, one part taking the branch composed of the resistances R and R_2, the other part taking the branch made up of R_1 and R_3. Both branches come together again at D, and return the current to the battery. A galvanometer G is now "bridged" between the two branches from A to C.

Assume R_3 to be the unknown resistance. R, R_1, R_2 are variable resistances, and are now adjusted until, on closing the key in the circuit of the galvanometer, it gives no deflection. This is called "balancing" the bridge. The points A and C are then at the same level or potential. For we have seen that where there is any difference of potential between two points, a current always flows when they are joined by a conductor. Since the points A and C are joined through the galvanometer, and no current flows, they must be at the same potential.

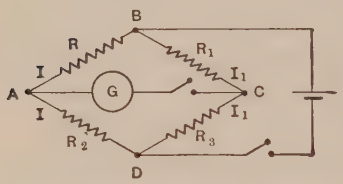

FIG. 106.—Wheatstone bridge, diamond form.

Let us now consider the voltages across the separate resistances. We have said that the current divides into two branches, BAD and BCD. Now the current in each branch need not be the same, so let us designate the current in the branch BAD, by the letter I, and the current in the branch BCD, by the letter I_1.

Then the voltage across BA (that is, R) $= IR$;
" " " " BC (that is, R_1) $= I_1R_1$.

But we have said that the points A and C are at the same level (since no current flows through the galvanometer) and therefore the voltage drop from B to A (IR) = the voltage drop from B to C (I_1R_1). That is,

(1) $$IR = I_1R_1;$$

For the same reasons, the voltage drop from A to D, (IR_2) must equal the voltage drop from C to D (I_1R_3); that is,

$$IR_2 = I_1R_3. \tag{2}$$

Dividing equation (1) by equation (2) we get,

$$\frac{IR}{IR_2} = \frac{I_1R_1}{I_1R_3}.$$

Cancelling the I's and I_1's,

$$\frac{R}{R_2} = \frac{R_1}{R_3}.$$

This is the fundamental equation of the Wheatstone bridge. Since R_3 is the only unknown, the equation can be solved and the value of R_3 found.

The ratio $\frac{R}{R_2}$ is generally made to equal some such fraction as $\frac{1}{10}$, and then it follows that $\frac{R_1}{R_3} = \frac{1}{10}$. R_1 is called the Rheostat arm. This makes R_3 10 times the value of R_1. If R_2 is made the same as R, then R_3 will be the same as R_1.

In writing the equation for the bridge, notice that we started at B (a point where the battery line came in). For one side of our equation we read along one branch. That is, we said, "R is to R_2" or $\frac{R}{R_2}$. For the other side of the equation we *went back to the point B again* and read along the other branch, saying, "R_1 is to R_3" or $\frac{R_1}{R_3}$. We did *not continue around* the loop and make the second side of the equation "R_3 is to R_1." Had we done so, the two ratios would have been unequal and we could have formed no equation.

It is to be noticed then, that in making the fundamental equation for a Wheatstone bridge, we read along one branch for one side of the equation, then go back and read along the other branch for the other side of the equation.

It is interesting, and of value, to know that it makes no difference at what point we start, providing we start at the *same point for each side of the equation.* Thus suppose we start at A. Reading along the upper branch, we have, "R is to R_1" or $\frac{R}{R_1}$, for one side of the equation. Then coming back to the point A and reading the lower branch we have, "R_2 is to R_3." The equation, then, $\frac{R}{R_1}=\frac{R_2}{R_3}$ is just as true as the first and as easy to solve.

If we start at the point C, we have "R_3 is to R_2" or $\frac{R_3}{R_2}$, "R_1 is to R," or $\frac{R_1}{R}$, that is, $\frac{R_3}{R_2}=\frac{R_1}{R}$, which equation is also true.

The important details about measuring resistance with a Wheatstone bridge are:

(1) Make a complete loop of *four resistances.*

(2) Connect battery terminals (through key) to two points that are not adjacent, that is, each branch of the current must always flow through *two resistances* before returning to the battery.

(3) Connect galvanometer through key across the two remaining points.

(4) In forming the equation, start at any point and read along two resistances for one side of the equation; then come back to *same point,* and read along the other two resistances for the other side of the equation.

Problem 27–5. In a Wheatstone bridge arranged as in Fig. 106, a balance is obtained when $R=10$ ohms, $R_1=45$ ohms, $R_2=100$. What is the value of R_3?

Problem 28–5. In Fig. 106, if $\frac{R}{R_2}=100$, and $R_3=4.26$, what is the value of R_1?

Problem 29–5. What value must R_2 have, if R is 1000 ohms and R_1 is 48 ohms and it is desired to measure R_3, which is known to be about 500 ohms?

83. Instructions for Use of Wheatstone Bridge.

1. Never hold the bridge plugs in the hand or place them where they are liable to come in contact with oils or acids. They may

become covered with oil and the "plug resistance" thus increased, or may be corroded so that they do not properly fit the sockets.

2. Insert all plugs firmly, with a twisting motion. Do not, however, insert them with force enough to strain the top.

3. Always put a key in the battery circuit and one in the galvanometer circuit. Close the *battery circuit first*, then the circuit through the galvanometer. In breaking the circuit, open the key in the galvanometer circuit first.

4. In closing the galvanometer circuit, *make only slight contact* with the key until balance is nearly secured. The galvanometer may thus be protected from heavy currents. If the galvanometer is very sensitive, put a shunt across its terminals until the bridge is nearly balanced.

5. In making a resistance measurement proceed in the same manner as in weighing with a chemical balance. First put in a coil estimated to be about correct for the resistance being measured. If this is too low, use one twice as great, if too high, one one-half as great. Fix in this way two limits between which the resistance lies. Then systematically bring these limiting values closer and closer together, until the nearest balance on the bridge is obtained. It will not, in general, be possible to secure an exact balance, but two values for the resistance in the rheostat arm may be found which will give steady deflections of the galvanometer needle in opposite directions. The correct value lies between. From the deflections of the galvanometer needle in the two cases, interpolate for the resistance which will give no deflection. Thus, suppose the smallest coil on the bridge is 0.1 ohm. With this added, a steady deflection of the galvanometer of 2 divisions to the *right* is given. Without it, the deflection is 3 divisions to the *left*. 0.1 ohm makes a difference, therefore, of 5 divisions, and the resistance which would give no deflection is therefore $\frac{3}{5} \times 0.1 = 0.06$ ohm.

6. When balance has been obtained, reverse the current through the bridge and see if the balance is maintained.

84. Slide-wire Bridge. Sometimes a single straight wire of uniform section and of high resistance takes the place of the resistances R and R_2 as in Fig. 107. The point A is then a sliding contact piece. R is the resistance of the straight wire from B to A, and R_2 the resistance from A to D. In this form the bridge is called the slide-wire bridge; a balance being obtained by sliding the contact maker A along the wire BD, thus decreasing or increasing R or R_2 as desired. The ratios are read as before, the lengths

BA and AD being used instead of the actual resistances, since the resistance is proportional to the length.

This form has the advantage that a balance can be obtained very quickly. It is not as accurate as the other forms, however, because of the wearing of the wire, and of the necessarily low resistance of R and R_2.

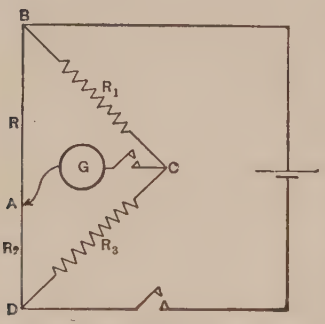

FIG. 107.—Wheatstone bridge, slide-wire form.

Sometimes R_1 is a permanent resistance coil, and the wire is so calibrated that the resistance of R_3 is read directly from the position of the slider A on the wire. It is then called an "Ohmmeter." A telephone receiver in this case generally takes the place of the galvanometer.

Resistance coils are often placed in R and R_2 to increase the accuracy of this form of bridge, though by so doing the range is lessened.

Problem 30–5. In a bridge arranged as in Fig. 107, BD is 100 cms. long, R_1 5 ohms. If bridge balances when BA is 37.5 cms., what value is R_3?

Problem 31–5. If $R_1=16$ and $R_3=90$ (Fig. 107), what must the values of BA and AD be? $BD=100$ cms.

85. Location of Faults in Cables, etc. It often happens that a telephone conductor becomes "grounded." It is then very desirable to locate the distance of this "fault" from one end or the other of the line before starting out from the office to repair it. Where the wire is laid underground, it would be too costly a proceeding to inspect any great length of cable, and thus very refined methods have been worked out for locating such faults to a great degree of precision. There are two methods in common use, called the Murray loop and the Varley loop. They are both modifications of the Wheatstone bridge, and therefore the underlying principles stated above apply to them.

86. Murray Loop. This test ordinarily makes use of the slide-wire form of the bridge and is arranged as in Fig. 108.

There is a "ground" at F in the conductor DC, of a line between two cities. A good wire BC, which happens to lie between the two cities and to be exactly like DC, is joined to DC at C. By means of the slide-wire BD, a complete loop of resistances is formed. The battery is connected to the slider A and to the ground. This is the same as connecting in the battery at A and F, since the ground serves as a conductor. The galvanometer is placed across BD. We now have a regular slide-wire Wheatstone bridge, as in Fig. 107; the resistance of the slide-wire R_2 and R forming one side of the bridge, while R_3 (the resistance out to the fault) and R_1 (the resistance through good wire to fault) form the other side.

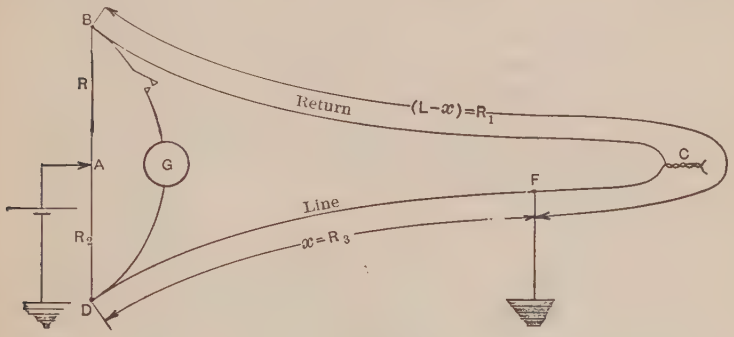

FIG. 108.

The bridge is balanced by moving A along the wire till there is no deflection of the galvanometer. Then the regular equation of the Wheatstone bridge holds true:

$$\frac{R_2}{R}=\frac{R_3}{R_1}.$$

If we let L equal the length of entire loop, which would be known, and let x equal distance out to fault, then the equation would become,

$$\frac{R_2}{R}=\frac{x}{L-x}.$$

As x is the only unknown quantity, it can easily be found as follows:

$$x=\left(\frac{R_2}{R+R_2}\right)L.$$

But $R+R_2$=the entire length of the slide-wire.

This shows that x is the same fraction of the total length of the loop that R_2 is of the total length of the slide-wire.

If the "good" wire BC is not of the same size and material as the "faulty" wire, the resistance of the loop BCD is first found in the usual way by a bridge. It is then connected up as in Fig. 108, and worked out as above, with the exception that L stands for the *resistance* of the entire loop instead of for the *length* of it, and x for the *resistance* out to fault. If the size of the "faulty" wire is known, the distance out to the fault is readily computed.

In the place of the slide-wire BD, two sets of resistance coils are often substituted.

87. Varley Loop. In the Varley loop, Fig. 109, instead of varying the ratio of R_2 to R as in the Murray form of bridge, these

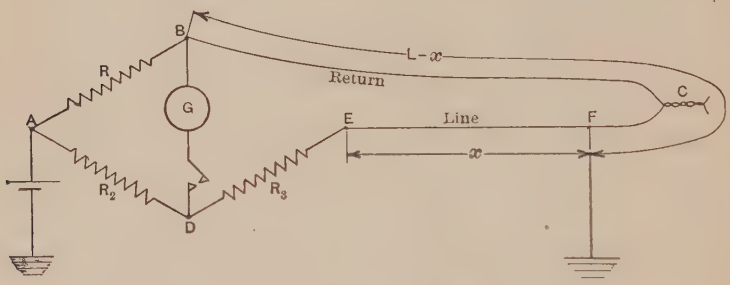

Fig. 109.—Varley loop for locating fault.

resistances are left constant and a balance is obtained by means of a third set of resistance coils R_3 inserted in series with the faulted cable. The resistance of the total length of the good and bad cables L, is either known or first found as in Murray loop.

The value of x (resistance out to fault) is then found by the equation of the Wheatstone bridge:

$$\frac{R_2}{R} = \frac{R_3 + x}{L - x};$$

$$x = \frac{R_2 L - R R_3}{R + R_2}.$$

As all the values in the right-hand member of the equation are known, the value of x may be found and the distance out to the fault F be computed from the resistance per foot of the cable.

An interesting variation in the use of the Varley loop is shown in Fig. 110.

The two cables must here be "twin" wires. Then N represents the resistance out to fault on the bad wire. Thus N would also represent the resistance of an equal distance out on the good cable. Now let x represent the resistance of the bad cable from fault F to *far* end. Then x will also represent the resistance of an *equal length* of the good cable. The resistances R_2 and R are made *equal* and the bridge is balanced by regulating R_3 as before. The equation then becomes:

$$\frac{R_2}{R} = \frac{R_3 + N}{2x + N}.$$

But as $R_2 = R$, we have,

$$R_3 = 2x.$$

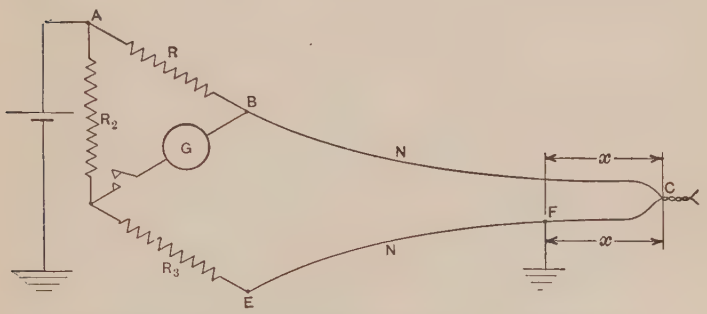

FIG. 110.—Varley loop; for locating a fault in "twin cables."

Thus x equals $\frac{1}{2}$ of whatever we have to make R_3 in order to balance the bridge. This gives a very rapid method of finding the location of a ground in a telephone circuit, and is as accurate as the resistances used in R, R_2 and R_3.

Problem 32–5. A Murray loop is arranged as in Fig. 108 with a meter slide-wire. $BA = 65$ cms. when bridge is balanced. The length of the line between the two cities is 2 miles. The return wire is exactly like the faulted wire. How far from test end is fault F?

Problem 33–5. In a Murray loop arranged as in Fig. 108, the resistance of the loop = 44.2 ohms. The line wire DC is No. 18 B. & S. copper. If bridge balances with AD equal to 32 cms., how far from test end is fault F?

Problem 34–5. A Varley loop in which the resistance of the twin cables is 44.8 ohms is arranged as in Fig. 109. A balance is

obtained when $R=10$ ohms, $R_2=100$ ohms, $R_3=357$ ohms. If the faulted wire is copper, No. 14 (B. & S.) how far from E is fault?

Problem 35–5. A Varley loop is arranged as in Fig. 110 with the return the same as the faulted line, No. 12 (B. & S.) copper. $R=R_2$; $R_3=4.12$ ohms. How far from C is fault F?

Problem 36–5. If wire in Problem 35–5 is iron and 50 ohms to the mile, how far will F be from C?

88. Voltmeter Method for High Resistances. It is often desirable to know whether a certain resistance comes within or exceeds a given value; as, for instance, the insulation resistance between the commutator and the frame of a motor. A voltmeter of known resistance R_v is placed across a constant source of power as AB, Fig. 111, and voltmeter reading v is noted.

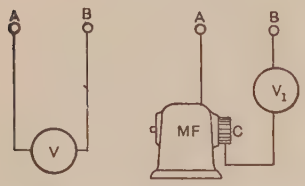

FIG. 111.—Voltmeter method of measuring insulation resistance.

The frame of the motor MF is then connected to terminal A, and commutator segment C is connected through the voltmeter to the other terminal B. The reading v_1 of the voltmeter is then taken. Since the motor-insulation and voltmeter are in series, the voltage of AB is distributed across the motor and voltmeter in direct proportion to their separate resistances. The voltmeter reads the voltage across itself as always, thus v_1 = voltage across the voltmeter. The voltage across the motor-insulation then equals the voltage across AB, v, minus voltage across voltmeter, v_1. That is,

$$\text{Voltage across voltmeter} = v_1;$$
$$\text{Voltage across motor-insulation} = v - v_1.$$

Since the two are in series, the resistances of each are proportional to the voltages across each.

$$\frac{\text{Insulation } R \text{ of motor}}{R \text{ of voltmeter}} = \frac{v - v_1 \text{ (volts across motor-insulation)}}{v_1 \text{ (volts across voltmeter)}}.$$

Let x = (insulation) resistance of motor. Equation then becomes,

$$\frac{x}{R_v} = \frac{v - v_1}{v_1}.$$

This equation can be solved for x, since all other values are known.

$$x = R_v\left(\frac{v}{v_1} - 1\right).$$

A deflection of 1 volt for v_1 would give x the following value:

$$x = R_v(v - 1).$$

If no deflection is noticed when the voltmeter is connected in series with motor-insulation, we know that the insulation is at least greater than $R_v(v-1)$. Thus if the resistance of the voltmeter R_v is 15,000 ohms, and the voltage across AB, (v), is 111 volts, we would know, that in case of no deflection, we had at least $15{,}000 \times 110 = 1{,}650{,}000$ ohms insulation resistance between frame and commutator.

89. Insulation Resistance of Covered Wire, etc. It is required by fire insurance companies that insulated wire of various grades shall be up to a certain standard insulation resistance per mile after being soaked in salt water for a stated length of time. The following method is the common practice for finding the ohmic resistance of the insulation. In Fig. 112 a known high resistance, as 100,000 ohms, is connected in series with a shunted galvanometer G across a constant potential source of power AB. The resistance of the galvanometer and of the shunt S are known. The deflection of the shunted galvanometer is noted, and by means of the resistances of the galvanometer and shunt it is possible to compute what the deflection would be without the shunt.

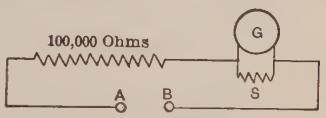

FIG. 112.—Insulation resistance. Standardizing the galvanometer.

For example, suppose the following data to be obtained:

Deflection = 80 scale divisions.
Resistance of G = 2000 ohms.
" " S = 10 "

Since the S and G are in parallel, the current will divide between in inverse proportion to the resistance; that is, the greater part of the current will go through the smaller resistance S.

Assume the current to be divided up into 2010 parts; then 2000 parts flow through the shunt and 10 parts through G. That is, $\frac{10}{2010}$ parts of the current go through the galvanometer, and therefore the deflection of 80 is only $\frac{10}{2010}$ (or $\frac{1}{201}$), as large as it would be if it were unshunted.

An unshunted galvanometer would then deflect $201 \times 80 = 16{,}080$ divisions, when connected in series with 100,000 ohms, and the same current were flowing in the line. If there were 1,000,000 ohms (1 megohm) in series, the unshunted galvanometer would deflect only $\frac{1}{10}$ as much (the more resistance the less current). That is, the deflection would be $\frac{1}{10} \times 16080 = 1608$ (the deflection for 1 megohm). This is called the *galvanometer constant*. The shunt is now removed from the galvanometer as in Fig. 113. The coil C, immersed in salt water, is put in place of the 100,000 ohms, one connection being made to the end y of the coil, and the other to the metal tub. The other end x of the coil is kept above the water, so that any current which goes through G must leak through insulation of the wire coil into the water.

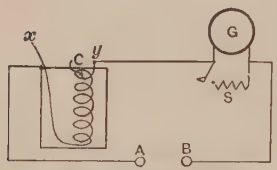

FIG. 113.—Insulation resistance.

Assume the galvanometer now reads 16. Since a deflection of 1608 means a megohm resistance in the line, a deflection of only 16 means a much greater resistance than 1 megohm in the line. In fact, the deflections are inversely proportional to the resistance of the line. Thus the unknown insulation resistance x is as much greater than one megohm as 1608 is greater than 16.

$$x = \frac{1608}{16} = 100.5 \text{ megohms.}$$

This is the insulation resistance of the coil.

If the coil were $\frac{1}{4}$ mile long the insulation resistance per mile would be only $\frac{1}{4}$ as great, or 25.1 megohms. This is because the current would have 4 times as much area to leak through in a mile as in $\frac{1}{4}$ mile.

Notice that the galvanometer constant 1608 is not used as a *multiplier* but as a *dividend*, which is *divided* by the deflections.

Note that in this test the resistance of the battery and galvanometer are neglected because they are too small in comparison with the resistance of the insulation and standard box to appreciably affect the results. When the standard 100,000-ohm box above

is in the line, the galvanometer is shunted so that it has less than 10 ohms resistance, which is one ten-thousandth of the total resistance of the circuit. When unshunted in the second part of the test, the resistance of the galvanometer is but one fifty-thousandth of the total line resistance. In fact, the standard resistance of 100,000 ohms might be left in the line in the second part as well, and its resistance neglected, so small is it in comparison with the insulation resistance.

Problem 37–5. In a test, as in Paragraph 89, for the insulation of a coil of wire the following data were taken:

Resistance of Gal. = 3000 ohms;
" " Shunt = 200 ohms.

Deflection of shunted Gal. with 200,000 ohms in series = 18.4.
Deflection of unshunted Gal. with the resistance of the insulation in series = 4.3.
Length of coil = 500 ft.
(*a*) What is the galvanometer "constant?"
" " " insulation resistance per mile of cable?

90. Construction of Various Types of Bridges. The appearance of the regular "Post-office" form of Wheatstone bridge is shown in Fig. 114.

FIG. 114.—Wheatstone bridge, "post-office" form.

The resistance wire is wound non-inductively on spools as in Fig. 116. The ends of the wire are now connected across the gap between two adjacent brass blocks, as shown

in Figs. 115 and 116. The construction of one such coil and

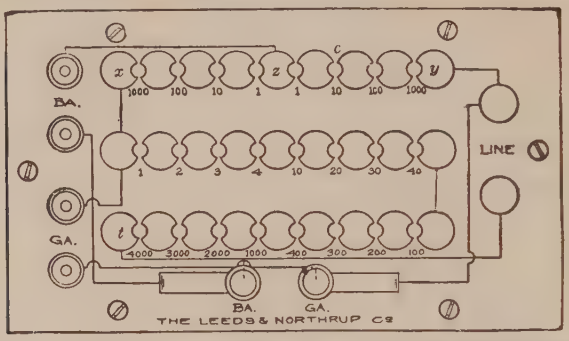

FIG. 115.—Diagram of connections of P. O. form of Wheatstone bridge.

set of blocks is shown in Fig. 116. When the brass plug is inserted in the tapered hole between the blocks, it forms a short circuit around the coil and "cuts the coil out" of the circuit. In order to put a certain amount of resistance into a circuit, say between z and y, Fig. 115, it is merely necessary to remove the plug, which short-circuits a coil of the desired resistance. Thus the plug being removed from hole C, the current must go through the coil C of 10 ohms in flowing from z to y or vice versa. Binding posts are connected to certain of the blocks as x, y, z and t, so that the bridge can be divided into three parts. Thus from z to y may be one ratio, from x to z the other ratio,

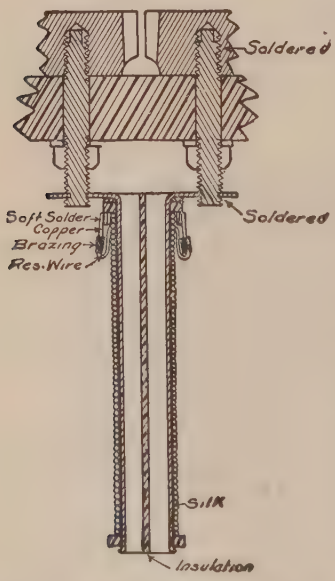

FIG. 116.—Details of connection of non-inductive coil to blocks on bridge top.

and from x to t, the variable resistance by which the bridge is balanced. Of course the unknown resistance has to be connected across the end from y to t of the bridge in order to complete the loop of four resistances.

In Fig. 117 is shown the appearance of the "Decade" form of bridge. In this type, resistance is introduced into

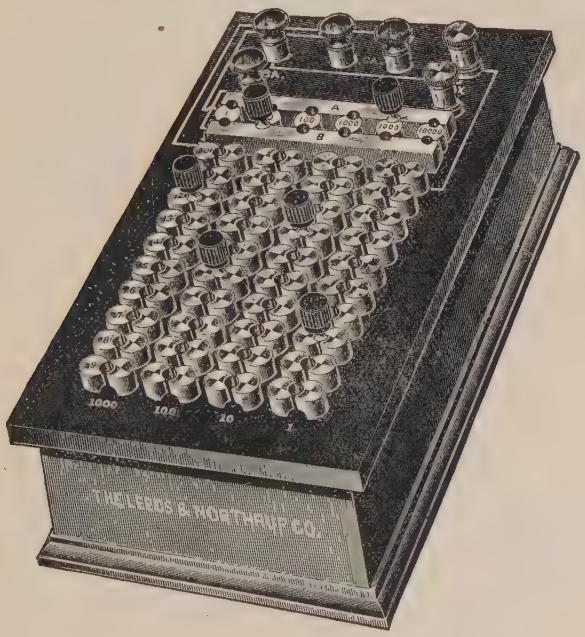

FIG. 117.—Wheatstone bridge, "decade" form.

a circuit by placing the *plug in the hole*, instead of by removing the plug. The details of the construction are shown in Fig. 118.

The coils composing the variable resistance for balancing the bridge are arranged in such an ingenious way that 10 variations in value in each "decade" or row are made by the use of only 4 coils. The diagram shows the details

of this scheme. The student will be interested in studying out for himself how the different values are obtained by "plugging in" at the proper place. In a bridge where there are four "decades," the same arrangement and ratios among coils is made for each decade, the difference

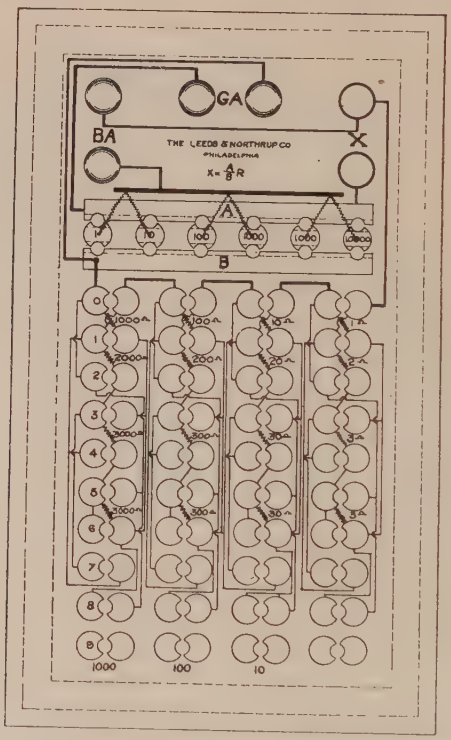

FIG. 118. Diagram of connections in decade form of bridge.

being that one decade represents 1000-ohm steps, the next 100-ohm steps, etc. This gives a total range in the four decades of 9999 ohms, by one-ohm steps.

The ratio arms are made by "plugging in" between bar *A* and any block for one arm, and between bar *B*, and any *other block* for the other arm.

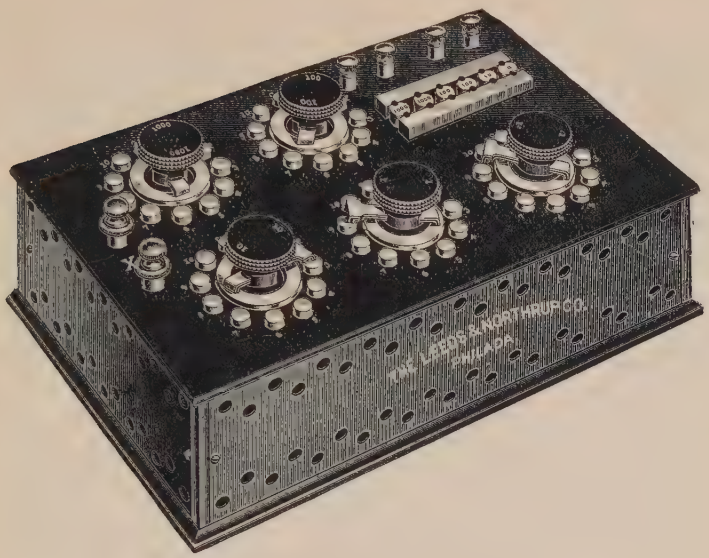

FIG. 119.—Wheatstone bridge, dial form.

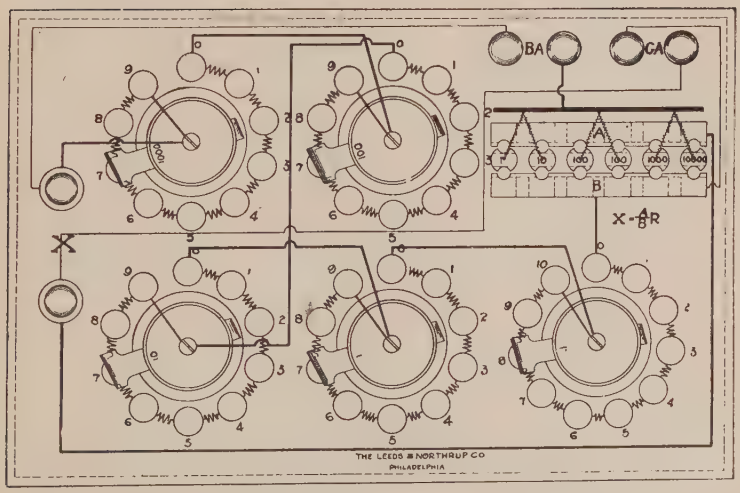

FIG. 120.—Diagram of connections in dial form of bridge.

The arrangement of the resistance coils which compose these arms is clear from the diagram.

The " Dial " form of bridge is shown in Fig. 119. It is seen by inspecting Fig. 120, that the ratio arms are formed as in the " decade " style. The balancing resistance is adjusted by turning the brushes, by means of the knurled head, from block to block on the dials. The resistance coils are connected between these blocks as per Fig. 120.

By means of this form of bridge a balance may be obtained in the shortest possible time.

The galvanometers used in connection with these bridges, are described in detail in Chapter XIV.

SUMMARY OF CHAPTER V

MIL-FOOT. A round wire one foot long and one circular mil cross-section area.

A circular mil is the area of a circle one mil, i.e., one-thousandth of an inch, in diameter.

The area of a circle in circular mils equals the square of the diameter in mils. $A = d^2$.

SPECIFIC RESISTANCE. The resistance of a mil-foot of any material. (Sometimes also taken as the resistance of a centimeter cube, from face to face opposite.)

The specific resistance of commercial copper at 20° C. = 10.4 ohms. Symbol = K.

RESISTANCE OF WIRE. The resistance of a wire equals the specific resistance (resistance per mil-foot) multiplied by the length in feet and divided by the section area in circular mils.

$$R = \frac{Kl}{d^2}.$$

For copper at 25° C.

$$R = \frac{10.4l}{d^2}.$$

TEMPERATURE COEFFICIENT OF RESISTANCE. For every degree centigrade rise in temperature, the resistance of a pure copper wire increases .0042 of its resistance at 0° C. This constant .0042 is called the Temperature Coefficient of Resistance. It has a much lower value in alloys, and a negative value in carbon, porcelain, etc. If coefficient of metals is based on the resistance at some point above 0° C. it becomes smaller. See table. Other pure metals have about the same coefficient as copper.

RESISTANCE OF COPPER AT TEMPERATURE (T). Based on resistance at 0° C.,

$$R_t = R_0(1 \pm .0042T).$$

Based on resistance at 25° C.,

$$R_t = R_{25}(1 \pm .00381T).$$

Based on any initial temperature (T_i),

$$R_t = R_i(1 \pm aT),$$

where T = change in temperature.

TEMPERATURE RISE MEASURED BY RESISTANCE RISE. Any one of the above equations can be used to find the temperature rise of copper wire, when hot and cold resistances are known.

$$T=\frac{R_t-R_0}{.0042R_0},$$

or

$$T=\frac{R_t-R_{25}}{.00381R_{25}},$$

or

$$T=\frac{R_t-R_i}{aR_i}.$$

METHODS OF MEASURING RESISTANCE.

(1) Fall of potential.
  (a) Standard resistance and voltmeter which need not be accurately calibrated to read volts.
  (b) Accurate method for measuring low resistance if voltmeter has high resistance or if a potentiometer is used.

(2) Voltmeter-ammeter.
  (a) Makes use of instruments in general use, but which must be accurately calibrated.

(3) Wheatstone bridge.
  (a) Three standard variable resistances and galvanometer. Has advantage of being a "no-deflection" method, thus galvanometer need not be calibrated. A slide wire may take the place of two of the resistances.
  (b) Modifications of this bridge are used in the Murray and Varley "loop" methods of locating faults in cables.

(4) Voltmeter, for high resistance.
  A simple method of ascertaining whether or not insulation resistance between different parts of a machine exceeds a certain minimum.

(5) Insulation resistance of covered wire.
  Galvanometer "constant" found by noting deflection when a standard resistance takes the place of the coil. Galvanometer deflections must be proportional to current through it. Source of power must be of constant potential. Wire must be soaked for 24 hours.

PROBLEMS ON CHAPTER V

38–5. A copper wire is 500 ft. long and .038 in. in diameter. How many volts will it take to send 1.5 amperes through it?

39–5. What will be the drop per mile in a line wire consisting of No. 10 (B. & S.) copper carrying 32 amperes?

40–5. What will be the line loss in voltage and in watts per mile in transmitting 10 K.W. at 550 volts if a No. 00 copper wire is used?

41–5. A group of incandescent lamps takes 12 amperes. The line loss is not to exceed 1.6 volts. What must be the size of the copper wire to be used if the lamps are 1500 ft. from the generator?

42–5. What size must line wires be in Problem 38–3 if motor is 500 ft. from generator?

43–5. If wire used in line in Problem 42–3 is No. 6 (B. & S.), how far must lamps be from generator?

44–5. Distance between L and M_2 is 200 ft., Problem 51–3. What size is line wire between motors? Temperature of wire = 50° C.

45–5. To what size wire, B. & S., are feeder and trolley, joined as in Problem 57–3 equivalent?

46–5. A rough rule for the safe carrying capacity of copper is, "1000 amperes per sq.in. cross-section." According to this rule what should be the diameter of a round wire capable of carrying 250 amperes?

47–5. According to rule in Problem 46, what should be safe carrying capacity of No. 0000 (B. & S.)?

48–5. At 65° C., what is the resistance per 1000 ft. of No. 6 copper wire?

49–5. A trolley wire consists of No. 0 hard-drawn copper. What will be the drop per mile on a day when the temperature of the wire is −10° C., if the wire carries 45 amperes?

50–5. A generator is supplying 10 amperes to 50 arc lamps in series. Drop across each lamp = 45 volts. Twelve miles of No. 10 wire are used for line. Find brush potential of generator if temperature of wire is 25° C.

51–5. If generator has 65 ohms resistance and line wire is at temperature of −5° C., what E.M.F. and brush potential must be used in Problem 50?

52–5. Compute efficiency of transmission in Problems 50 and 51.

53–5. A 110-volt system has an insulation resistance for each wire of 200 megohms per mile. What will the leakage be on a 5-mile line?

54–5. Insulation resistance should be high enough so that not more than one-millionth of the rated current leaks through the insulation. On this basis, what should be the insulation per mile of a 2-mile line, transmitting 120 K.W. at 550 volts?

55–5. According to rule in Problem 46, what size wire B. & S. should be used in system of Problem 54?

56–5. In Fig. 121 the wires used are No. 6 B. & S., copper.

$AB = 1500$ ft.;
$BC = HG = 700$ ft.;
$CD = GF = 700$ ft.;
$EF = 100$ ft.

Each lamp takes 2 amperes.
Find:

(1) Line loss in each section.
(2) Voltage across each group.
(3) Efficiency of transmission.

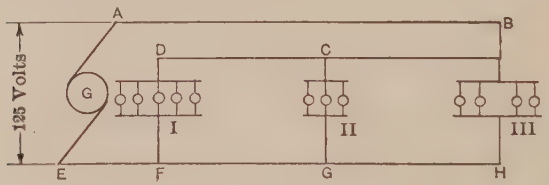

FIG. 121.

57–5. A line consisting of copper wire has a resistance of 2 ohms at 0° F. What is its resistance at 85° F.?

58–5. It is generally specified that the temperature of the field coils of a dynamo must not rise more than 50° C. on full load. The resistance of a set of field coils before running was 80 ohms at 20° C. After run of 3 hours at rated load the resistance became 92.4 ohms. Did machine meet the usual specifications?

59–5. The resistance of a car heater when cold (20° C.) is 120 ohms. If the temperature rises to 150° C. when hot, how much less current does it take when hot than when cold? Material of heater is iron wire. Voltage is 550.

60–5. It is desired to keep the current constant through a coil of copper wire, which is on a constant voltage circuit. The resistance of coil at 85° C. is 40 ohms. How many feet of Boker's

IAIA wire, No. 18 B. & S., must be added to copper wire when the temperature falls to 25° C. in order that current may not change?

61–5. A motor 2000 ft. from the generator requires 50 amperes at 550 volts. What wire should be used according to rule in Problem 46–5? What is brush potential of the generator and the power lost in line? Efficiency of transmission?

62–5. If wire of twice the cross-section is used in Problem 61–5, what would answers be?

63–5. A 110-volt 25-H.P. motor of 90 per cent efficiency is situated 500 ft. from the generator. No. 0000 B. & S. copper wire is used for line. What must brush potential of generator be? Efficiency of transmission?

64–5. What size wire might have been used in Problem 63 if a line drop of 4 per cent of brush potential of generator was desired?

65–5. What size aluminum wire could have been used in Problem 63–5 and have the same line drop?

66–5. What size of copper wire is required between a 115-volt generator and a 110-volt 10-H.P. motor of 85 per cent efficiency? The motor and generator are 800 ft. apart.

67–5. In Fig. 122, the numbers represent the ohms resistance in that part of the circuit. The battery has a terminal potential of 2 volts.

Find:

Current in AB.
" " AD.
" " BD.
" " BC.
" " DC.

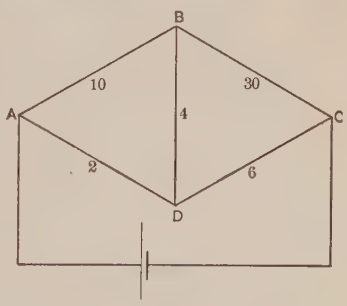

FIG. 122.

68–5. A building situated 1200 ft. from a 115-volt generator is to be supplied with sufficient current from the generator to light 500 lamps each taking .45 ampere. The efficiency of transmission must be 97 per cent. What size copper wire must be used?

69–5. In Fig. 111, voltmeter has resistance of 15,000 ohms. Voltage across $AB = 112$ volts. When connected as shown in diagram, voltmeter reads 4 volts. What is the insulation resistance between armature coils and frame?

CHAPTER VI

MAGNETIC FIELD DUE TO A CURRENT

Field within a Coil—Ampere-turns—Magnetomotive Force. Reluctance—Ohm's Law of the Magnetic Current —Relation between B and H—Permeability at Different Degrees of Magnetization—Three Stages of Magnetization—Saturation Point—Hysteresis.

91. Field within a Coil. We have seen in Chapter II that wherever there is an electric current there is always present a magnetic field. An electric current, then, possesses the property of producing and maintaining a magnetic field. This property of an electric current is called its magnetomotive force. The strength of this field depends upon the strength of the current, the shape of its path, and the permeability of the medium in which the magnetic field is set up.

For instance, the field about a straight wire in air is comparatively weak, while the field on the inside of a solenoid containing an iron core constitutes our most powerful electromagnet.

It is this magnetic field of a solenoid that we wish to consider here. A solenoid is a long coil, or a coil the length of which is great in comparison to its diameter.

It has been discovered that if we wind a solenoid so that there is one turn of wire to every centimeter of the length of the solenoid, and send 1 ampere through the wire, then the field intensity in the air on the inside of the solenoid is 1.26 gausses or $\frac{4\pi}{10}$ gausses. That is, there is an average of 1.26 lines per sq.cm. cross-section of the solenoid.

This is generally written in the form of an equation.

$$H = \frac{1.26NI}{l};$$

or,

$$\frac{4\pi NI}{10l};$$

where H = field strength in *gausses*;
N = total *number* of turns on solenoid;
I = current in *amperes*;
l = length of solenoid in *centimeters*.

Example. What is the field strength within a solenoid in the shape of a ring, 30 cms. long, consisting of 150 turns of wire carrying a current of 8 amperes? Air core.

$N = 150$ turns.
$l = 30$ cms.
$I = 8$ amperes.

$$H = \frac{1.26 \times 150 \times 8}{30} = 50.4 \text{ gausses.}$$

Since H, the number of lines per sq.cm. set up in air, is also the measure of the magnetizing force, it is possible to find the number of lines of force per sq.cm. set up in any material, providing its permeability μ is known.

For instance, if the coil in the above example contained an iron core, the permeability of which was 800, the flux density, B, in the iron would be 800×50.4 or 40,300 gausses. Of course, in order to compute the complete number of lines in the iron, it would be necessary merely to know the cross-section area of the iron. That is, $\phi = BA$.

This equation, then, $H = \frac{1.26NI}{l}$, is the fundamental equation on which the computation of all magnetic circuits is based. Its most important use is in the computation of the number of amperes and turns of wire necessary to

magnetize given magnetic circuits up to a certain flux density.

That part of the equation representing the product, $\overline{NI}$ (the amperes times the number of turns), is called the AMPERE-TURNS. As this expression is divided by the length l, it is evident that the value of H depends, not upon the total number of AMPERE-TURNS, but upon the AMPERE-TURNS *per centimeter* of length.

Note then in particular that the equation, $H = \frac{1.26\ NI}{l}$, gives the magnetizing force for one-centimeter lengths only. So in using it to compute the necessary ampere-turns to magnetize a piece of iron, remember that the value of NI obtained, will be sufficient for merely a centimeter of iron. If the iron has a greater length than one centimeter the result must be multiplied by this length.

Problem 1–6. What is the magnetizing force of a coil forming a closed ring having 200 turns to the centimeter and carrying .04 amperes?

Problem 2–6. What would be the flux density in a piece of iron placed within the coil of Problem 1? ($\mu = 500$.)

Problem 3–6. How many ampere-turns per centimeter are necessary to produce a field intensity of 16 gausses within a solenoid having a wooden core?

Problem 4–6. How many ampere-turns are necessary to produce a field strength of 16 gausses with a solenoid 40 cms. long? Wooden core.

Problem 5–6. It is desired to magnetize a piece of iron ($\mu = 1000$) to a flux density of 9000 gausses. (*a*) What magnetizing force is necessary? (*b*) How many ampere-turns per cm. are required?

Problem 6–6. If solenoid used to magnetize iron in Problem 5–6 has 50 turns per cm., how many amperes must be used?

Problem 7–6. If iron in Problem 5–6 is 10 cms. long, how many ampere-turns are necessary?

Problem 8–6. A ring of iron 25 cms. long is wound by a coil of equal length. What will be flux density within iron if the coil consists of 1250 turns of wire carrying .4 ampere? ($\mu = 750$).

Problem 9–6. How many ampere-turns are needed to magnetize 30 cms. of steel of a permeability of 1200 to a flux density of 16,000 gausses?

Problem 10–6. A ring of iron 24 cms. average length and 1.5 sq.cm. cross-section, is wound with 960 turns of wire. If .42 amperes are sent through wire what is flux density in the iron? ($\mu = 850$.)

Problem 11–6. (*a*) What is the total flux in iron of Problem 10–6? (*b*) What is its magnetic strength in unit poles?

Problem 12–6. A cylindrical piece of iron 30 cms. long, 3.6 cms. in diameter, is bent into a circle and wound with 1200 turns of wire. How many amperes must be sent through the wire to produce a magnet of 24,000 unit poles? ($\mu = 640$.)

92. Magnetomotive Force. In the above section we have seen that H, which equals $\frac{1.26NI}{l}$, is the force tending to magnetize one centimeter length of core, and that if we wish to magnetize a length l of more than one centimeter we must multiply the above force by the length l. This expression, Hl, is sometimes called the MAGNETOMOTIVE FORCE symbol, M.

This magnetomotive force M might be defined as the PRESSURE which forces the magnetic flux or current through the circuit, and corresponds to the electromotive force E, or *voltage* which forces the electric current through its circuit. They both are *pressure; M* is *magnetic* pressure, and E is *electric* pressure.

We have said that the magnetic pressure M of a long coil equals the *field strength* times the *field length.* That is, adding to the length l of the magnetic path, adds to the pressure needed to set up a given flux density throughout the circuit.

Likewise adding to the flux density to be set up throughout the circuit adds to the pressure needed to set up the flux.

Thus, as we have seen;

$$M = Hl.$$

But $$H = \frac{1.26NI}{l}.$$

Then $$M = \left(\frac{1.26NI}{l}\right) l = 1.26NI.$$

This may be written,

$$M = 1.26NI \text{ or } \frac{4\pi NI}{10},$$

Of course if we are considering one centimeter length of coil, the field intensity H equals the magnetomotive force M.

93. Comparison of Equations for H and M. We have then, these two very similar equations, and the distinction between them must always be kept clearly in mind.

The one:

$$H = \frac{1.26NI}{l}.$$

determines the field strength H in lines per sq.cm within a long coil with an air core, or the force necessary to magnetize one centimeter length of the air core.

The other:

$$M = 1.26NI,$$

determines the magnetomotive force (pressure) *tending* to set up the magnetic field throughout the entire path of any kind of material.

The equation for M is rarely used, it being customary to compute the magnetizing force, H, for one centimeter length by the equation $H = \frac{1.26NI}{l}$, and then to multiply the result by the length of the path (l), as in previous problems in this chapter.

However, the term magnetomotive force, M, is used when problems in magnetism are solved by means of what may

be called "Ohm's Law of the Magnetic Circuit" as explained below.

94. Reluctance. Ohm's Law of the Magnetic Circuit. The analogy between a magnetic circuit and an electric circuit is so complete that Ohm's law may be applied to both.

It has just been shown that there is a magnetic *pressure* M which causes a magnetic current to flow in a magnetic circuit, exactly as an electric *pressure* (voltage) E causes an electric current to flow in an electric circuit.

There is a magnetic current consisting of the lines of force ϕ just as there is an electric current consisting of the amperes I.

Similarly, there is a magnetic resistance in a magnetic circuit which opposes the flow of the magnetic current, just as the electric resistance opposes the flow of an electric current. This magnetic resistance is called RELUCTANCE, $\mathfrak{R}$, in order to distinguish it from electric RESISTANCE, R. There is no recognized name for a unit of reluctance.

The electric current flowing through a circuit equals the *electromotive* force divided by the *resistance*.

The magnetic *current* flowing through a magnetic circuit equals the *magnetomotive* force divided by the *reluctance*. By equations:

$$I = \frac{E}{R} \quad \text{(Electric)};$$

$$\phi = \frac{M}{\mathfrak{R}} \quad \text{(Magnetic)};$$

I = electric current in Amperes.
ϕ = magnetic current in Lines.

E = electric pressure in Volts.
M = magnetic pressure in (Gilberts).

R = electric resistance in Ohms.
$\mathfrak{R}$ = magnetic reluctance in (Oersteds).

Thus in order to find the number of lines, ϕ, threading a magnetic circuit, it is necessary to find the magnetic pressure M and divide it by the reluctance $\mathcal{R}$.

Example. The magnetic circuit of an electro-magnet has a reluctance of 0.002. How many lines of force thread the circuit if the magnetomotive force is 16 units.

By Ohm's law of magnetic circuits:

$$\phi = \frac{M}{\mathcal{R}};$$

$$\phi = \frac{16}{.002};$$

$$\phi = 8000 \text{ lines.}$$

Example. How many ampere-turns are necessary to supply the magneto-motive force in above example?

$$M = Hl = 1.26NI;$$

$$NI = \frac{M}{1.26}$$

$$= \frac{16}{1.26}$$

$$= 12.7 \text{ ampere-turns.}$$

Problem 13–6. The magnetic circuit of an electro-magnet has 0.4 unit of reluctance. How great a magnetomotive force is necessary to set up a flux 100,000 lines?

Problem 14–6. How many ampere-turns are necessary in Problem 13–6?

Problem 15–6. A certain electro-magnet has a flux of 200,000 lines. The coil has 450 turns carrying 1.4 amperes. What is the reluctance of the magnetic circuit?

Problem 16–6. A coil consists of 980 turns. The reluctance of the magnetic circuit is .008 unit. How many amperes must flow through coil to set up a flux of 1,000,000 lines?

95. Computation of Reluctance. In finding the reluctance of a magnetic circuit of known size and materials, we use the same method employed in finding the resistance of an electric circuit of known size and materials.

To find the *resistance* of a conductor we use the equation,

$$R = \frac{Kl}{d^2 \text{ (area)}};$$

where K = resistance of unit (*mil-ft.*);
l = length in *feet*;
d^2 = section area in *circular mils*.

Similarly, to find the *reluctance* of a magnetic conductor we use the equation:

$$\mathfrak{R} = \frac{Kl}{A \text{ (area)}};$$

where K = reluctance of unit (*cubic centimeter*);
l = length in *centimeters*;
A = section area in *square centimeters*.

The magnetic tables or data, however, instead of giving K, the RELUCTANCE of a cubic centimeter, generally give μ, the PERMEABILITY, which is merely the inverse of the reluctance of a cubic centimeter. This is exactly analogous to the fact that the electric conductivity is the inverse of the resistance of a mil-ft.

We may write, then, the equation:

$$K = \frac{1}{\mu}.$$

In order to use the tables, we generally use $\left(\frac{1}{\mu}\right)$ instead of (K) and write the equation for reluctance:

$$\mathfrak{R} = \left(\frac{1}{\mu}\right)\frac{l}{A}.$$

Example. A piece of iron is 400 cms. long and has a cross-section area of 200 sq.cms. Permeability = 1000. What is the reluctance?

Since the permeability $(\mu) = 1000$, the reluctance of a cubic centimeter $(K) = \frac{1}{1000}$. Thus

$$\mathcal{R} = \frac{Kl}{A}$$

$$= \left(\frac{1}{\mu}\right)\frac{l}{A}$$

$$= \frac{1}{1000} \times \frac{400}{200}$$

$$= .002 \text{ units.}$$

Reluctances in series are added like resistances in series.

Reluctances in parallel are treated like resistances in parallel.

Example. A magnetic circuit, Fig. 123, is made up of a curved bar of iron 400 cms. long, cross-section 80 sq.cms., and an air gap 2 cms. long. What is reluctance of circuit? Permeability of the iron = 1000.

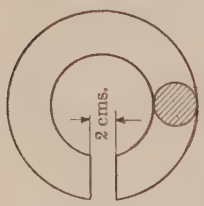

FIG. 123.

For the iron $\mathcal{R} = \frac{1}{1000} \times \frac{400}{80} = .005$;

for the air $\mathcal{R} = \frac{1}{1} \times \frac{2}{80} = .025$;

total reluctance $\mathcal{R} = .025 + .005 = .03$ unit.

Problem 17–6. What is the reluctance of a piece of cast iron 30 cms. long with a cross-section of 2×4 cms.? $(\mu = 200.)$

Problem 18–6. A wrought-iron ring, 2.4 sq. cms. cross-section, has a mean diameter of 20 cms. What is the reluctance if the permeability of the iron is 900?

Problem 19–6. A magnetic circuit is made up of the following parts in series: (1st) 18 cms. of 2×3 cms. cast iron, $\mu = 150$; (2d) 25 cms. of 3×2.5 annealed steel, $\mu = 1400$; (3d) 2 air gaps each 3.8 cms. long and 3.2×2.0 cms. cross-section. Find reluctance of circuit.

Such problems as the following can be solved, when the computation of reluctance is possible.

Example. How many ampere-turns are required to set up 500,000 lines in an iron ring 250 cms. long and 50 sq.cms. section area? Permeability = 500.

$$\phi = \frac{M}{\mathcal{R}};$$

$$\mathcal{R} = \frac{1}{500} \times \frac{250}{50} = .01;$$

$$M = 1.26NI;$$

$$\phi = 500{,}000 \text{ lines};$$

$$500{,}000 = \frac{1.26NI}{.01};$$

$$NI = 3980 \text{ ampere-turns.}$$

But note that this same example can also be solved as follows by the same method used in the first twelve problems in this chapter, which is the method generally used.

Lines per sq.cm. (B) in the iron must equal total lines (ϕ) divided by area (A) of ring.

$$B = \frac{\phi}{A}$$

$$= \frac{500{,}000}{50}$$

$$= 10{,}000 \text{ lines};$$

$$\mu = \frac{B}{H};$$

$$H = \frac{B}{\mu}$$

$$= \frac{10{,}000}{500}$$

$$= 20;$$

$$H = \frac{1.26NI}{l};$$

$$NI = \frac{Hl}{1.26}$$

$$= \frac{20 \times 250}{1.26}$$

$$= 3980 \text{ ampere-turns.}$$

Problem 20–6. How many ampere-turns are necessary to magnetize an iron ring 40 cms. long, 2.8 sq.cms. cross-section area, so that the total flux equals 50,000 lines? ($\mu = 950$.)

Problem 21–6. A magnetic circuit consists of three parts: (1st) 28 cms. of cast-iron, 5×1.5 cms. section, $\mu = 100$; (2d) 80 cms. of wrought iron, 4×2 cms. section, $\mu = 1200$; (3d) 1.5 cms. air gap, 4.5×3 cms. section. Find ampere-turns necessary to set up 100,000 lines in this circuit.

Problem 22–6. A flux of 40,000 lines is set up in magnetic circuit of Problem 19–6 by a coil consisting of 15,000 turns. What current flows in coil?

Problem 23–6. How many ampere-turns are necessary to set up a flux density of 10,000 gausses in ring of Problem 18–6?

96. Flux Density (B). Field Intensity (H). Permeability (μ). We have seen that by means of a form of Ohm's law, we can find the total flux (ϕ) threading a magnetic circuit, and that the number of lines per square centimeter of any part of the circuit is the FLUX DENSITY of that part. We use the symbol B to denote flux density in any material except air. For instance, a 20-sq.cm. cross-section iron circuit might have a total flux ϕ of 100,000 lines. The flux density would be $\frac{100000}{20}$ or 5000 gausses.

There are more lines set up in iron than in air by the same magnetizing force, H, because the permeability of iron is larger. In fact, the number of lines per sq.cm., B, is as much greater in iron than in air, H, as the permeability of the iron is greater than the permeability of the air.

Thus the permeability of a material may be found by comparing the flux density produced in it with the field intensity produced in air. The field intensity in air, H, is also called the magnetizing force. We often define permeability of a material, therefore, as the ratio of flux density B set up in the material to the magnetizing force H. That is, the permeability of a material is the ratio of the number of lines of force per sq.cm., set up in that material, to the number that would be set up in air under the same conditions.

For instance: A coil may have a field intensity, H, of 10 lines per sq.cm. when the inside of the coil is air. If the inside of the coil were an iron core, there might be 40000 lines per sq.cm., B, in the iron. There are then $\frac{40000}{10}$ or 4000 times as many lines per sq.cm. set up in the iron as in the air under the same conditions. The ratio of B to H is 4000, or in other words the iron has a permeability of 4000. It is often expressed in the form of an equation, as we have seen,

$$\mu = \frac{B}{H}.$$

This term H, which we know represents the field intensity in air, or the number of lines of force per sq.cm. in air, is often spoken of as the MAGNETIZING FORCE. This is not to be confused with M the magnetomotive force which is $H \times l$.

97. Permeability Depends upon Degree of Magnetization. Magnetic Permeability has been compared to Electric Conductivity. There is, however this difference: The conductivity of a given piece of copper, for instance, is constant, unless the temperature changes. We know that if we wish to send double the current through a wire we can count on the conductivity remaining the same as though we were not increasing the current.

On the other hand, if we wish to double the number of lines in a piece of iron, we find that we cannot do so by merely doubling the magnetizing force. Sometimes we need but increase the current a very little. Again, we must use many times the original current. In other words, the permeability of iron or steel depends upon how many lines it already contains. Another way of stating it is: the value of μ depends upon B, the magnetic condition of the iron. After a certain point is reached, as the flux density *increases*, the permeability *decreases* very rapidly, and it becomes difficult to set up a greater number of lines.

98. Saturation. This makes it impracticable to magnetize a piece of iron beyond a certain flux density, called the SATURATION POINT. Of course it is *possible* to magnetize iron beyond the saturation point, but for every small increase in flux density there must be a larger and larger increase in magnetizing force, which it is *unprofitable* to maintain.

This relation of B to H is clearly shown in curves plotted from some data taken by students at Pratt, on a sample piece of wrought iron.

WROUGHT IRON

B	H	μ	B	H	μ
1,000	.48	2,080	9,000	2.95	3,050
2,000	.61	3,280	10,000	4.32	2,310
3,000	.78	3,850	11,000	6.70	1,640
4,000	.92	4,340	11,500	9.46	1,220
6,000	1.20	5,000	12,000	12.40	953
7,000	1.40	5,000	12,500	16.00	781
8,000	2.00	4,000	13,000	23.80	546

In Fig. 124, the Magnetization Curve, as the curve showing the relation of B to H is called, the point x denotes the "knee" of the curve and is the point generally spoken of as the SATURATION POINT. It is the point where a small

increase of flux density B requires a large increase in magnetizing force H. In this specimen of iron, it occurs when a flux density of about 8000 lines per sq.cm. has been set up. This requires a magnetizing force of 2.00 gausses.

If the field cores, for instance, of a motor were made of this iron, the flux density at which it could be most profitably run, would be about 8000 lines per sq.cm. The number of ampere-turns to set up this flux density in each centimeter of the length of the magnetic circuit of the motor can be determined, as we have seen. Of course there must

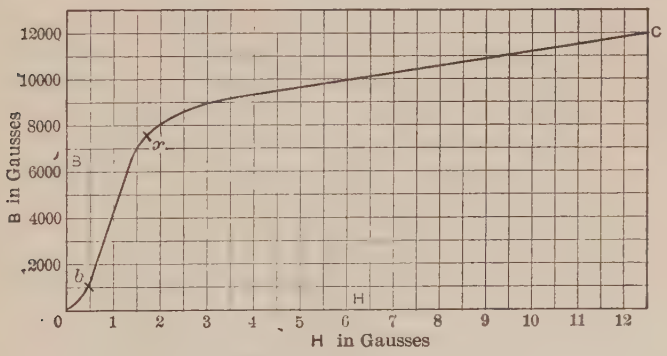

FIG. 124.—Magnetization curve of soft iron.

be added to this enough turns to overcome the reluctance of whatever air gaps, etc., there may be in the magnetic circuit.

The principal point to remember about the magnetization of the iron in any machine, is that there is a certain flux density called the saturation point, beyond which it is *possible, but not practicable,* to carry the magnetization. The reason for this is that it requires an ever-increasing proportion of power to magnetize iron or steel beyond this point, because of the rapid decrease in the permeability.[1]

[1] Physicists are accustomed to call the curve described above the "Induction Curve" and to draw as a Magnetization Curve, a

99. Three Stages of Magnetization. Three stages are generally noted in the magnetization of a piece of iron. If we divide up the curve, Fig. 124, into the sections *ab*, *bx*, and *xc*, the three stages will be fairly well represented.

First Stage. When there are very few, or no, lines in the specimen as represented by the part *ab*, it seems to be quite difficult to set up lines, and the flux density B is almost proportional to the magnetizing force H. According to the Magnetic Molecule Theory, this means that some difficulty is experienced in turning the first few molecules around into the magnetic position.

Second Stage. But once these molecules are turned, most of the others seem to turn more readily, as is evidenced by the great increase in flux density caused by a slight increase in the magnetizing force. This is shown by the section of the curve *bx* rising very abruptly.

Third Stage. When, however, the saturation point is reached, it is extremely difficult to turn the rest of the molecules into the magnetic position, and there is required an ever-increasing magnetizing force. The curve, therefore, falls off rapidly along the section *xc*.

100. Relation of Permeability to Flux Density. If we plot a curve, as in Fig. 125, between permeability μ and flux density B, we see that the permeability starts small and increases as the flux density increases, until the latter has reached a value between 6000 and 7000 gausses. Then suddenly, the permeability begins to decrease rapidly,

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curve between the magnetizing force and the *poles* set up in the iron. This differs but very little from the one used here, except in the portion which we have called *xc*. As this part of the curve is never used in practical construction, we have followed the general practice in calling the above "*BH* curve" the "MAGNETIZATION CURVE."

The term "Saturation Point," as used above, indicates the PRACTICAL SATURATION POINT, and not the THEORETICAL, which, of course, is much further out on the curve.

For a theoretical discussion of this subject see any book on the theory of magnetization.

which it would continue to do till it reached the value of μ for air.

Thus, in determining the value of μ for iron, we must know not only the quality of the iron, but also the flux density at which it is to be used. As stated previously, it is in this particular that magnetic permeability differs

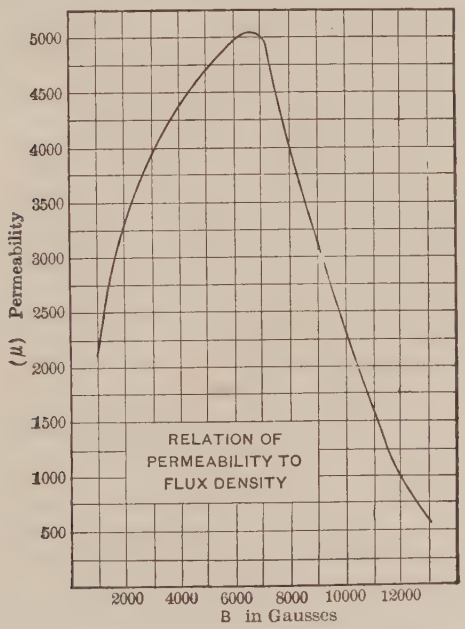

FIG. 125.—Relation of permeability to flux density in iron.

from electric conductivity. In electrical circuits, it makes no difference whether the current density is 1 ampere or 100 amperes, the *conductivity* remains the same (barring temperature changes). But, as has just been shown, the magnetic permeability changes every time the current (or flux density) changes.

In using any equation containing the quantity μ, it is

necessary, therefore, to select the value to be applied to it, with due regard to the magnetic conditions of the material.

101. Practical Computation of Ampere-turns. It is on account of this change in permeability, that in computing the ampere-turns necessary to produce a certain flux in a magnetic circuit, it is not customary to use the equation $\phi = \frac{M}{\mathcal{R}}$. Instead, the flux density (B) is computed $\left(B = \frac{\phi}{A}\right)$, and the value of H to produce this density is found in a table. The number of ampere-turns to produce this value of H is then computed from the equation

$$H = \frac{1.26NI}{l}, \qquad \text{or} \qquad NI = \frac{Hl}{1.26},$$

as explained earlier in this chapter.

Example. How many ampere-turns are required to set up a flux of 1,200,000 lines in a cast-iron circuit 200 cms. long, with a cross-section area of 300 sq.cms.?

$$B = \frac{1{,}200{,}000}{300} = 4000 \text{ gausses.}$$

From table, in Appendix, for relation of B to H in cast-iron,

$$H(\text{for } B \text{ of } 4000) = 8.5;$$

$$NI = \frac{8.5 \times 200}{1.26} = 1350 \text{ ampere-turns.}$$

The design of an electro-magnet includes many details in addition to the number of ampere-turns. The size of wire and the amount of surface required for radiation are two factors which affect the design fully as much as the ampere-turns.[1]

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[1] For practical directions in the design of electro-magnets, the Varley Electro-Magnet Co. of Jersey City has published a most excellent booklet. It contains a mass of information and practical formulas which will be found invaluable to one designing a magnet for a definite use.

Solve the following problems by use of BH tables in Appendix.

Problem 24–6. A wrought-iron magnetic circuit has a length of 65 cms. How many turns must be wound on it to give a flux density of 6000 gausses if the current through the turns is .75 ampere?

Problem 25–6. If circuit in Problem 24 has section area of 8 sq. cms., what total flux is set up?

Problem 26–6. A magnetic circuit consists of wrought-iron ring and air gap of the dimensions in Fig. 123. It is desired to set up a flux of 1,280,000 lines in this circuit. How many ampere-turns must be wound upon it?

Problem 27–6. If ring in Problem 26 were cast-iron, how many ampere-turns would be necessary to set up 1,000,000 lines in circuit?

Problem 28–6. In generator of Fig. 149, how many amperes must be sent through field coils containing 4000 turns each to set up 1,200,000 lines per pole? Poles and yoke are of wrought iron. Poles 40 cms. long, area 100 sq. cms. Path of flux through yoke from pole to pole, 100 cms. long, 80 sq. cms. area. Consider each magnetic path through the armature 10 cms. long, 120 sq. cms. area. Air gap .8 cm. long, 130 sq. cms. area.

102. Hysteresis. In Fig. 124, we saw how the flux density B increases when the magnetizing force H increases, even when the curve was carried to a point considerably beyond saturation. If now we gradually take away the magnetizing force, let us see what happens.

In Fig. 126, the line *abxc* is the magnetization curve of Fig. 124. At point C the flux density is 11,800 gausses and the magnetizing force 12.5 gausses.

When, however, the magnetizing force H is decreased, the flux density B does not decrease along the same line by which it had increased, but follows the curve *cdef*. At the point *e*, although the magnetizing force H has become zero, there still remains a flux density, B, of 8800 gausses. The flux B, therefore, is said to Lag behind the force H. This lagging is called Hysteresis, and is the cause of a certain loss in every alternating current machine and in the armatures of D.C. machines. The reason for this is seen if we consider the rest of the curve.

At the point e, as has been said, although H has become zero, the value of B has only decreased to 8800, represented by the line ae. This value is called the REMANENCE. To get this magnetic Remanence out of the iron, it is necessary to set up a magnetizing force H in the opposite direction, of 1.2 gausses. This force is represented by the line af, and is called the COERCIVE FORCE.

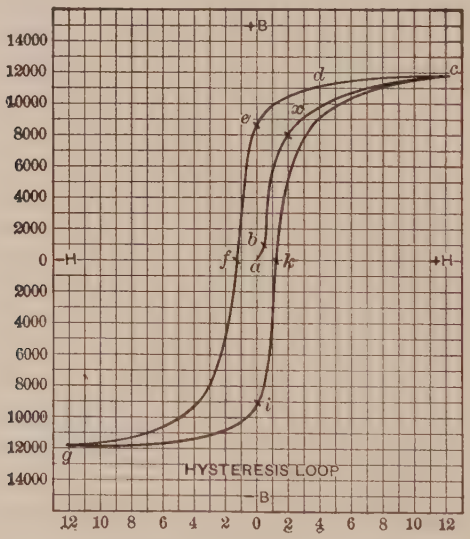

FIG. 126.—Hysteresis loop for soft iron.

If now this magnetizing force in the opposite direction, $-H$, is increased, a flux, $-B$, will be set up in the opposite direction and a curve of magnetization in this direction can be drawn as fg, until B has as large a negative value at g as it had a positive value at c.

The magnetizing force is again gradually decreased, and again the flux "lags" behind the force, until when H has again become zero, the value of B has decreased to i only. There is thus a remanence of about 9000 gausses, ai, left

in this direction, to remove which again requires a coercive force *ak*.

To bring the flux density up to the value *c* again, we merely have to increase this force to its original value for density at this point. The cycle thus completed is called the " Hysteresis Loop."

It can be shown that the area of this loop represents the work done in overcoming the Hysteresis effect on a cubic centimeter of this specimen of iron, just as the area

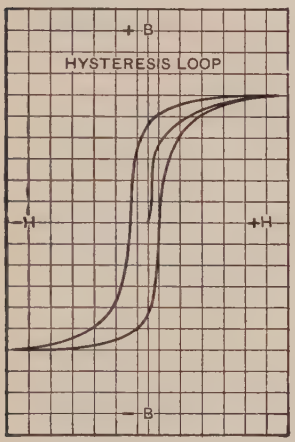

FIG. 127.—Hysteresis loss in annealed steel.

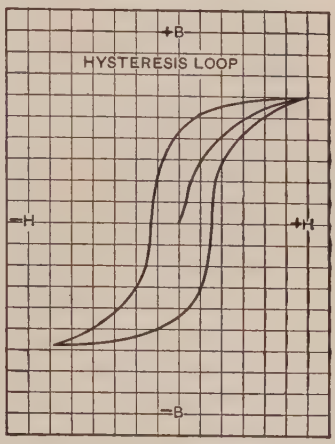

FIG. 128 —Hysteresis loss in hard steel.

of a steam indicator card represents the work done in overcoming the resistance to motion offered by the piston. Only, in this case, all work done, as represented by the area of the loop, is wasted.

This can be shown as follows:

Since the value of *B*, ie., the magnetic density of the iron, always lags a little behind the magnetizing force, some extra force must be exerted continually to urge on the change in the magnetization. When the flux density is being increased, the value of *H*, or magnetizing force, has to be a little greater than would be necessary, were there no hysteresis effect. Similarly, as the iron is

being demagnetized, the magnetizing force must be larger in the opposite direction in order to overcome the drag of the magnetic lines which seem to persist in the iron. Thus there is a constant action taking place between the magnetized iron and the magnetic field of the coil.

This action between a magnet and a magnetic field has been described in Chapter I and has been shown to be equal to the product of the magnetic strength of the iron (m) times the strength of the field H.

In the form of an equation this becomes

$$F = mH,$$

where F = force in dynes;
m = strength of magnet in unit poles;
H = strength of field in gausses.

The cubic centimeter of iron, which we are considering, is magnetized to a density of B gausses and must contain $\frac{B}{4\pi}$ unit poles. The action of the field H of coil upon the iron must, therefore, be represented by the equation,

$$F = \frac{B}{4\pi} \times H$$
$$= \frac{BH}{4\pi} \text{ dynes.}$$

Inasmuch as we are dealing with but one cubic centimeter of iron, this force, $\frac{BH}{4\pi}$ dynes, must be acting through a centimeter length only. That is, the magnetization is made to change throughout one centimeter length.

Now, when a force of one *dyne* acts through a *centimeter* space, it does one *erg* of work. Thus, $\frac{BH}{4\pi}$ dynes, acting through one centimeter, would do $\frac{BH}{4\pi}$ ergs of work.

But the value of $\frac{BH}{4\pi}$ is continually changing during the cycle of magnetization.

The total amount of work done during one cycle would then be the summation of all the different values of $\frac{BH}{4\pi}$ throughout the

cycle. The area of the loop, in BH values, represents this summation, and accordingly, the amount of energy consumed in overcoming the magnetic lag, or hysteresis during one cycle.

It is customary to compute this energy loss in watt-secs.; 1 watt-sec. $=10^7$ ergs.

The equation for hysteresis loss in 1 cu.cm. thus becomes

$$W=\frac{\text{area of loop } (BH \text{ units})}{4\pi\times10^7}.$$

Example 1. Find the hysteresis loss per cubic centimeter, per cycle, in the iron used to obtain loop in Fig. 126. Area of loop equals 24 sq.in.

Solution. On horizontal axis:

1 in. $=2$ gausses; magnetizing force H.

On vertical axis:

1 in. $=2000$ gausses; flux density B.

1 sq.in. $=2\times2000=4000$ (BH units).

Area of curve:

24 sq.in. $=24\times4000=96000$ (BH units).

Hysteresis loss:

$$W=\frac{96000\ (BH \text{ units})}{4\pi\times10^7}$$
$$=0.000764 \text{ watt-secs.}$$

Example 2. If 20 lbs. of the iron in Example 1 go through the cycle of magnetization 60 times per second, what power would be lost on account of hysteresis?

Solution.

$$20 \text{ lbs.}=\frac{20\times454}{7.4}=1230 \text{ cu.cms.}$$

From above example,

1 cu.cm. loses 0.000764 watt-sec. per cycle.

Then 1230 cu.cm. would lose

$0.000764\times60\times1230$ watt-sec. in 60 cycles

$=5.5$ watt-secs.

Since this loss occurred in one second the power lost

$=5.5$ watts.

With every cycle (double alternation) of an electric current in a machine, there is always this hysteresis loss

in the iron cores. The greater the number of alternations a second, the greater the total loss. For this reason, the iron to be used in Alternating Current machines, and in armatures in general, has to be selected with proper care as to its hysteresis qualities.

If a piece of annealed wrought iron or annealed silicon-steel is put to this test, the loops plotted will be very narrow and contain little area (Fig. 127), showing the loss to be small.

In the case of hard steel, the loop will widen out (Fig. 128), and contain a large area. For this reason, either annealed iron or annealed silicon-steel is used in transformer cores, armatures, etc.

103. Computation of Hysteresis Loss. Dr. Steinmetz has given the following equation for computing the hysteresis loss:

$$P = \frac{K B_{\max}^{1.6}}{10^7};$$

where P = loss per cu.cm. of iron for 1 cycle per sec. in *watts*.

$B_{\max}$ = maximum flux density in *gausses*.

K = constant, depending on material.

The following table of the values of K is taken from Foster's "Electrical Engineer's Pocket Book."

HYSTERESIS CONSTANTS FOR DIFFERENT MATERIALS

Material.	Hysteresis Constant. K
Best annealed transformer sheet metal	.001
Very soft iron wire	.002
Thin good sheet iron	.003
Thick sheet iron	.0033
Most ordinary sheet iron	.004
Transformer cores	.003
Soft annealed cast steel	.008
Soft machine steel	.0094
Cast steel	.012
Cast iron	.016
Hardened cast steel	.025

Example. What loss in power is due to hysteresis in a machine that contains 2500 cu.cms. of annealed iron, which is subjected to 60 cycles a second? $K = .003$. Maximum Flux Density = 12,000.

$$P = \frac{K B_{max}^{1.6}}{10^7};$$

$$P = \frac{.003 \times 12{,}000^{1.6}}{10^7}$$

$$= \frac{.003 \times 3{,}370{,}000}{10^7}$$

$$= .00101 \text{ watt per cu.cm. at 1 cycle per sec.}$$

$$\text{Total power lost} = .00101 \times 60 \times 2500$$

$$= 152 \text{ watts.}$$

Example. If cast iron, $K = .016$, had been used and same flux density and frequency maintained, what would have been the loss in above machine?

$$P = \frac{.016 \times 12{,}000^{1.6}}{10^7}$$

$$= .00539 \text{ watt per cc. at 1 cycle per sec.}$$

$$\text{Total loss} = .00539 \times 60 \times 2500$$

$$= .81 \text{ K.W.}$$

Since 2500 cu.cms. of iron weigh less than 45 lbs., it can be seen that the machine is not very large. A loss of .8 K.W. then, would be fatal to it.

For methods of obtaining the hysteresis loop, see the Laboratory Notes on the experiment with this title.

Problem 29–6. The armature of a motor contains 70 lbs. of sheet iron (specific gravity 7.7). The magnetic flux, which has a maximum value of 11,000 gausses, goes through 40 cycles per.sec. What is the K.W. loss?

SUMMARY OF CHAPTER VI

FIELD INTENSITY WITHIN A COIL; AIR CORE.

$$H=\frac{4\pi NI}{10l}=\frac{1.26NI}{l}.$$

MAGNETOMOTIVE FORCE.

$$M=Hl$$

$$M=\frac{4\pi NI}{10}=1.26NI.$$

These two equations must not be confused. The Magnetomotive Force is the magnetic pressure which sets up the magnetic current. It corresponds to the Electromotive Force which sets up the electric current.

RELUCTANCE. The reluctance of a given magnetic circuit corresponds to the resistance of an electric circuit.

The same form of equation is used to find the reluctance of a certain part of a magnetic circuit as is used to find the resistance of any part of an electric circuit.

$$\mathfrak{R}=\frac{1}{\mu}\times\frac{l}{A}.$$

Reluctances in series are added like resistances in series.

Reluctances in parallel are treated like resistances in parallel.

OHM'S LAW OF THE MAGNETIC CIRCUIT. Ohm's Law applies to the magnetic circuit as well as to the electric circuit.

$$\text{Current (in lines)}=\frac{\text{Pressure}}{\text{Reluctance}};$$

$$\phi=\frac{M}{\mathfrak{R}}.$$

VALUE OF μ DEPENDS ON B. The value of the Permeability μ of iron and steel is not constant like the value of the Conductivity of copper. Every change in the Flux

Density B changes the value of the Permeability μ. It is therefore necessary to know at what Flux density a piece of iron is to be used, before the correct value of μ can be determined.

THREE STAGES OF MAGNETIZATION. When a piece of iron or steel contains few or no magnetic lines, it requires considerable magnetizing force to set up an increase of flux density. After starting the building up of the lines within the iron, it becomes easy to increase the flux until a point called Saturation is reached. While it is possible to go on increasing the flux density beyond the Saturation point, it is highly impracticable, because with each succeeding increase in flux density, a greater and greater magnetizing force must be used, until soon a very great increase in magnetizing force gives but a small increase in flux density.

The Saturation point is located at the "knee" of the Magnetization curve. A machine should be run with the flux density of the magnetic circuit at about the Saturation Point.

HYSTERESIS. When we remove the magnetizing force, all the magnetic lines do not fall out, because there is a certain "lag" in the magnetic flux with regard to the magnetizing force. This "lagging" is called "Hysteresis."

In all armature cores and in A.C. machines it is necessary to expend a certain amount of power to overcome the effects of this "lagging." Since soft iron and annealed silicon-steel exhibit this property of hysteresis to a very limited degree, it is customary to use these materials wherever frequent and large changes in the magnetic flux are required.

Steinmetz gives the following formula for computing the "Hysteresis Loss" in Watts for 1 cu.cm. of iron for 1 cycle per second.

$$P=\frac{KB_{max}^{1.6}}{10^7}.$$

It can also be computed from the area of the Hysteresis Loop.

PROBLEMS ON CHAPTER VI

30–6. A wrought-iron ring 200 cms. long is to carry a flux of 800,000 lines. Flux density is to equal 6000 gausses. (*a*) What is the cross-section area? (*b*) What is the permeability at this density? (*c*) How many ampere-turns are required? (*d*) What is reluctance of circuit?

31–6. In order to magnetize an iron rod, a magneto-motive force of 400 units is necessary. How many volts must be applied to a coil of 200 turns and 68 ohms resistance?

32–6. If wire in Problem 31 is No. 24, B. & S. copper, how many feet of it are in the coil?

33–6. It is desired to magnetize a 40 cm. cast steel ring to a density of 16,500 lines per sq.cm. 800 turns of No. 23, B. & S. copper wire are used. Average length of each turn is 14 in. How many volts must be applied to coil in order to set up desired flux density?

34–6. How many ampere-turns are necessary to magnetize a 12-cm. cast-steel bar of 6 sq.cms. cross-section so that it has a strength of 2000 unit poles? Bar is bent into a ring.

35–6. A magnetic circuit is made up of 300 cms. of wrought iron 50 sq.cm. in cross-section; 1.5 cms. of air, 55 sq.cms. in cross-section. Find number of ampere-turns necessary to set up 600,000 lines of force in above circuit.

36–6. A magnetic circuit is made up of

200 cms. of cast steel, 40 sq.cms. cross-section
120 " " sheet " 50 " "
1.2 " " air, 50 " "

Find current that must be sent through a coil of 20,000 turns in order to set up 800,000 lines in above circuit.

37–6. If cross-section of each part of circuit in Problem 36, were doubled, what current would be necessary?

38–6. A magnetic circuit is made up of:

150 cms. of wrought iron, $B = 10{,}000$ gausses.
200 " " sheet steel, $B = 14{,}000$ "
50 " " cast iron, $B = 7{,}500$ "
1 " " air, $H = 12{,}000$ "

Find ampere-turns necessary to maintain the circuit at these flux densities.

39–6. If the total flux in circuit of Problem 38 is 1,000,000, what must be the area of each part of the circuit?

40–6. What is the reluctance of circuit in Problem 39–6?

41–6. A cast-iron ring has a mean diameter of 50 cms. and a cross-section area of 36 sq.cms. How many ampere-turns are required to set up a flux of 460,000 lines?

42–6. What is the reluctance and permeability of ring in Problem 41?

43–6. What would be the reluctance and permeability of ring in Problem 41–6, if 230,000 lines were set up?

44–6. Fig. 129 represents a magnet used to hoist steel rails. Magnet core *A* is of wrought iron, 120 sq.cms. cross-section. Rail *B* has a cross-section of 250 sq.cms. There is a space of 0.15 cm. between ends of core *A* and rail *B*, due to rust. The flux required is 1,800,000 lines. How many ampere-turns are required?

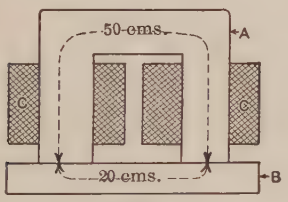

FIG. 129.

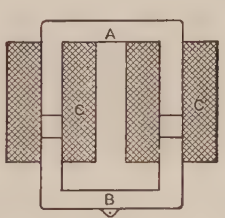

FIG. 130.

45–6. The lifting magnet Fig. 130 has the following dimensions: Core *A*, mean length of 11 inches, area of section 1 sq.in. Core *B*, mean length 6.5 inches, area of section .9 sq.in. Material of both is sheet steel. Air space between ends of *A* and *B* $\frac{7}{8}$ in. If 100,000 lines are required in circuit to lift *B*, how many ampere-turns must be wound on *C*?

46–6. The core of a transformer weighs 280 lbs. If it is used on an alternating current circuit of 133 cycles per sec., what will the hysteresis loss be? $B_{max} = 10{,}000$ gausses.

47–6. What would the hysteresis loss be in Problem 46–6, if the transformer were used on a 60-cycle circuit?

48–6. In Fig. 156, the field cores are of wrought iron each 50 cms. long and an average cross-section area of 140 sq.cms. Yoke is of cast steel 25 cms. long, 80 sq.cms. cross-section; armature

core is of sheet steel 14 cms. long, average cross-section of 120 sq.cms. Two air gaps each .8 cm. long, of 250 sq.cms. cross-section area. If 1,250,000 lines are desired in air gap, of how many ampere-turns must the field coils consist?

49–6. The coils in Problem 48, each have 8000 turns of wire. Each lamp in a circuit supplied by this generator takes .4 ampere. How many amperes must armature deliver?

50–6. Armature of generator in Problem 48 makes 1200 R.P.M. What is hysteresis loss?

51–6. Assume 1 sq.in. of curved surface of coil is needed to radiate 1 watt to prevent a coil from becoming overheated. What must be outside diameter of coil in Problem 33–6 if it is to be used continuously? Length = 25 cms.

52–6. On basis stated in Problem 51, what must diameter of coil in Problem 36 be if length is 78 cms.? Resistance = 40 ohms.

53–6. On basis stated in Problem 51, the temperature of a coil rises 55° C. above surrounding temperature on continuous run. If the curved surface of Problem 52 is 2750 sq.cms., how high will temperature rise?

CHAPTER VII

THE GENERATOR

Electromagnetic Induction—Direction of Induced E.M.F.—Amount of Induced E.M.F.—Current in Revolving Loop—Sine Curve of E.M.F.—Collecting Rings—A.C. Power—Commutator—D.C. Power—Ring and Drum Armatures—Action within an Armature—Magnetization of the Core—Neutral Axis—Axis of Least Sparking—Voltage and Resistance of D.C. Armature—Field Excitation of Generator—Separately Excited, Series Wound, Shunt Wound, Compound—Losses in Generator—Eddy Current Loss.

Thus far we have been studying the laws controlling the action of electric and magnetic pressures and currents. We are now to consider how electric pressures and currents are generated and maintained.

104. Electromagnetic Induction. It has been shown that we always set up a magnetic field around a conductor whenever we set up an electric current in the conductor. This magnetic field lasts as long as the current lasts.

On the other hand, it has been found that if we have a magnetic field present, and wish to set up an electric current in a conductor, it is necessary to *move* the conductor in such a way as to cut the magnetic lines. For instance, if a wire is swung in the earth's magnetic field in such a way as to cut across the lines, we find that an E.M.F. is induced in the wire. This is shown by placing a galvanometer across the ends of the wire and noting the deflection every time the wire is swung. As long as the wire remains stationary no current flows. In fact, even if the wire moves, but in

a direction parallel to the lines of force, no current flows. This shows that the conductor must *move and cut lines of force* in order to have an E.M.F. induced in it.

105. Direction of Induced E.M.F. Consider Fig. 131. If the wire AB is moved *down* across the lines of force as marked, an electric current will flow along the wire from A to B. If the wire were moved up across the lines, the current would flow in the opposite direction, from B to A.

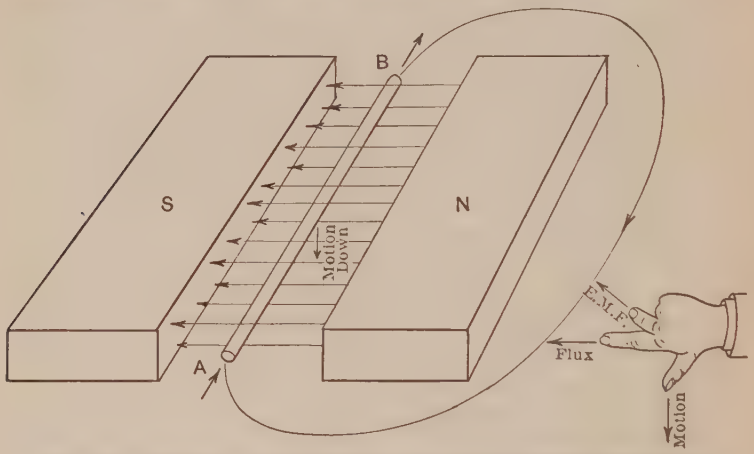

Fig. 131.—Direction of E.M.F. induced in wire cutting magnetic lines of force.

The rule for finding the direction of the Induced E.M.F. which sets up this current, is as follows.

Extend the Thumb, Forefinger and Middle Finger of the Right hand at right angles to one another. Let the Thumb point in the direction of the motion, the Forefinger in the direction of the magnetic flux, then the Middle Finger will be pointing in the direction of the induced E.M.F.

The hand in Fig. 131 shows the application of this rule to the case of the wire being moved *down*.

Fig. 132 brings out this relation more clearly.

The direction of the induced E.M.F. then, is seen to

depend upon the direction of the flux and the direction of the motion of the conductor.

106. Amount of Induced E.M.F. The amount of the induced E.M.F. depends upon the number of lines cut per second. It has been found that if a conductor cuts 100,000,000 lines per sec., that an E.M.F. of 1 volt is produced between the terminals. This number 100,000,000 is usually written 10^8, and we read it "ten-to-the-eighth-power."

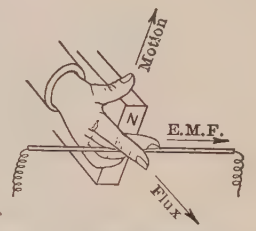

FIG. 132.—Right-hand rule for induced E.M.F.

Example 21. A wire passes 40 times a second across the pole face of a field magnet the flux density of which is 15,000 gausses. The dimensions of the face are 30×20 cms. What E.M.F. is induced in the wire?

$$\phi = 600\times15{,}000 = 9{,}000{,}000 = 9\times10^6.$$

$$\text{Lines cut per sec.} = 9\times10^6\times40 = 3.6\times10^8.$$

$$\text{Volts} = \frac{3.6\times10^8}{10^8} = 3.6 \text{ volts.}$$

Problem 1–7. How many volts are induced across the ends of a wire which cuts 3×10^{10} lines in 2.4 secs.?

Problem 2–7. A wire cuts through a field of 2,000,000 lines, at an average rate of 2400 times per minute. How many volts (average) are set up across the wire?

Problem 3–7. A wire 100 cms. long passes through a magnetic field where the flux density is 8000 gausses. If the velocity at right angles to the lines is 1200 cms. per sec. what voltage is induced in the wire?

Problem 4–7. If a wire which is cutting through a magnetic field at the rate of 100 ft. per sec. induces 4 volts across its terminals, how fast must it move to set up 6 volts?

107. Current in Revolving Loop. The E.M.F. of a generator is the E.M.F. induced in wires revolving so as to cut the lines of force of a powerful magnetic field. Let us consider the case of a single loop of wire so revolved.

In Figs. 133 and 134, the loop of wire is revolving as shown by the arrow.

In Fig. 133, the side of the loop BD is moving *down* across the lines. By applying the Right-Hand Rule, the induced E.M.F. is found to be in the direction of D to B as marked. The side AC is moving up and the induced E.M.F. is from A to C as marked. Thus a current will flow around the loop in the direction $ACDB$, as marked.

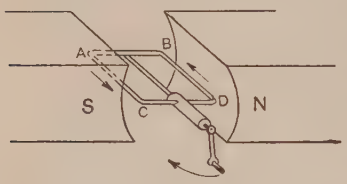

FIG. 133.—Model single loop generator.

In Fig. 134, the wire BD has now turned to the opposite side, and is moving *up* instead of *down*. The induced E.M.F. has now reversed and is from B to D. For the same reason, the E.M.F. in AC is reversed and is from C to A. This now causes a current to flow around the coil in the direction $BDCA$, which is the reverse of the direction of the current when the coil was in the other position.

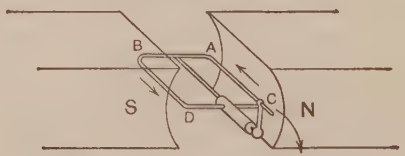

FIG. 134.—E.M.F. in loop, after one-half turn.

Thus the current in a revolving coil flows one way during half the revolution and in the reverse direction during the other half.

There is then an alternating current in a closed loop of wire revolved in a magnetic field.

The current will be at its maximum value in *one direction* when the coil is in the position of Fig. 133, and at its maximum value in the *opposite direction*, when in the position of Fig. 134, because at these positions the sides of the coils are cutting the greatest number of lines per sec. When

the loop is vertical, as in Fig. 135, the sides of the loop are moving *parallel* to the magnetic lines and are *not cutting them*. There is then no E.M.F. induced in the loop. It is then said to be in the "neutral position."

108. Sine Curve of E.M.F. If we consider the neutral position of the coil (Fig. 135), as the "zero" position,

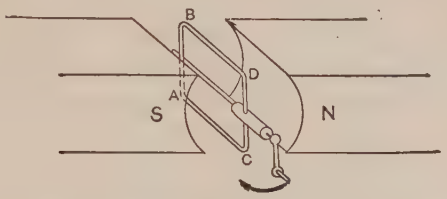

FIG. 135.—Loop in neutral position. The sides are not cutting lines.

then the horizontal position of Fig. 133 would be 90° from the zero position. When the coil had become vertical again, it would be again in a neutral position, at 180° from the first neutral position. When again horizontal, as in Fig. 134, it would be 270° from the first neutral position.

We may plot a curve using the *position* (in degrees from the neutral position) as the abscissæ and the induced E.M.F. as the ordinates.

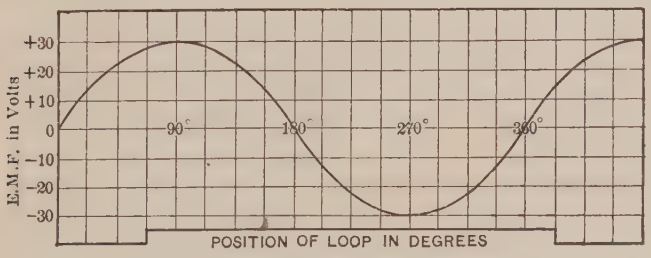

FIG. 136.—Sine curve of induced E.M.F.

When the E.M.F. is in one direction, we call it +, and in the other, when the field is uniform −.

We obtain a curve as in Fig. 136.

This curve shows that the voltage (induced E.M.F.) rises rapidly to a maximum at the 90° position (horizontal)

where the lines are being cut at the greatest rate. It falls again to zero at the 180° position (vertical) when the sides of the loop are moving parallel to the lines and are thus not cutting them. Then the sides begin cutting in the opposite direction and a voltage is thus induced in the opposite direction (which we have agreed to call "negative.") The voltage in this direction increases rapidly to a maximum at the 270° position (horizontal) where the lines are again being cut at the greatest rate (only in the opposite direction). Again it decreases as it begins to approach the neutral (vertical) position where the sides again move parallel to the lines, and thus again at this point no voltage is induced. This "cycle" of events, as it is called, takes place every complete revolution in a two-pole machine, the maximum voltage reached in each direction depending upon the number of lines cut per sec., at the instant the sides are cutting at the greatest rate.

In order to get a high maximum voltage in a generator:

First. The inside of the loop is filled with an iron core to increase and concentrate the magnetic field.

Second. A great many loops of wire are wound in series on this core, so that the voltage across the ends at any time is the sum of the voltages in each loop.

The "loops" and the "core," together with the device for taking out current from the loops, form the armature of most Direct Current and many Alternating Current Generators.

The current within the armature of a DIRECT as well as of an ALTERNATING CURRENT GENERATOR is always an ALTERNATING Current, and goes through much the same "cycle" of values described by the curve in Fig. 136.*

Whether a machine delivers an Alternating or a Direct Current, then, must depend upon the device used for taking

* The one exception to this statement is the UNIPOLAR generator, which has come into some use with the advent of the high-speed steam turbine.

off the current from the armature. This device may consist of either COLLECTING RINGS, which deliver an alternating current, or a COMMUTATOR, which delivers a direct current.

109. Collecting Rings. If we have the outside circuit *continually connected* with the two ends of the revolving coils, we must get the same kind of voltage induced across the outside circuit that we do across the inside. That is, we would get an alternating current from the machine, since the *inside current is always alternating.* This is done by connecting the ends of the wire of the coils to rings, called COLLECTING RINGS. Brushes bearing on these rings will lead the alternating current to any desired outside circuit.

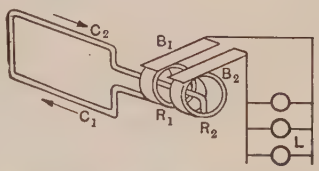

FIG. 137.—Single loop fitted with collecting rings. An alternating current is delivered.

Fig. 137 shows the principle of Collecting Rings. C_1, C_2 represent the sides of a coil of wire in the armature; R_1 and R_2 the collecting rings joined one to each end of the coil; B_1 and B_2, the brushes, and L, a bank of lamps as outside circuit.

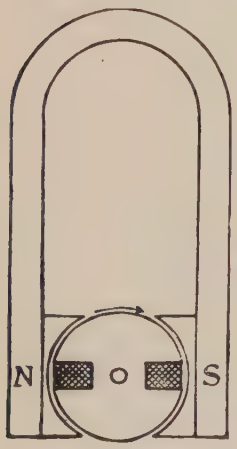

FIG. 137*a*.—Magneto generator.

Since the lamps always form a continuous circuit with the coil, they will always have the same E.M.F. that the coil has. The sine curve of Fig. 136 will then be a fair representation of the E.M.F. across the lamps as well as across the armature. A single-coil MAGNETO is an example of a very simple Alternator, and is shown in Fig. 137*a*.

110. Commutator. If we can connect each brush first with one end and then with the other end of the revolving

coil, making the change just at the instant the current in each side is reversing, we shall then have the current on the outside always in the same direction. This is accomplished by means of a COMMUTATOR. Fig. 138 shows the principle of the commutator. The segments S_1 and S_2 are attached each to an end of the revolving coil C_1, C_2. The brushes B_1 and B_2 collect the current and deliver it to the lamp bank L.

Assume that, at the instant, the current in C_1 is coming to the brush B_1. The current in the side C_2 must be going in the opposite direction or away from the brush B_2. B_1 is then + and B_2 is −. The brush B_1 bears on the segment S_1 as long as the current in C_1 is in this direction. As soon as the current in C_1 changes to the opposite direction, the segment S_1 has left the brush B_1 and the segment S_2 has come into contact with B_1. But just as the current in C_1 has reversed, so the current in C_2 has also reversed, and the current *now flows to segment* S_2. S_2 now delivers current to brush B_1 which keeps it still the positive brush. Thus B_1 is always positive. In the same way it is seen that B_2 will always be the negative brush. The current delivered to the lamps then will always be in the same direction.

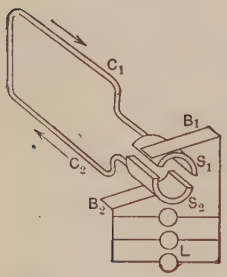

FIG. 138.—Single loop fitted with commutator. A direct current is delivered.

A single loop of wire, of course, does not deliver a steady current, even with the commutator, although the current is always in the same direction. The brushes are so placed that the change from one segment to another is made when there is no current flowing in the coil. At this moment, also, there would be no current flowing in the lamps. So we would have a Pulsating Current in the lamps as represented in Fig. 139. The current on the inside of the coil is still that represented by the curve in Fig. 136. The values

of the voltages on curve in Fig. 139 are the same as those on curve in Fig. 136, the voltage across the lamps being zero when the voltage across the coil is zero, and greatest when greatest across the coil. The sign, however, for the voltage across the lamps is always positive (+), since the current is always in the same direction. Curve 139 then is Curve 136, with the negative loops turned into positive ones.

To obtain a steady direct current, a large number of coils is placed around the core, and the commutator divided up into a correspondingly large number of segments. The loops of wire of the different coils are distributed so that at

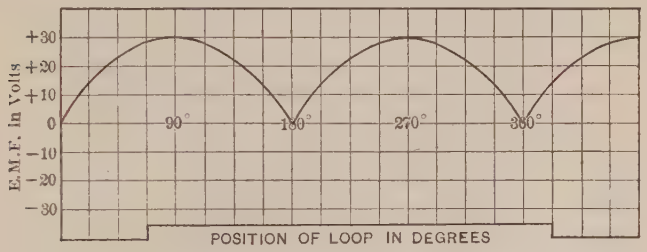

FIG. 139.—Curve of pulsating E.M.F. across single coil armature fitted with commutator.

every instant some of them are cutting lines at the maximum rate. Examples of the more important of these arrangements will be given later.

It can be stated as a general rule that an armature always maintains an alternating pressure across its coils. When used with COLLECTING RINGS, the brushes are then placed always across the two ends of the same coil and thus the machine delivers an alternating current. When used with a commutator, each brush is placed so that it takes the current from one side of the coil only so long as that side is in a certain part of the field. When the current reverses in one side of the coil, that side is passed on to the other brush. In this way a current is delivered which is always

in the same direction. Machines using or delivering such a current are called Direct Current Machines. It is this type that we will now study more in detail.

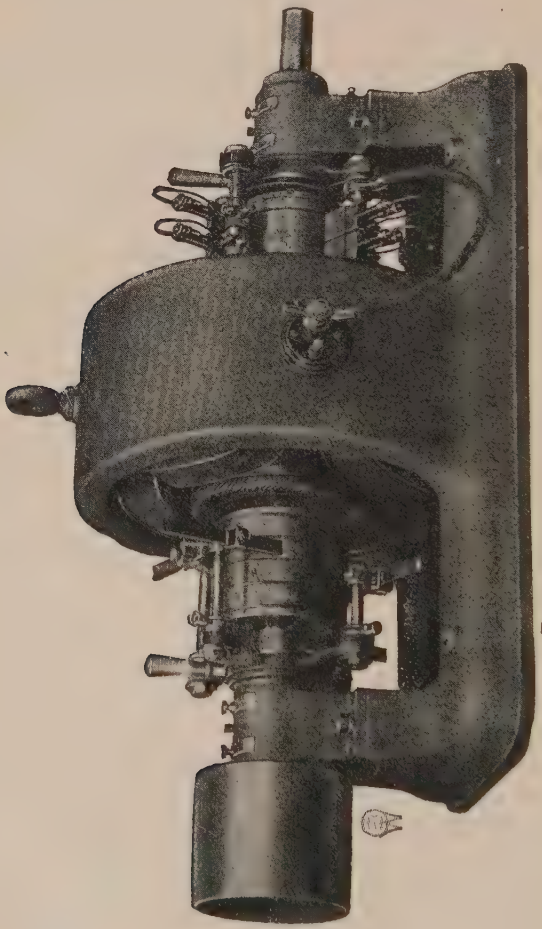

FIG. 140.—A double current generator.

Fig. 140 shows a mounted armature fitted so that it can deliver both direct and alternating current. On the left-hand side of the machine are seen the collecting rings. Brushes

placed on these rings will have an alternating E.M.F. across them when the armature is revolved in a strong field as explained above. On the right is a commutator with a large number of segments. Brushes placed on the commutator at proper positions will have a direct E.M.F. across them, and since there is a large number of coils in the armature, the E.M.F. will be very steady.

Fig. 141 shows a partly-wound Direct Current armature.

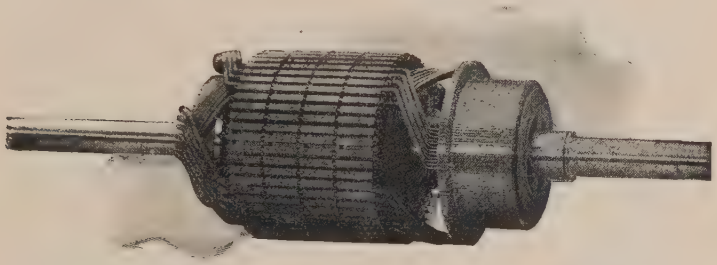

FIG. 141.—Partly-wound armature of D.C. generator. Allis-Chalmers.

111. Ring and Drum Armature. In making an armature, there are two general types of cores on which the loops of wire are wound, (1) the DRUM and (2) the RING. The DRUM type consists of a cylinder of iron built up of thin sheets cut to proper shape in a punch press.

The wires are then wound around the outside of this drum, as in Fig. 142, which represents a four-loop armature.

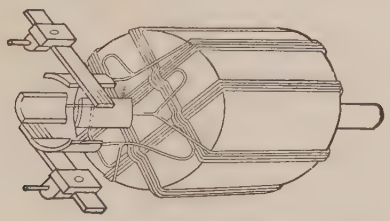

FIG. 142.—Drum-wound armature.

The ring core is similar to the drum, except that it is hollow and the wires are wound in and out and do not pass

all around the outside. Fig. 143 shows the ring method of constructing an armature.

Note that in a DRUM-wound armature *both sides* of each loop cut the lines of force, while in the RING-wound one side only cuts lines, the other side being on the inside of the ring, which is a space practically free from magnetic lines. See Fig. 145 for illustration of this fact.

Most cores are now made of the drum type, because it is simple in construction and mechanically stronger. The surface of such a core is generally not smooth, but slotted so that the loops of wire lie in grooves between projections

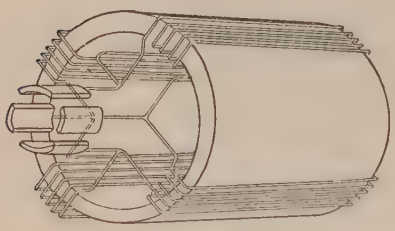

FIG. 143.—Ring-wound armature.

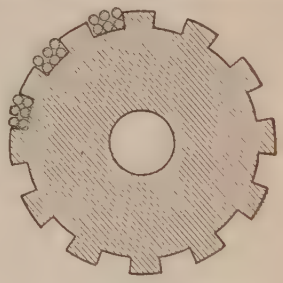

FIG. 144.—Slotted armature core, drum type.

called teeth. Fig. 144 shows an ideal cross-section of such a slotted armature.

Although the ring type is very rarely used, it affords the simplest diagram from which to explain the action in an armature. The action in a drum armature is essentially the same, but it is more difficult to represent it without complicated diagrams.

112. Actions within an Armature. Magnetization of the Core. When a soft iron ring is placed in the magnetic field between the poles of a 2-pole machine, the magnetic field may be represented as in Fig. 145.

Suppose now this soft iron ring were wound with six coils of wire and connected to six commutator segments as in Fig. 146. If this armature were rotated in the direc-

tion shown, the outside wires would cut lines of force, and there would be a tendency for an electric current to flow through the coils in the directions marked. Test this by the Right-Hand Rule.

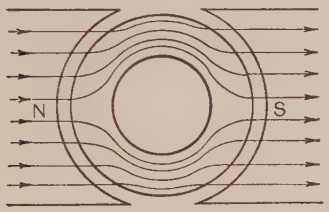

FIG. 145.—Field in core of ring armature when there is no current in the armature coils.

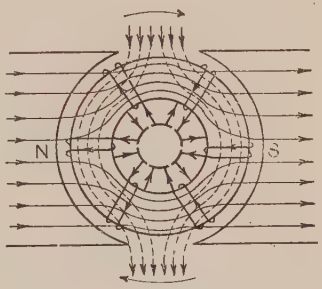

FIG. 146.—Dash lines show direction of field set up by current in armature coils.

A magnetic field would then be set up within the iron ring in the direction of the dash lines, i.e., the bottom of the ring would tend to become a north pole. We now

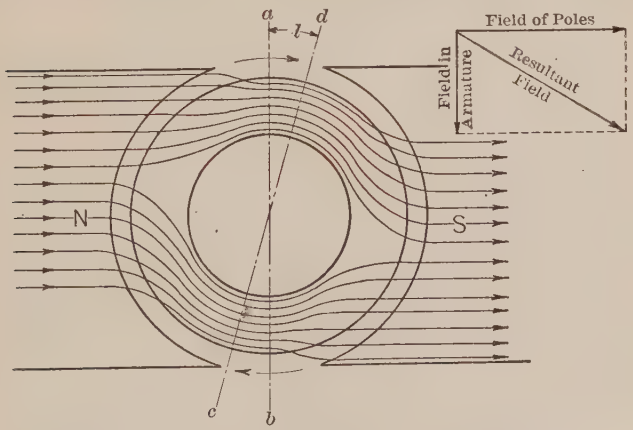

FIG. 147.—Resultant or distorted field in armature core.

would have two fields at right angles to each other. The resultant field is then the combination of the two, shown

in Fig. 147, just as in mechanics a single force is often the resultant of two forces at right angles to each other.

When any current flows in the armature and sets up a cross field in the armature core, the field is then said to be distorted.

The angle l measures the amount the field is distorted. This angle lies between *ab*, a line drawn perpendicular to the *original field*, and bisecting angle between axes of poles, and *cd*, a line drawn perpendicular to the *resultant* field.

113. Setting the Brushes. Neutral Axis. Axis of Least Sparking. The line *ab*, Fig. 147, is called the Neutral Axis, because it is perpendicular to the field at no load. It indicates the places in which a wire would be cutting no lines of force, if the armature were revolving but delivering no current. The line *cd* indicates the approximate positions of the brushes, in order that there be no sparking when the armature is delivering a current. This is called the Axis of Sparkless Commutation and is best explained by referring to Fig. 148.

Note the arrangement of the six coils in Fig. 148. Each is connected across the gap in the commutator, the ends being joined to adjacent segments. The segments then serve to connect all the coils in series.

The brushes B_1 and B_2 are placed 180° apart on about the axis *CD*, perpendicular to the resultant field. Notice that in a generator, this axis *cd* is *ahead of the neutral* axis *ab* which is perpendicular to line joining poles. This axis of commutation *CD* will be seen to be *behind the neutral axis ab* in a motor. The words "ahead" and "behind," refer to the position *relative to the motion* of the armature. If the motion were in the opposite direction, then the line *cd* would be said to be behind the line *ab*.

By applying the Right-Hand Rule to the outside wires in coils 2 and 3 (the inside wires cut no lines) we find that the current tends to flow from *right* to *left* across the face of the ring, coming *out of the face* of the ring in such a way

as to flow away from brush B_1, and toward brush B_2. The current in coils 5 and 6 goes *into the face* of the ring, but away from B_1 and toward B_2. Thus there are two currents, one in each half of the armature flowing in parallel *away* from B_1 and *toward* B_2. The brush B_1 would then be negative and B_2 positive, since B_2 would deliver current to the outside circuit, and B_1 receive it back to the armature.

The brushes, however, form a short circuit across the coils 1 and 4, when they are in just this position. But

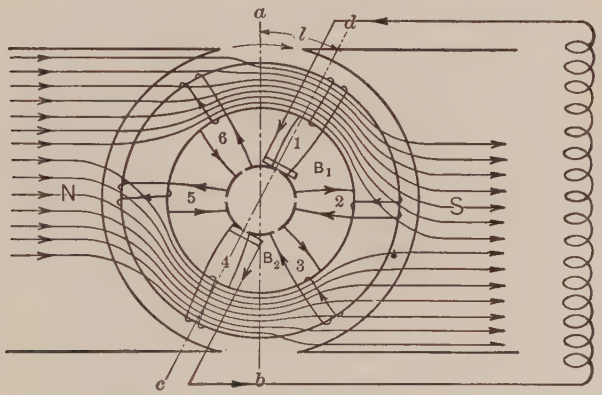

FIG. 148.—Position of brushes in D.C. generator.

note that in just this position *these coils are cutting no lines of force* and there is, therefore, no current flowing in them. This is the chief reason for the brushes being placed at this particular place, i.e., at axis of sparkless commutation.

Suppose, for instance, that the brushes were placed on the neutral axis *ab*. When the coils reached this position, they would be cutting lines of force and as the brushes would momentarily short-circuit them, a current would flow, which might be large enough to damage the coil. The greatest harm, however, would be done when the shorted coil moved out from under the brush and this current in it were broken. The breaking of the current would

cause an arc between the brush and commutator segment which would soon roughen and destroy the commutator.

It would appear from the above that in order to secure sparkless commutation, the brushes must be placed in such a position that such coils only are short-circuited as are cutting no lines of force, and thus when the short circuit is broken, there is no current to be broken and thus no arc is formed. This would place the brushes *exactly on the axis of sparkless commutation.*

There is, however, another point to be considered which causes this axis to be slightly off from a line perpendicular to the resulting field.

Consider brush B_2. Coil 3, which has not yet reached the brush, is delivering current to it. Coil 5, which has just passed away from the brush, is also delivering current to it. Yet the current in Coil 5 is flowing in the opposite direction to the current in Coil 3. The current in 3 is coming *out* of the face of the ring, the current in 5 is going *into the face* of the ring. Thus the current in a coil must not only *stop flowing* when it reaches the brush, but also must *reverse in direction* on leaving it. In order to stop the current flowing in this coil it has been found advisable to set up an E.M.F. in the opposite direction. See Chapter X on INDUCTION. This can be done easily if the brush is set a little ahead of the neutral axis, so that the coil has begun to cut lines in the opposite direction before it is short-circuited. The E.M.F. thus induced is in the opposite direction to the current that is flowing and thus quickly reduces that current to zero. Before this E.M.F. can set up a current of its own in the opposite direction in the coil, the brush has ceased to short-circuit it, and thus there is no "short-circuit current" to be broken. In this position the brush is said to lie on the "AXIS OF COMMUTATION," which in a generator is usually a little ahead of the line *CD*. The angle between the NEUTRAL AXIS and the AXIS OF COMMUTATION is called the "ANGLE OF LEAD."

For certain reasons concerning the magnetic disturbances which this *Lead* of the brushes causes, it should always be made as small as possible.

The actions taking place inside of a 6-coil armature in a 2-pole generator have been described because they are typical of all D.C. Generators, whether Drum or Ring

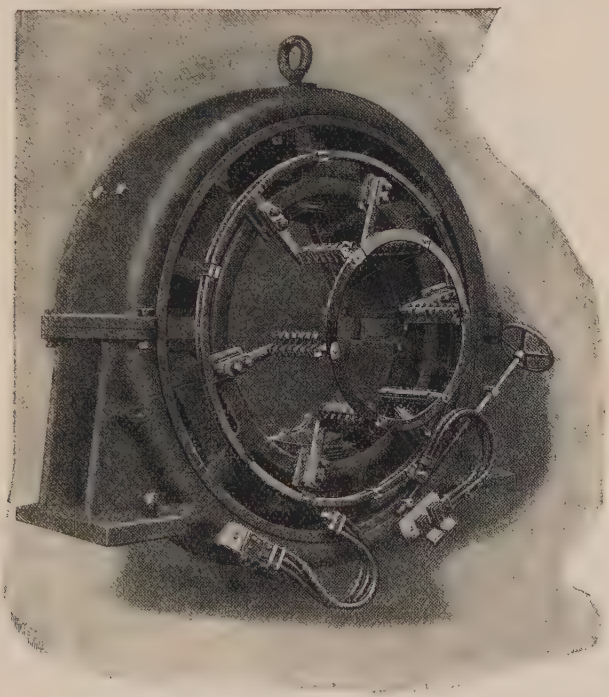

Fig. 149.—Arrangement of poles in a six pole Westinghouse generator.

wound, bipolar or multipolar. It would not be advisable to go into the different details of armatures of other types or into further details in this type, since these belong to a more advanced course.

114. Multipolar Generators. In order to obtain a voltage high enough for practical purposes, the armature

of a 2-pole generator has to be run at speeds which are prohibitive for machines of any size. Since the voltage depends upon the number of lines cut per sec., when great speed is not permissible, more lines of force must be added to the field. This is usually done by adding more poles. Fig. 37 shows a 4-pole generator. Notice that when the armature makes one revolution, each conductor cuts the

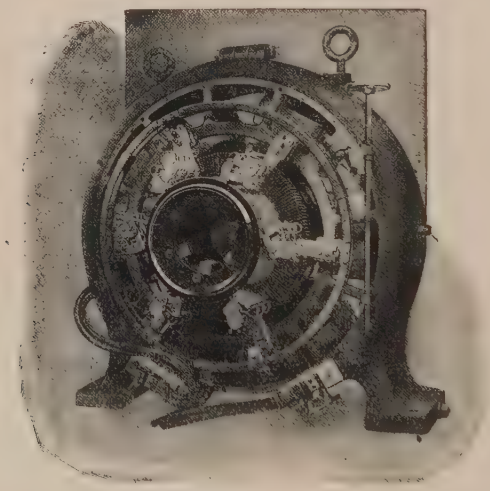

Fig. 150.—A six pole generator with armature in place. Westinghouse.

lines in the air gap *four times* instead of *twice*, as in a 2-pole machine. The speed can therefore be slowed down in proportion and practically the same voltage obtained, other conditions remaining unchanged. Fig. 149 shows the arrangement of the poles in a 6-pole generator.

Fig. 150 shows a 6-pole generator with armature in place. Notice that every other brush is connected to the same terminal, making three positive brushes and three negative. Fig. 150*a* shows this diagrammatically.

115. Resistance of a D.C. Armature. It has been shown that in a 2-pole generator, the current flows in two

parallel circuits within the armature, since the brushes put the two halves of the armature in parallel. Thus the resistance of the armature is one-half the resistance of either of these two circuits. If there were 4 poles to the machine and 4 brushes, the current would flow in 4 parallel circuits, and the ARMATURE RESISTANCE would be one-fourth that of any single path or circuit through the armature.

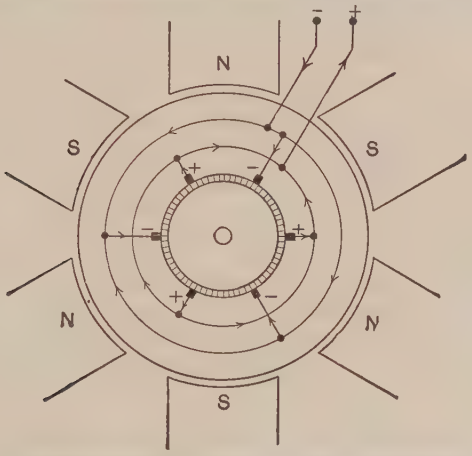

FIG. 150*a*.—Brush connection in a six pole generator.

Example. The armature of a 4-pole 4-brush D.C. generator is wound with 360 ft. of No. 18 (B. & S. gauge) copper wire. What is the armature resistance at 68° F.?

Resistance per 1000 ft. of No. 18 = 6.37 ohms.
" of 360 " " " " = 2.29 ohms.
" " 1 circuit $\frac{2.29}{4}$ = .573 ohm.
" " armature $\frac{.573}{4}$ = .143 ohm.

Problem 5–7. A six-pole 6-brush generator has 800 ft. of No 4 B. & S., copper wire wound on the armature. (*a*) What is the total resistance of the wire wound on the armature (20° C.)?
(*b*) What is the resistance of one path through the armature?
(*c*) What is the armature resistance from brush to brush?

Problem 6–7. What will armature resistance of generator in Prob. 5 become when the machine has been running a while and the temperature has risen 50° C.?

Problem 7–7. The armature resistance of a bipolar generator was measured at 20° C. and found to be 2.3 ohms. Size wire used was No. 14 B. & S. How many feet of wire were there on the armature?

Problem 8–7. What is armature resistance of generator Fig. 150, if 740 ft. of No. 6 B. & S. copper are used in winding armature?

116. Voltage across a D.C. Armature. The voltage induced in a D.C. armature is the sum of the voltages induced at a given instant in the separate coils forming *one circuit of the armature.*

In Fig. 148, the voltage across the brushes at the instant would be the voltage across coil 2 plus the voltage across coil 3.

Generally all the coils are not cutting lines at the same rate, and thus the voltage across each coil is different. Each coil then does not contribute an equal amount to make up the voltage across the brushes.

The average voltage across the brushes depends upon how many lines are cut per sec. by the wires forming *one* circuit of the armature between the brushes.

Therefore, to find the voltage which any D.C. armature is generating, it is merely necessary to find the number of lines of force that is being cut per sec. by those conductors only which lie between two adjacent brushes, since these conductors form one path through the armature.

Example. Assume that there are 198 active conductors on armature of Fig. 150. Speed is 800 R.P.M. Each pole has 5,000,000 lines. What E.M.F. is generated?

The number of conductors between any two adjacent brushes, that is, in one path in armature, must be:

$$\frac{198}{6} = 33 \text{ conductors.}$$

Voltage induced in these conductors equals lines cut per sec. divided by 10^8.

$$E = \frac{33 \times 5{,}000{,}000 \times 6 \times 800}{10^8 \times 60}$$

$$= 132 \text{ volts.}$$

Problem 9–7. Assuming resistance found in Prob. 8 as correct for armature in above example, what will be the brush potential of the generator when delivering 280 amperes?

Problem 10–7. The armature of a 4-pole 4-brush generator consists of 800 active conductors. Speed = 1200 R.P.M. ϕ for each pole is 6,000,000 lines. Find E.M.F.

Problem 11–7. Armature in Prob. 5–7 has 200 active conductors. R.P.M. = 740. ϕ = 4,000,000 for each pole. What is the E.M.F.?

Problem 12–7. What is the terminal voltage of generator in Problem 11 when delivering 400 amperes? Temperature is 50° C.

Problem 13–7. What is the terminal voltage of a bipolar generator, the drum armature of which has 960 active conductors? Speed = 1800 R.P.M. ϕ = 8,000,000.

117. Excitation of the Field of Generators. D. C. generators and motors are often classified with regard to the manner in which the field magnets are excited, as Separately Excited or Self Excited. The Self Excited are again divided into Shunt, Series, and Compound according to the manner of connecting the field to external circuit. When current for the field coils of a generator is taken from an outside source, the field is then said to be Separately Excited. When the exciting current is drawn from the armature of the machine itself, the field is said to be Self Excited.

The current through the field coils of a self-excited generator depends upon the brush potential. It is often desirable to maintain a field strength independent of brush potential, and separately excited generators are then used. This happens most frequently in the testing department of a manufacturing plant.

118. Separately Excited D.C. Machines. Self-excited machines sometimes change their polarity on starting and stopping. It is customary, therefore, to use a separately excited generator whenever it is desired that the current shall never change its direction. This point is of greatest importance in electroplating.

Fig. 151 represents the connections for a separately excited generator supplying current to a set of electroplating vats. The fields are excited from a storage battery, thus insuring a permanent polarity. Fig. 152 shows the

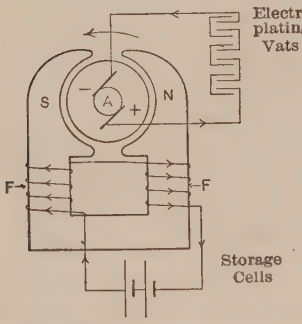

FIG. 151.—Separately excited generator supplying current to electro-plating vats.

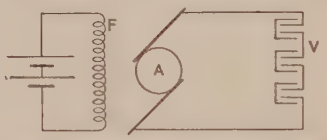

FIG. 152.—Conventional way of representing separately excited generator feeding electro-plating vats.

standard way of representing a separately excited generator, connected to electroplating vats *V*.

119. Self Excitation. Series-Wound. Fig. 153 shows a series-wound generator. Fig. 154 is the standard dia-

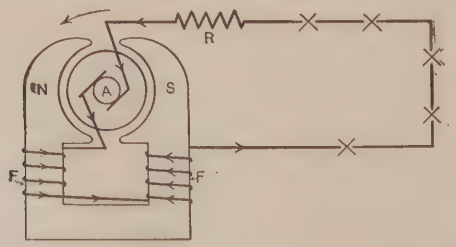

FIG. 153.—Series wound generator feeding series arc lamps.

gram for the same. The same current that goes through the main line of arc lamps also goes through the field coils.

The field is thus in *series* with the external circuit. When a series generator is not delivering current there is only the residual magnetism in the fields. Thus the voltage is almost zero.

The more current that is used by the line, the more highly magnetized the field becomes and thus the higher the voltage induced in the armature. The external circuit must therefore be arranged to keep about the same current flowing through the circuit and thus maintain the field constant. It would not do to have a great number of incandescent lamps in multiple on the line. Every time someone turned off a set, the voltage in all the rest would drop.

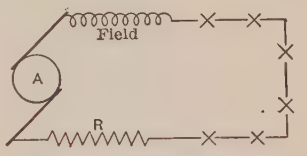

FIG. 154.—Conventional diagram for series generator feeding series arc lamps.

For this reason a series generator is called a Constant Current Generator, not because it, of itself, delivers a constant current, but because it works best on a circuit where a constant current is desired and maintained.

Arc lamps in series form one of the best loads of this nature. Several thousand volts are generated in order to force the current through these lamps. When a lamp is not in use, it is short-circuited so as not to break the main circuit. When several lamps are thus "shorted" the current (and thus also the voltage) is likely to become excessive, on account of the decreased resistance of the circuit. A resistance (R) may be introduced to take the place of the idle lamps. Another device for controlling the voltage is to put a variable resistance in multiple with the field coils, and thus make the amount of current going through the field coils any desired *fraction of the current* in the outside circuit, instead of all of it.

Owing to the high brush potential of the commercial

series generator, great care must be observed in handling such a circuit.

If we plot a curve between the amperes delivered by a series generator and the brush potential while running at a constant speed, we obtain a curve like that of Fig. 155. Notice that the curve does not start from the origin on account of the fact that there must be always some magnetism (residual) in the fields, even with no current through the field coils. If there is no residual magnetism in the field, the machine will have no induced voltage with which to start a small current flowing through the field coils in order to excite the field. Thus the machine would fail to "build up" a brush potential.

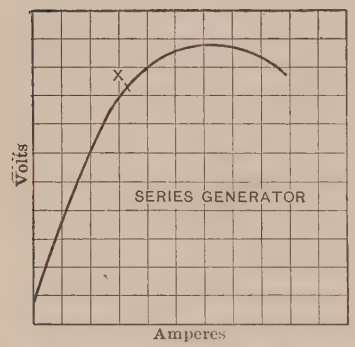

FIG. 155.—Relation of brush potential of series generator to current delivered.

Note also that the voltage increases very rapidly with any increase of the amperes delivered, until the point x is reached. At that point the fields are saturated, so that an increase in current through the field coils does not thereafter give a very large increase in magnetization, and, therefore, no great increase in voltage.

The fact that the voltage actually falls off if too much current is taken from the generator is due, in part, to the great magnetic disturbances produced in the armature by excessive currents through the armature windings, and in part to the increased armature drop.

The field coils of a series generator should be wound with a few turns of large wire, because the necessary ampere-turns must be obtained with as little resistance to the flow of the current as possible. This is because the current used in the field is the same as the current used in the line,

and as little voltage as possible should be used in forcing it through the field coils.

120. Shunt Generator. When the fields are excited by a current shunted around the main circuit as in Figs. 156 and 157, many turns of fine copper wire are used. In this case it is not desirable to have a large current going

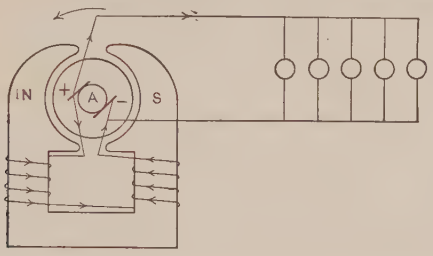

Fig. 156.—Shunt wound generator supplying incandescent lamps in parallel.

through the coils, because every bit thus used is really stolen from the outside circuit. The resistance of the coils is therefore high and the necessary ampere-turns are obtained by the large number of turns which make up for the small current.

A shunt generator "builds up" in the same way that a series generator does. The small amount of residual magnetism in the fields causes a small induced voltage in the armature. This voltage sends a slight current through the field coils, which increases the magnetization. Thus the induced voltage in the armature is increased. This, in turn, increases the current through the fields, which still further increases the magnetization, and so on, until the saturation point and normal voltage of the machine are reached. This "building up" action is the same for any self-excited

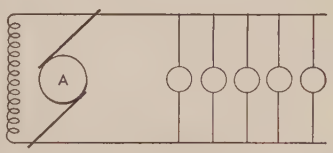

Fig. 157.—Conventional diagram of shunt generator supplying incandescent lamps.

generator and the process often requires from 20 to 30 seconds.

121. Voltage Regulation and Control. A shunt generator is used to supply a circuit where only fairly constant voltage is required, as it has only fair VOLTAGE REGULATION.

By voltage regulation is meant the percentage of "Full Load" VOLTAGE, that the VOLTAGE falls as the load rises from "no load" to "full load" (the speed remaining constant).

$$\textit{Voltage Regulation} = \frac{\text{no load } \textit{voltage} - \text{full load } \textit{voltage}}{\text{Full load } \textit{voltage}}.$$

By REGULATION is always meant, some change which a machine makes of its own accord when the load is changed. This change is then inherent in the machine, and depends upon the construction of the machine.

By CONTROL is always meant some change which an attendant brings about in a machine, as the raising of the voltage by cutting out resistance from field circuit.

If the shunt generator is run at constant speed, as more and more current is drawn from the generator, the voltage across the brush falls, though but slightly. This fall is due to the fact that it requires more and more of the generated voltage to force this increased current through the windings in the armature. This necessarily leaves a smaller part of the total E.M.F. for brush potential. And then, when the brush potential falls, there is a slight decrease in the field current, which depends upon the brush potential. This causes the total E.M.F. to drop a little, which still further lowers the brush potential.

These two causes combine to gradually lower the brush potential to a noticeable degree, especially at heavy overloads. The curve in Fig. 158 shows these characteristics of a shunt generator. At first, for small loads, the curve runs nearly horizontal, but at heavy loads it shows a decided drop.

The Voltage (brush potential) may be kept fairly constant by automatic devices for regulating the speed of the "prime mover," which may be a water wheel or a steam engine. By this means the speed may be increased whenever the brush potential falls a little, thus increasing the total E.M.F. and thereby raising the brush potential.

Another method is to have an extra resistance in the field circuit, which may be cut out as the brush potential falls. This will allow more current to flow through the field coils and increase the number of lines set up in the magnetic circuit. If the speed is kept constant, the armature conductors cut through the stronger magnetic field at the same speed, and thus induce a greater E.M.F. and restore the brush potential to its former value. This resistance may be cut out either automatically or by hand.

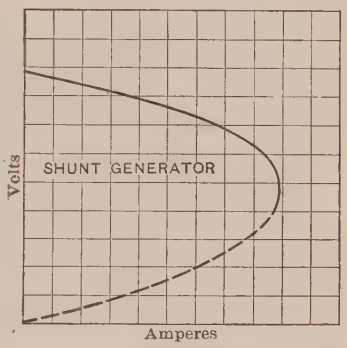

Fig. 158.—Relation of brush potential of shunt generator to current delivered.

Example. The resistance of the armature of generator in Fig. 156 is .16 ohm. When running with no load at rated speed the terminal voltage is 130 volts. (*a*) What will the terminal voltage be when the generator is delivering 50 amperes, assuming field remains unchanged? (*b*) What is the voltage regulation on this basis?

Armature drop $= 50 \times .16 = 8.0$ volts.

E.M.F. $-$ arm. drop $=$ terminal voltage.

$130 - 8 = 122$ volts.

$$\text{Voltage regulation} = \frac{\text{volts at no load} - \text{volts at full load}}{\text{volts at full load}}$$

$$= \frac{130 - 122}{122}$$

$$= \frac{8}{122}$$

$$= 6.5\%.$$

Of course the field would also decrease, due to the current in the field coils becoming less as the terminal voltage fell. This would cause the terminal voltage to fall to a point still lower than 122 volts.

Problem 14–7. In the generator of Fig. 156, the brush potential falls from 130 volts at no load to 116 volts at full load. What is the voltage regulation?

Problem 15–7. Assuming 12 volts of the fall in voltage in Problem 14–7 to be due to pressure used in forcing current through armature, what is the full load current of machine? Resistance of armature = .16 ohm.

Problem 16–7. Assume the resistance of armature in Fig. 150 is .020 ohm. Voltage at no load is 135 volts. When shunt field alone is used, the voltage regulation is 7 per cent. What is the full-load voltage?

Problem 17–7. Assuming 90 per cent of the change in voltage from no load to full load in generator of Problem 16 is due to armature drop, how much current does each path in armature carry at full load?

122. Compound Generators. Another method for keeping the voltage constant is to add a set of series windings to the field, as in Figs. 159, 160 and 161. These series coils

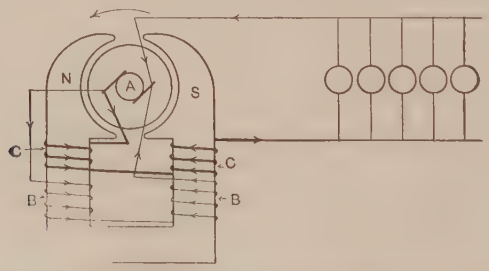

Fig. 159.—Compound generator feeding incandescent lamps in parallel.

will tend to increase the magnetization of the field as the current increases, just as the coils of a series generator. This increase in the magnetization of the fields as the current is increased will cause the voltage to rise and thus

compensate for the fall in voltage that would take place if the machine were a simple shunt generator.

If the series coils are adjusted so that the voltage remains practically constant at all loads, the generator is said to be *Flat-Compounded.*

It is sometimes desirable for the brush potential to actually rise in order to make up for the increased "line drop," etc., when the load increases. In this case, the machine is said to be *Over-Compounded.* By this method a constant voltage might be maintained at a point some distance from the generator under varying loads.

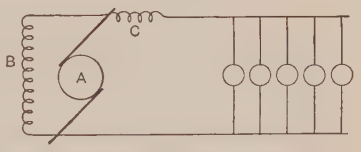

FIG. 160.—Diagram of short shunt compound generator.

There are two ways of connecting the shunt field in compound generators. One method, shown in Fig. 160, is to connect the shunt coils *B* around the armature *A* only. This is called a Short Shunt.

The other method, shown in Fig. 161, is to connect the shunt coils *B* around both the armature *A* and the series coils *C*. This is called a Long Shunt.

If three generators of the same shape and K.W. output were to have the fields wound in the three different ways, *series, shunt,* and *compound,* the number of *ampere-turns* would be the same in each. As far as efficiency goes, one type has no advantage over the others. The question of selecting any one of the three depends upon the use to which it is to be put.

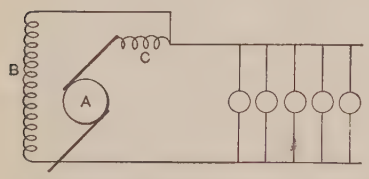

FIG. 161.—Diagram of long shunt compound generator.

When several shunt or compound machines are used in one station, as is very common, they are almost always

operated in parallel. Series machines are operated in series whenever used in combination. This, however, is not a common practice.

Example. Assume that there are 9000 turns in the shunt field of generator, Fig. 150. No load voltage equals 120 volts. Current through shunt field coils at 120 volts, is .8 ampere. In order that the full load voltage equal 120 volts, it is found that 9600 ampere-turns are necessary. If the full load current is 100 amperes, how many turns must be wound on the series fields to flat compound the machine?

Ampere-turns in shunt field $= 9000 \times .8 = 7200$

Ampere-turns to be added in series field $= 9600 - 7200 = 2400$

Turns to be added $= \frac{2400}{100}$

$= 24$

Problem 18–7. How many series turns must be added to the field of above example if the full load current is 200 amperes?

123. Losses in a Generator. In Chapter IX it is stated that there are three sources of loss in a generator.

(1) Mechanical losses: friction, windage, etc.

(2) Copper loss (I^2R): in field and armature windings.

(3) Iron losses: hysteresis and eddy currents.

The value of the mechanical losses may be obtained by mechanical means.

The copper loss we have seen is computed by multiplying the square of the current in the armature by the resistance of the armature and adding to that, the product of the square of the current in the field multiplied by the resistance of the field, i.e.:

$$P_a = I_a^2 R_a$$

$$P_f = I_f^2 R_f$$

$$P_a + P_f = \text{Copper loss.}$$

Of the iron losses we have seen that the part due to hysteresis may be determined from the equation

$$P = \frac{KB_{max}^{1.6}}{10^7}$$

This gives the loss in watts for each cubic centimeter of iron going through the magnetic cycle once every second.

It remains to discuss the EDDY CURRENT LOSS.

124. Eddy Current Loss. We have seen that when any conductor cuts lines of force, there is always set up an induced electromotive force which tends to send an electric current along the conductor. This is the fundamental principle underlying the construction of the generator.

The copper wires of the armature which are to carry this induced current are wound on an iron core. This iron core is, itself, an electrical conductor, and when it revolves in a magnetic field, it cuts lines of force. This sets up an E.M.F. across certain portions of the core which tends to send an electric current through the core.

Currents thus set up are called FOUCAULT or EDDY CURRENTS, and if they are allowed to flow around in the iron they will soon heat up the core and damage the windings on it. Also, it is always true, that there is required an expenditure of power to set up an electric current. As we cannot use these eddy currents within the armature core, the power used in generating them is a dead loss. It is therefore necessary to reduce these eddy currents to as small a value as possible.

This is done by laminating the core; that is, building it up of thin sheets of iron, insulated from one another by japan or iron rust. This does not much affect the magnetic circuit, but cuts down these electric currents, since the laminations are " transverse " to the direction in which they tend to flow. The thinner the laminations, the more effective is this method in decreasing the eddy current loss. Fig. 162 is a photograph of a pole face laminated in this way.

There is no formula sufficiently accurate to determine the eddy current loss in the core of a given generator, owing largely to the impossibility of knowing the insulation resistance between the plates. For this reason the designer of a generator usually "estimates" rather than "calculates" the probable eddy current loss of his machine by means of tables made up from data taken on similar machines.

It is well to remember, however, that the eddy current loss is proportional to the *square of the speed*, to the *square*

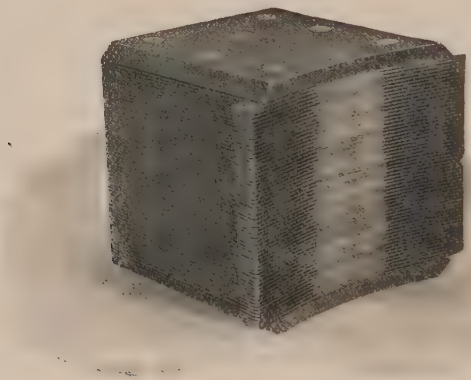

Fig. 162.—Laminated pole piece. Westinghouse generator.

of the flux density, and to the *square of the thickness* of the laminations. When slotted armature cores are used, it is customary to laminate the pole faces as well as the armature core. Fig. 163 explains the reason for this.

The teeth, between which the conductors lie, become more densely magnetized than the slots, and act like small poles. When they move across the pole face, therefore, the pole face is really cut by lines of force and small eddy currents are set up as shown in Fig. 163(*b*). Laminating the pole face breaks up the electric circuits and diminishes these currents.

For a discussion of the use of EDDY CURRENTS in damping watt-hour meters, etc., see Chapter XIV. For their

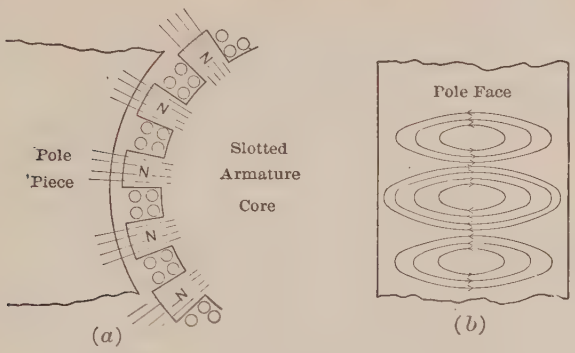

FIG. 163.—Action of slotted armature on pole face.

effect upon the efficiency of generators and motors see Chapter IX.

SUMMARY OF CHAPTER VII

ELECTROMAGNETIC INDUCTION. When an electric conductor cuts magnetic lines of force, an E.M.F. is set up across the conductor, proportional to the rate of cutting.

DIRECTION OF INDUCED E.M.F. (RULE.) If the Thumb, Forefinger, and Middle Finger of the Right Hand are held at right angles to one another, with the Thumb extended in the direction of the motion of the Conductor, the Forefinger in direction of the Flux, then the Middle Finger will indicate the direction of the Induced E.M.F.

AMOUNT OF E.M.F. When 10^8 magnetic lines are cut per second, one volt of E.M.F. is induced.

MODERN GENERATORS are, fundamentally, loops of wire revolved so as to cut through a strong magnetic field. An Alternating E.M.F. is induced in these loops, the curve of which approximates a Sine Curve.

If Collecting Rings are attached to ends of these coils, an alternating current is delivered by the Generator.

If a COMMUTATOR is used, a direct current is delivered. Since the armature coils are each connected across a gap between two commutator segments, the brushes must continually short circuit some two coils.

ANGLE OF LEAD. AXIS OF COMMUTATION. The brushes must be so placed that the coils short circuited by them are not cutting lines of force at the instant of the short circuit. This position, owing to the cross magnetization due to the current in the armature coils, is on a line ahead of the neutral axis by the angle of lead. This position is called the AXIS OF COMMUTATION.

THE NEUTRAL AXIS may be defined as a line which bisects the angle between the axes of the poles.

THE E.M.F. INDUCED IN a D.C. ARMATURE equals $\frac{1}{10^8}$ of the number of lines cut per second by the conductors in series between any two adjacent brushes; that is, in any single armature path (there being as many paths as brushes.)

THE ARMATURE RESISTANCE, from plus terminal to negative terminal, equals the resistance of the wire wound on the armature divided by the square of the number of brushes.

D.C. GENERATORS are divided into two classes: 1st, Separately Excited; 2d, Self Excited.

SEPARATELY EXCITED Generators have their fields excited by some outside source of current, and are used (1st) wherever the field strength must be independent of the terminal voltage; (2d) wherever a given polarity is the essential factor. They have very limited use.

SELF-EXCITED machines are divided into three types, according to the method of connecting the field coils to the receiving circuit:

(1st) Series.

(2d) Shunt.

(3d) Compound.

SERIES. All the current delivered to the line flows through field coils, which consist of a few turns of heavy wire. Voltage rises as load increases. Operates satisfactorily on a constant current line.

SHUNT. Field is connected in parallel with line and only a small current goes through coils, which consist of many turns of fine wire. Is a nearly constant potential generator, the voltage falling slightly as the load increases. Voltage may be controlled somewhat by field rheostat.

COMPOUND. The field consists of two sets of coils; one series and the other shunt. When enough series turns are wound on, to offset exactly any fall in terminal voltage due to increased armature drop, etc., the machine is said to be Flat-Compounded. When terminal voltage rises slightly with load, it is said to be Over-Compounded.

EDDY CURRENT LOSS. The losses in a generator are

1st, Mechanical losses; (friction and windage.)

2nd, Copper losses; (I^2R losses.)

3rd, Iron losses; (hysteresis and eddy currents.)

Eddy currents are local currents set up within the core of the armature and of the poles, when they cut magnetic flux. They cause heating and consume power; are greatly diminished by the use of laminated parts.

There is no method by which the eddy-current losses in modern machines may be computed to any degree of accuracy. The losses are, accordingly, estimated by comparison with machines of similar design and characteristics.

PROBLEMS ON CHAPTER VII

19–7. An armature has 100 conductors connected in series. If these conductors cut 800 times a minute through a field of 5,000,000 lines, what average voltage is induced?

20–7. Assuming that the resistance of the armature conductors in Problem 19 is 20 ohms, and if the external circuit is zero, what power is consumed in the armature circuit?

21–7. Assume the external resistance to be 100 ohms, instead of zero, what power is consumed in the complete circuit in Problem 20?

22–7. (*a*) How much power is consumed by the armature alone in Problem 21–7? (*b*) What is the brush potential?

23–7. Assuming no friction, how many H.P. would be required to drive armature of Problem 21?

24–7. The drum armature of a 4-pole generator has a speed of 1200 R.P.M. Number of active conductors = 418. Number of brushes = 4. ϕ, per pole, = 4,000,000. What E.M.F. does it generate?

25–7. If the armature in Problem 24, is wound with 5600 ft. of No. 10. B. & S., what is the armature resistance?

26–7. What current will generator in Problem 25 deliver when connected to an external circuit of 2.4 ohms?

27–7. What will be the brush potential of generator in Problem 26?

28–7. How would you rate the generator of Problem 27, if this is its full load; (*a*) as to K.W. output; (*b*) as to its voltage regulation?

29–7. How much power is used in each armature winding of Problem 27?

30–7. A ring armature of a 2-pole generator has 480 turns, a speed of 1000 R.P.M. ϕ, per pole, = 8,000,000 lines. What E.M.F. is induced?

31–7. The armature of a bipolar generator has 450 conductors. ϕ = 6,000,000 lines. Induced E.M.F. = 120 volts. What is the speed in R.P.M.?

32–7. The armature of a 6-pole generator has 1200 active conductors. The speed is 900 R.P.M. Area of each pole face is 800 sq.cms. Flux density in air gap = 7500 gausses. Number of brushes 6. What E.M.F. is induced?

33–7. Armature resistance in Problem 32 is .12 ohm. What power is generator delivering to external circuit at full load if the brush potential falls 24 volts?

34–7. What is voltage regulation in Problem 33?

35–7. If ring armature of Problem 30 were used in an 8-pole, 8-brush generator, what voltage would be induced, other data remaining unchanged?

36–7. What would the armature resistance be if armature of Problem 25 were used in a 6-pole 6-brush generator?

37–7. What voltage would be induced in armature of Problem 24, if used on a 6-pole, 6-brush generator, and other data of problem remained unchanged?

38–7. The resistance of the field coils of a 1 K.W. shunt generator is 220 ohms. Full load brush potential is 110 volts. (*a*) What current is used by field coils? (*b*) Power consumed in field coils equals what?

39–7. (a) How many turns are there in shunt field of Problem 38, if the ampere turns equal 1000? (*b*) If machine were a series generator of same output, how many turns would be required?

40–7. A 12-pole, 12-brush, 750-K.W. generator, has a terminal voltage of 660 volts on full load. There are 5000 ft. of No. 4, B. & S. copper wire wound on armature. What E.M.F. must be induced in armature?

41–7. If there are 100 active conductors in each circuit of generator in Problem 40, what must the flux per pole be? Speed = 120 R.P.M.

42–7. How much would the flux of generator in Problem 31–7, have to be increased in order to have a terminal voltage of 115 volts, when delivering a current of 20 amperes? Armature resistance = 1.8 ohms.

43–7. The total resistance of a series generator is .12 ohm. When delivering 10 amperes, the terminal voltage is 100 volts. Find the terminal voltage when machine runs at same speed, but delivers 20 amperes. Assume the field strength increases 50 per cent with increased current.

44–7. In compound generator, Fig. 164;
Resistance of arm. $(A) = .04$ ohm;
" " shunt field $(L) = 550$ ohms;
" " series " $(S) = .03$ ohm.
When delivering 60 amperes to line at 550 volts, (*a*) What is E.M.F. of generator? (*b*) How much current flows through armature?

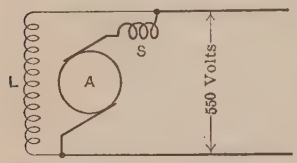

FIG. 164.—Long shunt compound generator.

45–7. The drum armature of a bipolar generator is wound with 600 turns of No. 8, B. & S. copper wire. Each turn is 3.5 ft. long. What is the armature resistance?

46–7. Allowing 1000 amperes per sq.in. of cross-section of conductor, how many amperes should generator of Problem 45 deliver?

47–7. Assuming that 580 of the turns on the armature of Problem 46 are active, what E.M.F. must be generated to maintain a brush potential of 110 volts at full load?

48–7. What must be the value of ϕ in Problem 47? R.P.S.=20.

49–7. What would be the resistance of armature and the current delivered to the brushes in Problem 46, if the armature were used in an 8-pole, 8-brush generator?

50–7. An 8-pole, 8-brush generator has 640 active conductors on armature which makes 180 R.P.M. No load voltage is 115 volts. Area of each pole face is 580 sq.cms. What is the flux density?

51–7. When generator in Problem 50 is delivering 2000 amperes the brush potential is 110 volts. If the armature resistance is .004 ohm, what must the flux density be at this load? Same speed.

52–7. The resistance of the armature in a bipolar shunt generator is .40 ohm. The field resistance is 50 ohms. If the generator delivers 40 amperes to the line at a brush potential of 100 volts, what E.M.F. must it generate?

53–7. What current flows in each armature circuit of generator in Problem 52?

54–7. The resistance of the armature of a series generator is .20 ohm; of the field, .25 ohm. The generator supplies 25 arc

lamps, each of which takes 9.5 amperes at 55 volts. What must be E.M.F. of generator?

55–7. Assume that 1 mile of No. 10, B. &. S., copper wire is used in line of arc lamps in Problem 54. (*a*) What E.M.F. must be generated? (*b*) What is brush potential?

56–7. The resistance of the armature of a 10-pole, 10-brush generator is .005 ohm. ϕ, per pole, = 8,000,000 lines. Speed = 180 R.P.M. Generator delivers a current of 1200 amperes at 115 volts. How many active conductors are there in the armature?

57–7. Each lamp in Fig. 164*a* takes 25 amperes at 110 volts. Armature has .04 ohm resistance.

Find:

(1) E.M.F. of generator.

(2) Current in armature.

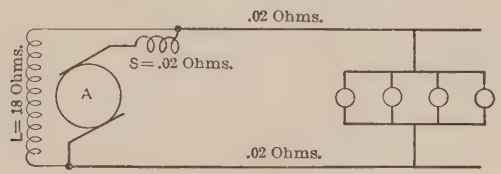

FIG. 164*a*.

58–7. Find E.M.F. and armature current of machine in Problem 57, when connected as a short-shunt compound generator and supplying same circuit.

59–7. Assuming that the same power is lost in the field windings of both machines in Problem 39–7, what must be the resistance of the series field?

60–7. In designing a generator to have an E.M.F. of 230 volts, a drum armature is wound with 400 conductors. Two brushes are used which reduce the number of active conductors to 360. The flux per pole is 3,000,000 lines. How many poles must generator have in order to run as nearly as possible at a speed of 480 R.P.M.?

61–7. What would the flux per pole in Problem 60 have to be to produce exactly the 230 volts desired?

62–7. A compound generator, short shunt, is delivering power to the following pieces in multiple. 1000 20-c.p., 110-volt

tungsten lamps, taking 1.25 watts per candle. One 110-volt 15-H.P. motor, with an efficiency of 90 per cent. The resistance of shunt field of generator = 35 ohms; of series field, .007 ohm; of armature, .014 ohm; of line wire, .03 ohm.

Find:

(*a*) Armature current.

(*b*) Field current.

(*c*) E.M.F.

63–7. What are the terminal voltage and K.W. rating of generator in Problem 62?

64–7. If a long-shunt generator were used, what would be the answers to Problems 62 and 63? All other data as given in Problem 62.

65–7. From the following data, find the number of ampere-turns required in the field coils of a 115-volt 2-pole generator. Armature has 280 active conductors. Speed = 1800 R.P.M. Magnetic circuit is made up of the following parts in series: 120 cms. of cast steel, 250 sq.cms. cross-section; 20 cms. of wrought iron 240 sq.cms. cross-section; 1.4 cms. air gap, 260 sq.cms. cross-section. Assume no leakage of magnetic lines.

CHAPTER VIII

MOTORS

Force on Wire in Magnetic Field—Torque of Motor—Power Necessary to Drive Generators—Damping of Electrical Instruments—Counter E.M.F.—Armature Reaction in Motors—Direction of Rotation of Motor—Shunt Motor—Starting Box—Speed Regulation and Control of Shunt Motor—No Voltage Release—Overload Release—Starting and Stopping Shunt Motor—Series Motor: Starting—Series-Parallel Control—Compound: Differential and Cumulative—Motor and Generator Characteristics Compared.

In order to understand the operation of a motor, it is necessary to refer to Figs. 27 and 28, Chapter II.

Fig. 27 shows the circular magnetic field about a wire carrying a current. Fig. 28 shows the effect produced

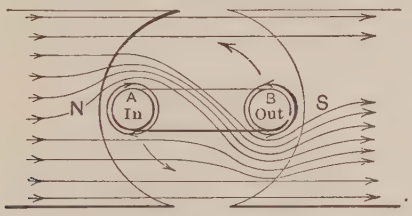

Fig. 165.—Field about a loop carrying an electric current in a magnetic field.

by placing this wire in a magnetic field. The result is a condensation of the lines above the wire and a thinning out of the lines below the wire. The "rubber-band" effect of the lines then causes a downward push on the wire.

125. Force on Wire in Magnetic Field. Torque. Fig. 165 shows a cross-section of a single loop in a magnetic

field. If a current is sent *in* at A and *out* at B, as marked, the flux would be weakened *below* A and *above* B. Thus the loop would tend to revolve counter-clockwise as marked.

The force with which it tends to revolve is proportional to the *length of the wire, the field strength, and the current in the wire.* This force, in dynes, may be computed from the following equation:

$$F = \frac{IHl}{10}$$

where F = force on wire in *dynes;*
I = current in *amperes;*
H = field intensity in gausses;
l = length of wire in *centimeters.*

Example. A wire 15 cms. long carrying 40 amperes, is in a field of 20,000 gausses. What is the force acting on the wire?

$$F = \frac{IHl}{10}$$
$$= \frac{40}{10} \times 20{,}000 \times 15$$
$$= 1{,}200{,}000 \text{ dynes}$$
$$= 1225 \text{ grams}$$
$$= 2.7 \text{ lbs.}$$

Example. Loop in Fig. 165 has a mean diameter of 16 cms. and is carrying 12 amperes. Each side of loop is 20 cms. long. Field intensity between the poles is 15,000 gausses. What torque is developed with the loop in this position?

$$F = \frac{IHl}{10}$$
$$= \frac{12}{10} \times 15{,}000 \times 20$$
$$= 360{,}000 \text{ dynes, on each wire}$$
$$= 368 \text{ grams}$$
$$= .81 \text{ lb.;}$$

$$16 \text{ cms.} = \frac{16}{2.54 \times 12} \text{ ft.}$$

$$= .525 \text{ ft.};$$

$$\text{Torque} = \text{force} \times \text{arm}$$

$$= \text{one of couple} \times \text{distance between them}$$

$$= .81 \times .525$$

$$= .426 \text{ lbs. ft. torque.}$$

It is a torque built up in this way that causes the armature of a motor to rotate. There are many loops, each adding its share to make up the total torque. When there is no current flowing in the loop there is no force acting on the sides of the loop.

Problem 1–8. A wire 20 cms. long lies at right angles to the lines in a magnetic field. The force on the wire is 1.4 lbs. when 25 amperes flow through it. How strong is the magnetic field?

Problem 2–8. A 28-cm. wire is urged perpendicular to lines in a magnetic field of 20,000 gausses by a force of 2 lbs. What current is flowing in wire?

Problem 3–8. (*a*) Indicate resultant field if poles and current are as marked in Fig. 166.

(*b*) Indicate direction in which loop will tend to turn.

(*c*) If flux is 8000 gausses, length of sides of loop 25 cms. each; current through loop, 80 amperes.; what force in lbs. is there on each wire?

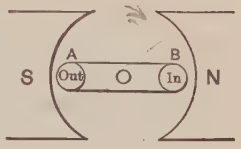

Fig. 166.

Problem 4–8. Loop in Problem 3 has a diameter of 15 inches, what torque does loop possess?

Problem 5–8. If the field strength of Problem 4 were doubled, what would be the torque?

126. Power Necessary to Drive Generator. The fact that there is a force on a wire in a magnetic field when it carries a current explains why it takes power to drive a generator when it is delivering current and why it does *not take power* when the generator is *not delivering current*, even though a high voltage may be developed.

When the generator is not delivering current, practically no current is flowing in the armature windings, thus there is no force to be overcome. But as soon as a current flows through the armature, this force is present, as explained in Fig. 165, and must be overcome. In other words, the torque that we get in a motor when we send a current through the armature, is exactly the torque we have to overcome when we desire to run the machine as a generator and draw this amount of current from it. This fact may be illustrated as follows:

Suppose we wish to use the loop in Fig. 165 as a generator to *set up a current in the direction marked.* By applying the right-hand rule, it is seen that the direction of *rotation must be the reverse* of the direction it has as a motor. This shows very clearly that the torque, which it has as a motor, must be overcome, when the machine is run as a generator and supplies the current to the line.

Problem 6–8. A single coil armature as in Fig. 166 is delivering 10 amperes. Wires are 15 cms. long and the field strength is 12,000 gausses; diameter of coil is 9 inches. What torque is needed to turn armature when in this position if there is no friction?

Problem 7–8. In what direction would coil in Problem 6 have to turn in order to *deliver* the current as marked in Fig. 166?

127. Power Lost in Setting up Eddy Currents. The above also explains how laminating the armature cores decreases the eddy current loss, by decreasing the amount of current in the "eddies." Thus little torque is wasted in overcoming any opposing torque, set up by these currents.

128. "Damping" of Electrical Instruments. Again it explains the "damping" of electrical measuring instruments. In the Weston D.C. Ammeters and Voltmeters, for instance, the moving coil is wound on an aluminum bobbin. When this bobbin is moved in the magnetic field, currents are generated in it by the cutting of lines of force. These currents then produce a torque on the bobbin which

opposes the motion, and thus the tendency to keep oscillating is constantly opposed by the torque of the current induced in the bobbin.

129. Counter Electro-Motive Force. We have just been considering the "*motor-effect*" that is present *in a generator* when it is delivering a current. We have seen how this "motor-effect" is produced by the current flowing in the armature coils and that it is opposite to the motion of the armature of a generator.

Similarly, there is a "*generator effect*" *in a motor* which is called the COUNTER or BACK ELECTRO-MOTIVE FORCE.

This will be clear when we consider that a motor consists of a number of conductors turning in a magnetic field. Now if conductors move in a magnetic field so as to cut lines of force, an E.M.F. will be set up, whether we call the machine a generator, a motor, or anything else.

If we apply the RIGHT-HAND RULE to the motor coil turning as in Fig. 165, we find that there must be a voltage induced in *A*, which would tend to send a *current* OUT. The current is *actually sent* IN at *A* to cause the coil to turn. By applying the rule to *B*, we find that here, too, there is a voltage induced which would tend to send a *current* IN, while the current is *really flowing* OUT of *B*.

This pressure, which we find in each case opposes the flow of the current through the armature, is the COUNTER E.M.F.

It is called the Counter E.M.F. because it is in the opposite direction to the E.M.F. impressed across the armature to cause it to turn as a motor.

There are then two sources of pressure in the windings of a motor that is running: one, the voltage which is impressed on the terminals from an outside source; the other, the voltage which is set up by the windings in cutting lines of force. These two pressures are in opposite directions, and thus the current that flows must be proportional to their difference.

We might represent this graphically as in Fig. 167. The arrow E_x represents the impressed voltage across the motor. The counter E.M.F., generated when the motor is running would be represented by a slightly shorter arrow, E_c, pointing in the opposite direction. The voltage E_x tends to send a current *down* through the armature, while the counter E.M.F. tends to send a current *up*. The current will flow *in the direction* of the greater pressure E_x. The pressure under which it flows, however, will be the difference between the two forces $(E_x - E_c)$. This is analogous to the way any mechanical body behaves when acted upon

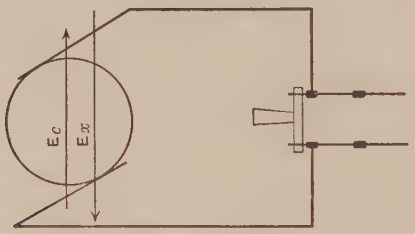

FIG. 167.—*Ex* represents impressed E.M.F. *Ec* represents counter E.M.F.

by two opposing forces. It moves in the direction of the greater force, and accelerates at a rate proportional to the difference between the opposing forces.

Thus if the impressed voltage $E_x = 110$ and the counter E.M.F. $= 106$ volts, the resulting voltage, forcing the current through the armature of the motor, equals $110 - 106$, or 4 volts. And the result, as to current flow, is just as though there were but 4 volts drop across the armature. That is, if the armature resistance of this motor were .5 ohm, the flow of current would be $\frac{4}{.5} = 8$ amperes.

Example. The armature resistance of a motor is .24 ohm. If it has a counter E.M.F. of 108.8 volts when running on a 110-volt circuit, what current does it take?

$$\text{Effective volts} = 110 - 108.8 = 1.2;$$

$$I = \frac{E}{R};$$

$$I = \frac{1.2}{.24} = 5 \text{ amperes.}$$

Problem 8–8. The impressed voltage across a motor armature is 115 volts; the counter E.M.F. = 112 volts; current = 6 amperes. What is resistance of armature?

Problem 9–8. It is desired to find the counter E.M.F. in a motor armature at a certain speed. Armature resistance = .2 ohm. Impressed voltage = 112 volts. Current = 16 amperes.

Problem 10–8. If speed in Problem 9 were reduced one-half and field remained the same, what current would flow in the armature?

Problem 11–8. Suppose armature in Problem 9 were stopped altogether, what current would flow?

130. Reaction in Armatures of Motors. Any generator may be used as a motor.

Assume the armature of a bipolar generator to run in a certain direction. Instead of taking a current from it, suppose we send the current through it, only *in the opposite direction* to the one it is generating. The machine will then run as a motor and in the same direction in which it was being run as a generator. That is, if current in the machine as a motor flows in the opposite direction to its current as a generator, the machine turns in the same direction in both. Of course, the change in direction of the armature current will magnetize the armature core in the opposite direction. Thus the field will be distorted in the opposite direction, and the axis of commutation will be shifted back *behind the neutral axis.* The brushes therefore must be shifted to about this line in order to maintain sparkless commutation.

A comparison of Figs. 148 and 168 will make this clear. Fig. 148 represents a six coil armature of a bipolar D.C.

generator. Fig. 168 represents a six coil armature of a bipolar D.C. motor. Notice that the direction of rotation is the same. The direction of the current through the coils is opposite. The field distortion is *ahead* in Fig. 148, and *back* in Fig. 168. The axis of commutation *cd* of the generator (Fig. 148) is *ahead of the neutral axis*. The axis of commutation *cd* of the motor is *behind the neutral axis ab*.

The brushes of the generator are thus said to have a FORWARD LEAD.

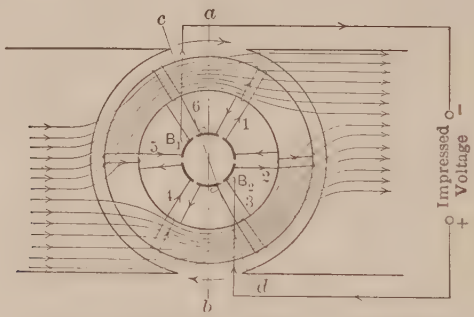

FIG. 168.—Position of brushes in D.C. motor.

The brushes of the motor (Fig. 168) have a BACKWARD LEAD.

The brushes of the generator in Fig. 148, it was said, are set a little *forward even of cd*. This is in order to bring the current in the shorted coil to zero as soon as possible, by causing a slight voltage to be set up in it in the opposite direction to which this current is flowing. For a similar reason the brushes of the motor are set a little *behind cd*.

Consider coil No. 6 shorted, at a certain instant, by the negative brush B_1 of the motor. As it reaches the brush there must be a current flowing in it in the direction of the current in coil No. 5. In order that this current may stop flowing while the brush shorts the coil, the brush is made to short it just *before* it moves to a neutral spot and

ceases to cut lines. Of course the cutting of the lines in a motor always sets up a back E.M.F. which opposes the current actually flowing. So the E.M.F. now set up in this shorted coil No. 6 opposes the current that is flowing in it and quickly brings it to zero. Before this E.M.F. has succeeded in actually setting up a current in the opposite direction the brush has ceased to short-circuit the coil and therefore there is no current to be broken. Thus no sparking results as the segment passes from under the brush.

131. Rule for Direction of Rotation of Motors. When all other conditions are the same, *the current in a motor* flows in the *opposite direction to the current in a generator.* In other words, the *back* voltage, the (voltage which the machine produces as a generator) is opposite to the *impressed* voltage—the voltage which causes it to run as a motor.

The rule for finding the direction of rotation of a motor is the same as that for finding the direction of rotation of a generator. We must, however, deal with the back voltage, which opposes the impressed voltage.

Therefore, point the forefinger *of the right hand* in the direction of the *flux,* the *middle finger* OPPOSITE the direction of *impressed voltage,* and the *thumb* will point the direction of *rotation.*

NOTICE THAT ALL RULES FOR DIRECTION OF MAGNETIC FIELD, CURRENT, INDUCED VOLTAGE, MOTION, ETC., APPLY ONLY TO THE RIGHT HAND.

To change the direction of rotation of a motor, it is necessary to change the *relation* between the direction of the flux and the direction of the armature current. If we change the direction of *both* the flux and the armature current we do *not change the relation* between them and therefore the direction of rotation remains as before.

The rule, then, for reversing the direction of rotation of a motor, is to reverse *either the field or armature* connections, *not both.*

132. The Shunt Motor. There are three general types of motors, as there are three general types of generators, SHUNT, SERIES and COMPOUND. They are named according to the way the fields are connected. The shunt motor is by far the most common type of the three and will be described first.

133. Starting Box. We have seen that the current which a motor takes in the armature depends upon the difference between the impressed voltage and the counter E.M.F. This counter E.M.F. keeps the current from being excessive although the armature resistance is very low. We have seen that this counter E.M.F. depends upon the speed, since it is caused by the armature conductors cutting lines of force.

When, therefore, the armature is not rotating, there is no counter E.M.F. and the current through the armature depends upon the impressed voltage and the armature resistance only. Suppose then, that full voltage were thrown on to the armature. Since the resistance is small, the current through the armature would be excessive and might burn out the windings.

Example. The armature of a shunt motor contains .2 ohm resistance. The motor is to run on a 110 volt circuit. (*a*) Suppose that it is thrown on the circuit suddenly while the armature is standing still, what current will it take?

$$I = \frac{110}{.2} = 550 \text{ amperes.}$$

With the machine running at normal speed on 110 volts there is a Counter E.M.F. of 107 volts. (*b*) What current does it take at this speed?

$$I = \frac{110 - 107}{.2} = 15 \text{ amperes.}$$

In order to avoid this excessive current on starting, a STARTING RESISTANCE is introduced into the circuit which

cuts down the voltage across the motor armature at first, and allows but a small current to flow through it. By slowly cutting out this resistance as the motor speeds up and sets up its counter E.M.F., it is possible to throw the full voltage across the motor. It is safe to have this high voltage across the motor as long as the speed is high enough to oppose it by a high counter voltage, but at low speed (and thus a small counter voltage) too much current is likely to be forced through the armature.

Fig. 169 is a simple diagram of the starting resistance used with a shunt motor.

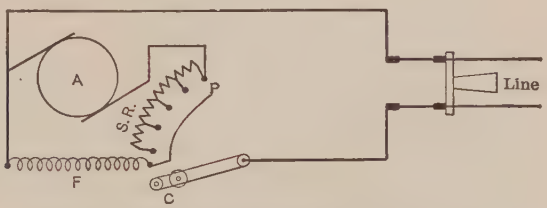

FIG. 169.—Diagram of starting resistance for shunt motor.

When the line switch is thrown and C is swung to first point, it merely puts the shunt field F on to the circuit, thus building up the field immediately. Then when the arm C is swung to the next contact point, the starting resistance SR is put in series with the armature across the line. The resistance SR prevents too large a current from entering the armature. As soon as some speed is acquired by the armature (and therefore a counter E.M.F.) the arm C is swung to the next contact point, cutting out some of the resistance. As the motor gets up its full speed the rest of the resistance SR is gradually cut out. Finally the armature is put directly across the line, by swinging the arm C to the point P.

134. Speed Regulation of Shunt Motor. A shunt motor has the advantage of having a definite speed at "no load;" that is, it does not tend to "run away" or "race" when

the load is taken off. The "full load" speed is but slightly lower than the "no load" speed. Thus the motor possesses a good "speed regulation."

The percentage of the *full load speed*, that the speed changes of its own accord as the load changes from "no load" to "full load" is called the "SPEED REGULATION."

$$\text{SPEED REGULATION} = \frac{\text{No Load Speed} - \text{Full Load Speed}}{\text{Full Load Speed}}.$$

Thus if the speed of a machine fell from 1280 R.P.M. at no load to 1200 R.P.M. at full load, the regulation would be $\frac{80}{1200} = 6.7$ per cent.

By *regulation* is meant always some change which the machine itself makes as the load is changed. By *control* is meant some change that an attendant brings about on the machine, as by throwing of more resistance into the field, to raise the speed. See Chapter VII.

135. Speed Control of Shunt Motor. The usual method of changing the speed of a shunt motor is to change the strength of the field, although resistance is sometimes put in the armature circuit to decrease the speed.

Suppose by means of a variable resistance in the shunt field, we increase the resistance of the field coils. This causes a decrease of current through the coils, which in turn causes a decrease in the magnetization. Then fewer number of lines are cut per second by the armature coils, and thus the counter E.M.F. is lowered. This allows more current to flow into the armature coils.

This increase of current in the armature coils is much greater in proportion than the decrease in the strength of the magnetic field. Thus the actual force or the torque of the motor is greater, and the armature speeds up until a balance is again obtained. Of course, if the current increased only in proportion as the field decreased, the product of current times field strength (which is always

the force on the armature conductors) would remain constant and there would be no gain in torque, hence no increase in speed when the field strength was decreased.

Therefore, strange as it appears at first sight, the weaker the field, the faster a shunt motor tends to run. This method of changing the field strength by means of a variable resistance in series with it is very common.

When a slower speed is desired, some resistance is cut out and more current allowed to flow through the field coils. The armature coils then have to cut through a denser field, and a higher counter E.M.F. is developed, which cuts down the current in the armature and causes it to run at a lower speed. The machine will keep approximately this speed at any load within the range of its rating.

There are limits to the changes in speed which can be effected by this means. (1) It is not feasible to increase the flux density of the iron much beyond saturation, and thus the machine can be slowed down to a certain point only. (2) On the other hand, when the flux density gets low in the field, the magnetic disturbances caused by the armature are so great that serious sparking at the brushes results.

Another scheme which takes advantage of this change in field strength to bring about a change in speed, is that used by the Stow Mfg. Co., illustrated by Fig. 170.

The cores of the fields are made of plungers which can be drawn away from the armature, thus increasing the air gap. This of course, increases the reluctance of the magnetic circuit and thus the flux density decreases, and an increase of armature speed results.

By this method most of the sparking at high speeds is avoided. The apparatus, however, for changing the position of the plungers in the field coils is complicated and expensive. A motor equipped with either device is called an "ADJUSTABLE SPEED MOTOR."

A somewhat simpler scheme which allows a wide range

of speeds, is the COMMUTATING POLE, sometimes called the INTER-POLE. The arrangement of the field of this motor is shown in Fig. 179. On the small commutating poles, placed between the regular poles, the coils are in series with the armature. The strength of these extra poles then depends upon the current which the motor is taking. Their action is to neutralize the excessive

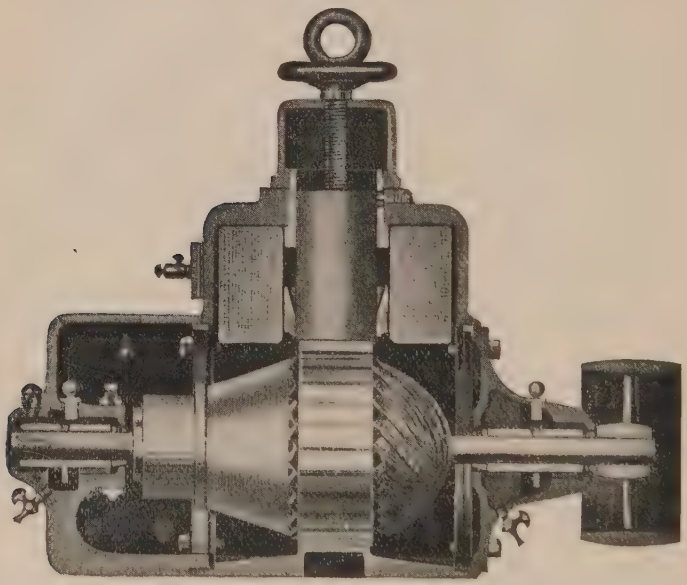

FIG. 170.—Stow variable speed motor, two pole.

magnetic disturbances which take place when the field is considerably weakened, in order to obtain high speed. This method is so successful in maintaining the proper shape of field that sparkless commutation is possible at speeds varying 600 per cent from slowest to fastest.

136. "No Field" Release. What would happen if the current in the field coils *were reduced to zero*, and only the residual magnetism were left in the fields?

We have seen that the armature increases in speed as the field decreases in strength. We would thus expect the armature speed to become excessive if the field became very weak. And this is just what happens to a shunt motor when the field is broken, and the load is light. It immediately races and destroys the armature windings; the centrifugal force pulling them away from the core. Of course, if the machine is heavily loaded, it stops and the greatly increased current in the armature coils causes them to burn up. This event of "no field" must therefore be guarded against by some device, which will break the armature circuit automatically as soon as the field circuit is broken, and thus avoid the racing or burning of the armature coils.

Fig. 171 shows a device of this kind. The field is led through a small electro-magnet *M* on the starting box

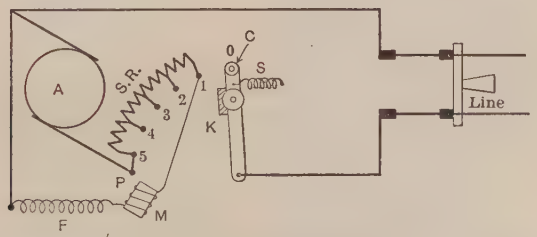

FIG. 171.—"No-field" release.

The swinging arm *C* has a soft iron keeper *K* attached to it. When the arm has come into the running position and all the starting resistance *SR* is cut out, the keeper *K* comes in contact with the electro-magnet *M* which holds the arm in this position, acting against the tension in the spring *S*.

If anything happens to break the current in the field coils *F*, the current in the electro-magnet is also broken, and the swinging arm is released and pulled away by the spring *S*. This action breaks the armature circuit and thus stops the motor.

137. "No Voltage" Release. There is also sometimes danger that the voltage will go off the line, and, a few minutes later, will be thrown on again. In the meantime, the motor will have slowed down and possibly stopped. If

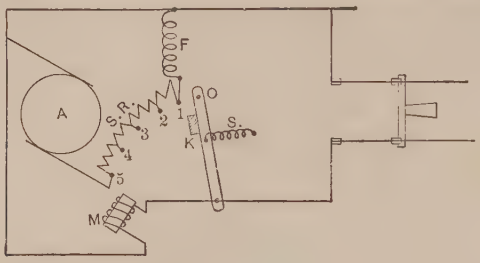

FIG. 172.—"No-voltage" release.

the voltage is now thrown on with the arm in the running position, with all the starting resistance cut out, the armature might be burned out. The above "no field" release is designed to work also as a "no voltage" release; that

FIG. 172*a*.—Cutler Hammer starting box. "No-voltage" release.

is, to release the arm and throw the motor off the line if the voltage drops.

It would appear that if the line voltage were shut off there would be no current through the electro-magnet *M* and the arm *C*, Fig. 171, would be released. The motion of the armature, however, is always setting up a counter E.M.F.

which will send a current through the fields and electro-magnet *M* as long as it is turning fast enough. Thus this device doesn't act immediately as a "no-voltage" release, though it will do so, if the line remains "dead" long enough for the motor to slow down to a point where the counter E.M.F. will not excite the electro-magnet *M*.

No-voltage releases are often arranged as in Fig. 172. Fig. 172*a* shows the construction and appearance of such a box. For diagram of connections see Fig. 229.

138. "Overload" Release. There must also be some arrangement to prevent putting too much load on a motor

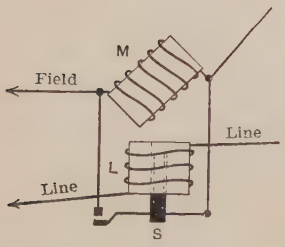

FIG. 173.—"Overload release" for a "no-field"-release box.

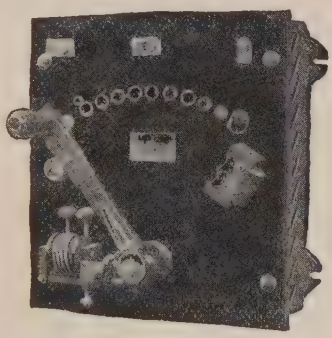

FIG. 173*a*.—Ward Leonard starting box with "Overload" and "no-field" releases.

and thus causing the speed to become so slow that the armature current is excessive. An OVERLOAD RELEASE as shown in Fig. 173 takes care of this emergency. A coil *L* of low resistance is placed in the motor line so that all the current taken by the motor must pass through it. When the current in the motor becomes excessive, the coil becomes so strongly magnetized that it sucks up the plunger *S* and short circuits the magnet coil *M*. This destroys the magnetizing force of *M*. The arm *C* is thus released and the current shut off from motor armature and field. Fig. 173*a* shows the appearance of this box. For diagram of connections for another style, see Fig. 230.

Problem 12–8. The line voltage is 115 volts. The resistance of the shunt field, *F*, Fig. 171, is 200 ohms; of magnet coil *M*, 20 ohms. In starting resistance *SR* the resistance from 1 to 2 is 5 ohms; from 2 to 3, 4 ohms; from 3 to 4, 3 ohms; from 4 to 5, 1 ohm; armature resistance is 2 ohms. The switch to power is thrown and contact *c* is swung to point No. 1. (*a*) How many amperes flow through the armature? (*b*) How many amperes flow through the field?

Problem 13–8. When the armature of Problem 12 has attained enough speed to set up a counter E.M.F. of 25 volts, the contact *c* is swung to point 2. (*a*) How many amperes flow through the armature? (*b*) How many amperes flow through the field?

Problem 14–8. When the counter E.M.F. of armature in Problem 13 is 50 volts, contact *c* is swung to point 3. (*a*) How many amperes flow through the armature? (*b*) How many amperes flow through the field?

Problem 15–8. The armature of Problem 14 attains a counter E.M.F. of 75 volts and contact *c* is swung to point 4. Answer (*a*) and (*b*) of Problem 14.

Problem 16–8. Armature of Problem 15 attains a counter E.M.F. of 107 volts and contact *c* is swung to 5. Answer (*a*) and (*b*) of Problem 14.

139. Directions for Starting and Stopping a Shunt Motor. Start slowly, moving the swinging arm slowly until the motor comes up to speed. Do not leave it on any intermediate point for any length of time, or the starting resistance may be burned out.

Always stop the motor by pulling the line switch Fig. 171, and allowing the arm *C* to snap back when the electromagnet releases it. *Never pull* the arm *C* back, as this causes bad arcing across the points 1 and 0 which roughens the copper contacts.

140. Series Motors. No-load Speed. Series motors, unlike shunt motors, have not a definite " no-load " speed. When unloaded, a series motor will continue to increase in speed, until it destroys the armature by the great centrifugal force set up.

This may be seen by considering the behavior of the field in an unloaded series motor. The motor starts up as the current is sent into the armature and field coils. There being no opposing torque, since the motor is unloaded, the speed tends to increase until the counter E.M.F. equals the impressed voltage. But the ever increasing counter E.M.F. decreases the current in the field and armature. As the field weakens, it requires an increased speed to set up the desired counter E.M.F. The field continues weakening and the speed continues increasing in an effort to build up a counter E.M.F. equal to the impressed voltage, until the centrifugal force wrecks the machine. Thus a series motor is used only where *the load is always attached* to the armature, as in a fan, a railway car, an electric crane, etc.

We have seen that an unloaded shunt motor tends also to speed up till its counter E.M.F. becomes equal to the impressed voltage. But since the field remains constant, this equality is approximately reached and maintained at a certain definite speed.

141. Comparison of Shunt and Series Motors. Starting Torque. We have seen that the force on the armature conductors, and therefore the torque, of any motor is proportional to the current I in the armature times the field strength H. Other things remaining constant, we may write the proportion:

$$\text{Torque} \propto HI.$$

But in a series motor, increasing the current I in the armature increases the current in the field exactly as much, and thus increases the field strength H nearly as much.

The magnetization curve approximates a straight line until the saturation point is reached.

Thus we may write the proportion for the *series* motor

$$\text{Torque} \propto I^2.$$

So, if at any time the current in the armature of a series motor be *doubled*, the field strength is also *doubled*, providing saturation is not exceeded, and the torque is increased *four* times.

In the case of a shunt motor, the field current remains practically constant, so that doubling the current means merely *doubling the armature current*. The torque is then only *doubled*.

In other words, the proportion for a *shunt* motor is:

$$\text{Torque} \propto I.$$

Accordingly, when, as in starting a motor, we allow a heavy current to flow momentarily through the machine, if it is a series motor, the torque is proportional to the square of the current. In the case, however, of a shunt motor, the torque is proportional to the current only.

Thus, of two motors of same size, the series has a much greater starting torque. In order that the shunt motor have the same starting torque as the series motor, it would be necessary to force so much current through the armature that it would be burned up. In other words, if the two motors are to have the same starting torque, the shunt motor must be built much heavier, to stand larger currents.

142. Torque-Speed Characteristics. The torque of a series motor varies greatly with the speed, being greatest when the speed is slowest. As explained above, this is because the slow speed sets up a small counter E.M.F. and the impressed voltage can force a large current through both armature and field. Similarly, a high speed reduces the current in both armature and field, and cuts down the torque. So when a heavy load is thrown on a series motor, the speed decreases, and the armature current and field strength both increase, with the result of a great gain in torque, to take care of the increased load.

On the other hand, we have seen that the field being constant, shunt motors have but little change of speed

with a variation of load from no load to full load. So, when a heavy load is thrown on a shunt motor, it has to gain its increased torque entirely by the increased current in the armature. In order to take care of the same load, therefore, the shunt motor would have to be much larger than the series motor.

The shunt motor cannot slow down and take the heavy load at a slower rate, but must rush through the hard job at about the same speed it maintains on a light load.

Wherever a fairly constant speed is desired, a shunt motor can be used, care being taken to get one large enough to take care of the heaviest load.

When the speed is not an important factor, but high torque is required, a series motor is preferable. Take for instance the most common use of a series motor, electric railway work. If a shunt motor were used, the car would have to climb the hills at about the same rate as it goes along a level, because the shunt motor is very nearly a constant speed machine. This would require a heavy current to do work at this rate and would burn out the motor on a long hill unless the motor were several times the size of the *series motor* used. The series motor, however, decreases its speed and thus works at a slower rate and still gains torque to be used in overcoming the large opposing torque offered by the grade.

On the other hand, a series motor would not do to run a lathe in a machine shop. Every time the load changed, the speed would change, and the most efficient cutting speed could not be maintained. Moreover, as soon as the load was taken off, the speed would become so great as to wreck the lathe.

143. Starting a Series Motor. Series motors are started by placing a resistance in series with them and gradually cutting it out, as motor gets up speed. This starting resistance serves the same purpose as the starting

resistance which is placed in series with the armature of a shunt motor.

144. Series-Parallel Control for Electric Cars. Most electric cars have at least two motors. The controller shown in Fig. 174 is in the front of the car and is operated by the motorman as follows:

FIG. 174.—Westinghouse controller.

When the controller handle is advanced to the first notch, it places the two motors *A* and *B* in series with each other and with the starting resistance *SR* as in Fig. 175. As the handle is advanced, it gradually cuts out the starting resistance until the car gets up a speed of about 10 miles an hour on the level. Then the next notch puts the two motors in parallel with each other and again in series with the resistance *SR* as in Fig. 176. If greater speed is desired, the resistance is again cut out by advancing the handle.

The scheme of putting the two motors in series at the start, allows the car to be started on half the current it would take to start with them in parallel and thus preserves a more even distribution of current in the trolley system, and wastes much less power. It really makes one motor act as starting resistance for the other, at the same time helping it to supply tractive effort.

145. Caution in use of Series and Shunt Motors. A shunt motor *races* when the field is broken, if the armature circuit is not also broken. Therefore:

NEVER PULL THE FIELD OF A SHUNT MOTOR.

A series motor *races* when there is no load connected to it. Therefore:

NEVER START AN UNLOADED SERIES MOTOR AND NEVER REMOVE ALL THE LOAD FROM A SERIES MOTOR WHILE IT IS RUNNING.

146. Compound Motors. Differential and Cumulative. When absolutely constant speed is desired under wide

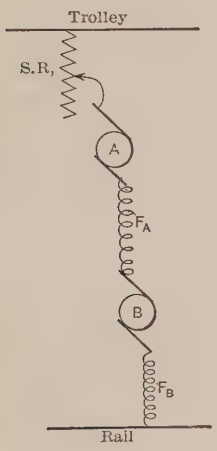

FIG. 175.—Motors in series at start.

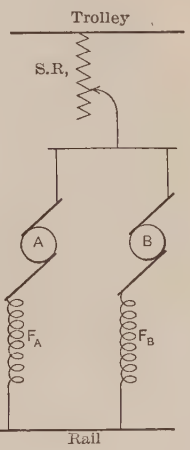

FIG. 176.—Motors in parallel when running.

changes in load, as in a machine shop, a DIFFERENTIAL COMPOUND MOTOR is sometimes used. This has a set of series field coils "bucking" the shunt coils. The ampere-turns of the series coils *D*, Fig. 177, oppose the ampere-turns on the shunt coils *S*. The field strength due to these series coils increases as the load increases and acting against the field due to the shunt coils, weakens the total field enough to allow an extra current to flow through the armature. This current will create enough torque to overcome the extra drag on the pulley and keep the speed constant.

Of course the shunt field *S* is much stronger than the series field *D*. It, however, is possible, by making the series field large enough, to cause the speed to increase with the load. In the main, shunt motors furnish for most purposes a speed which is constant enough at all loads.

A DIFFERENTIAL COMPOUND motor then has exaggerated shunt characteristics; the starting torque is comparatively low (lower even than the starting torque of a shunt motor); the speed regulation is exceedingly good (being better than the speed regulation of a shunt motor).

As the starting torque of a series motor is greater than that of a shunt motor, series coils are sometimes added which "aid" the shunt coils instead of "buck" them.

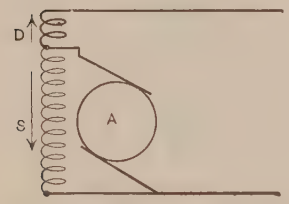

FIG. 177.—Differential compound motor.

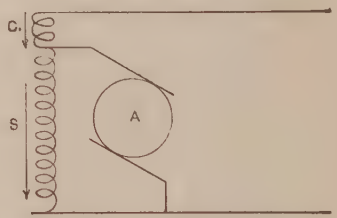

FIG. 178.—Cumulative compound motor

The motor is then called a CUMULATIVE COMPOUND MOTOR. The ampere-turns of the series coils *C*, Fig. 178, aid the ampere-turns on the shunt coils *S*. Such an arrangement gives the motor the advantage of the large starting torque of a series motor, without the disadvantage of the machine "racing" on no-load, since the shunt coils maintain the field. The characteristics are those of the series motor exaggerated; the speed slowing down as the load increases, more than does the speed of a straight series motor. To avoid this, the series field is often short circuited, after the motor has been started. The motor then starts as a series motor but runs as a shunt motor.

147. Comparison between Generators and Motors. It is to be noted that generators and motors of the same type have very similar characteristics. We have merely to read

FIG. 179.—Westinghouse commutating pole motor.

speed in motors for *voltage* in generators. The following table is intended to bring out this relation:

Shunt

GENERATORS. Constant *voltage;* good *voltage* regulation. *Voltage* controlled by field variation.

MOTORS. Constant *speed;* good *speed* regulation. *Speed* controlled by field variation.

Series

GENERATORS. *Voltage* depends entirely upon load.
MOTORS. *Speed* depends entirely upon load.

Compound

GENERATORS. *Voltage* may be made absolutely uniform throughout wide range of loads.
MOTORS. (*Differential*). *Speed* may be made absolutely uniform throughout wide range of loads.

SUMMARY OF CHAPTER VIII

FORCE ON WIRE IN MAGNETIC FIELD. When a wire carrying an electric current lies in a magnetic field at right angles to the lines, there is a force on the wire due to the reaction between the circular field about the wire and the field in which the wire lies. The amount of this force can be found from the equation:

$$F = \frac{IHl}{10}.$$

TORQUE. The torque of the armature of a motor is due to the above force, and is computed from the following equation:

$$T = r\left(\frac{IHl}{10}\right),$$

where I = current in armature wire in amperes;
H = field strength perpendicular to wire in gausses;
l = total length of active wire in centimeters;
r = radius of coil in centimeters;
T = torque in dyne-centimeters.

MOTOR EFFECT IN GENERATORS. This armature torque in a motor is the opposing torque, which the machine has to overcome when running as a generator with same field strength and delivering same current. This helps us to understand how power is lost in producing eddy currents.

The opposing current set up by Eddy Currents in the bobbin, on which moving coils of galvanometers, ammeters, etc., are wound, is used to damp the oscillations of the coil.

Eddy currents in motors and generators have a torque which opposes the motion and consumes power.

GENERATOR EFFECT IN MOTORS, COUNTER E.M.F. The armature conductors of a motor cut lines of force and set up an E.M.F. opposite in direction to the impressed E.M.F.

The current flowing through the armature is, then, under a pressure equal to the difference between the Impressed Voltage and Counter E.M.F. The value of the current can be found from the equation:

$$I = \frac{E_x - E_c}{R}.$$

AXIS OF COMMUTATION. BACKWARD LEAD. The position of the brushes for a motor for sparkless commutation is on an axis back of the neutral axis, instead of ahead, as in a generator.

DIRECTION OF ROTATION. Extend the thumb, forefinger and middle finger of the right hand, at right angles to one another, as in the rule for the generator. When the middle finger points in the direction of the counter E.M.F., the forefinger in direction of the flux, the thumb will indicate the direction of necessary rotation.

Any motor must be started slowly by means of a starting box, which puts resistance in series with armature, to keep the current in the armature from becoming excessive, until a counter E.M.F. is set up by the motion of the armature.

Motors are divided into three general types.

(1) Shunt; field shunted around the armature.
Nearly constant speed.
Low starting torque.
Races when field is broken, thus the necessity of "no-field" release.
Speed controlled by varying field strength.

(2) SERIES; field in series with armature.
Speed varies with load.
Large torque at slow speeds, thus large starting torque.
Races on "no-load," thus the necessity of having load permanently attached.

(3) (a) COMPOUND (Differential); series field coils bucking shunt coils.
Exaggerated shunt characteristics.
Low starting torque.
Constant speed at all loads within limits.
(b) COMPOUND (Cumulative); series field coils aiding shunt coils.
Exaggerated series characteristics.
Will not race at "no-load."
Large torque at low speeds, hence high starting torque.
Speed varies greatly with load.

The characteristics of motors and generators of same type are very similar if speed of motors is compared to voltage of generators.

PROBLEMS ON CHAPTER VIII.

17–8. A motor is running on 115 volts and taking 7.4 amperes in the armature. The armature resistance of the motor is .32 ohm. What counter E.M.F. is being generated?

18–8. From following data compute the armature resistance of motor:

Impressed voltage = 220 volts.
Counter E.M.F. = 214 volts.
Armature current = 12 amperes.

19–8. Draw resultant field if poles and current, Fig. 179*a*, are as marked. Indicate direction of force on each side of loop and direction of tendency of rotation.

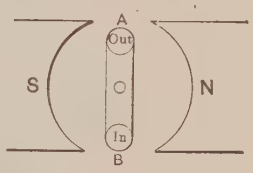

Fig. 179*a*.

20–8. *A* and *B*, Fig. 165, are each 14 inches long and carry 20 amperes. The field intensity in which they lie is 20,000 gausses. What is the force tending to make each move?

21–8. Assume that the distance from *A* to *B*, Fig. 165, is 10 inches. What is the torque of loop?

22–8. Suppose machine in Problem 18 were running as a generator at same speed, and delivering the 12 amperes instead of receiving them; field unchanged. (*a*) What would the brush potential of the machine be? (*b*) What would be its output in K.W.?

23–8. There are 400 conductors on a D.C. armature. Assuming 70 per cent of these lie in a magnetic field of 6000 gausses, how many pounds are acting on armature when each conductor carries 20 amperes? Length of each conductor is 12 inches.

24–8. There are 600 conductors on D.C. armature, Fig. 179*b*, 80 per cent of which lie in the magnetic field. Diameter of armature is 15 inches. Width of pole face parallel to shaft is 25 inches. Average intensity of field is 5000 gausses. Armature takes a total current of 60 amperes. What is force in pounds on each active conductor?

25–8. (*a*) What is total force on active conductors in Problem 24? (*b*) What torque in lb.ft. is developed?

26–8. (*a*) If speed of motor in Problem 24 is 1200 R.P.M. what horse-power is transmitted to pulley, allowing 10% for mechanical losses? (*b*) What voltage must be applied to motor, if field takes 2 amperes and efficiency is 85%?

27–8. In Problem 24, the width of the pole face parallel to shaft is 25 inches. Assume width in the other direction to be 9.2 inches. What is the counter E.M.F. at speed given?

28–8. (*a*) What is the resistance through one path in armature of Problem 24? (*b*) What is armature resistance from brush to brush? (Use data obtained in Problems 24, 25 and 26.)

29–8. In a shunt motor the speed falls from 1600 R.P.M. at no load to 1480 R.P.M. at full load. What is the speed regulation?

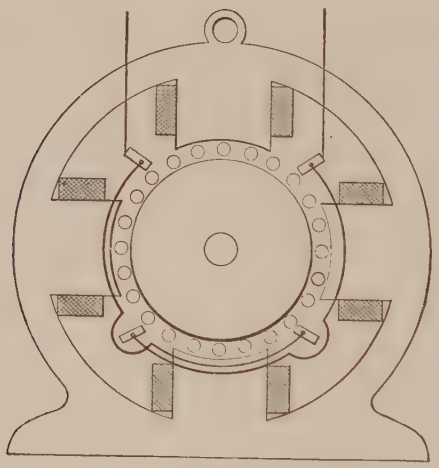

FIG. 179*b*.

30–8. The armature of a shunt motor has a resistance of 2 ohms. The field has a resistance of 200 ohms. What current will flow in armature when 110 volts are applied to terminals with machine at rest? (*b*) Through fields with machine at rest?

31–8. If motor in Problem 30 were turning fast enough to develop 108 volts counter E.M.F., what current would flow in the armature circuit? (*b*) In the field circuit?

32–8. Armature, Fig. 172, resistance is 3 ohms. Field resistance = 224 ohms. Resistance of starting box is divided as follows:

1 to 2 = 2 ohms
2 to 3 = 2 "
3 to 4 = 3 "
4 to 5 = 2 "
M = 1000 ohms

Voltage of line = 112 volts. (*a*) What is starting current of motor? (*b*) If counter E.M.F. of motor is 75 volts when running on point 2 of starting box, what current is armature then taking?

33–8. If motor in Problem 32 takes 3 amperes when running on point 4, what is its counter E.M.F.?

34–8. A shunt motor takes a total current of 80 amperes from 115 volt mains. The resistance of the armature is .04 ohm. Resistance of the field is 60 ohms. (*a*) What current does each take? (*b*) What power is used up in heating the field and armature?

35–8. What current does field take in Problem 32 (*a*) and in Problem 32 (*b*)?

36–8. (*a*) What is the total power taken by motor in Problem 34? (*b*) Why is this greater than the power used in heating the field and armature as computed in Problem 34? (*c*) Compute the counter E.M.F. of motor in Problem 34.

37–8. It requires 1000 ft. of No. 30 (B. & S. Gauge) copper wire for the field coils of a 4-pole, 4-brush shunt motor. The armature has 600 ft. of No. 25 copper wire. If the motor takes a total current of 17 amperes at 112 volts (*a*) What is its counter E.M.F.? (*b*) What power is lost in heating armature and field?

38–8. (*a*) What voltage would motor in Problem 37 deliver if run at the same speed, and 17 amperes were flowing through the armature? Field separately excited to same degree of magnetization. (*b*) What power would it be delivering to the line?

39–8. A 15-K.W. 110-volt shunt generator has a speed of 900 R.P.M. at full load. Field resistance is 35 ohms; armature resistance is .06 ohm. (*a*) What are the armature and field currents at full load? (*b*) What is E.M.F. of generator?

40–8. If machine in Problem 39 were run as a 110-volt motor and had same current flowing through the armature, at what speed would it run?

41–8. A 2-pole 220-volt shunt motor takes 50 amperes at full load. The field resistance is 100 ohms. Armature consists of 200 active conductors and has a resistance of .36 ohm. $\phi = 4{,}000{,}000$ lines. At what speed does it run at full load?

42–8. If motor in Problem 41 were a 4-pole 4-brush machine, and all other data were the same, what would be the full load speed?

43–8. If motor in Problem 41 were a 4-pole 2-brush machine and all other data were the same, what would be the full load speed?

44–8. If motor in Problem 41 requires 8 amperes at no load, what is the speed regulation?

45–8. The armature of a 2-pole, 110-volt shunt motor has 180 active conductors and a resistance of .05 ohm. It takes 6.5 amperes to run motor at no load and 100 amperes at full load. The field has a resistance of 55 ohms. $\phi = 3{,}000{,}000$ lines. What is the speed regulation of the motor?

46–8. How much resistance would have to be placed in series with armature in Problem 45 in order that the starting current may not exceed twice the full load current?

47–8. A 550-volt series motor having 4 poles and 2 brushes, requires 180 K.W. at full load. Field and brush together have resistance amounting to .041 ohm. There are 350 active conductors on armature, which has a resistance of .042 ohm. Speed is 600 R.P.M. What is the flux per pole?

48–8. A 110-volt shunt motor has an armature resistance of .8 ohm. The field resistance is 220 ohms. The full load speed is 1200 R.P.M. and the motor is taking 10 amperes. At what speed must this machine run as a generator in order to deliver 10 amperes at 110 volts?

49–8. (*a*) What is counter E.M.F. of machine in Problem 48 when running at full load as a motor? (*b*) What is E.M.F. when run as a generator and delivering 10 amperes at 110 volts?

50–8. The motor of Problem 41 delivers 12 H.P. at full load. What torque does it develop at full load?

CHAPTER IX

FURTHER APPLICATIONS

Solution of Some of the More Difficult Problems Encountered in Electrical Practice

Efficiency of Electrical Machinery and Processes—Transmission of Electrical Power—Efficiency: Effect of Voltage—Relation of Size of Conductor to Voltage of Transmission—Feeders—Three-wire System — Kirchhoff's Laws—Current Distribution in Parallel Combinations of Battery Cells or Generators—Stray Power Losses —Commercial Efficiency of Generators and Motors—Electrical Efficiency of Generators—Mechanical Efficiency of Motors.

148. Advantages of Electrical Transmission. The advantages of electrical transmission of power are almost too well known to be mentioned. The simplicity of the mechanism required for electrical transmission and the immense distance that electricity can be transmitted are the most striking advantages. A couple of small *stationary wires* are all that is required to convey enormous quantities of power in this form from one end of the state to the other. Mechanical power can be transmitted at best but a few yards, and then only by such cumbersome *moving parts* as belts, ropes, chains, etc.

The use of compressed air for conveying mechanical power in small quantities from one part of a building to another, is perhaps the nearest rival that electricity has, but here too the machinery is cumbersome.

The efficiency of electrical transmission can be made remarkably high, even over great distances, if certain conditions are complied with. It will be instructive to

find out just what these conditions are, and to what extent they affect the efficiency.

149. Relation of Voltage of Generator to Efficiency of Transmission. Suppose it is possible to generate 30 kilowatts at a certain place and that it is desired to use the power to run a motor, say two miles distant. The power must be transmitted in a practical form and at as small a loss as possible.

In Fig. 180, *G* represents the 30 K.W. generator, *M* the motor at a distance of about two miles from *G*. Assume for convenience, that the line wires have a total resistance of 3 ohms. Let us try the effect of using various voltages

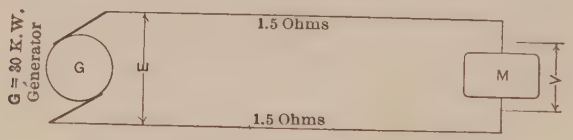

FIG. 180.—Generator delivers 30 K.W. to line.

ranging from 100 to 10,000 volts, at which we might generate the 30 K.W. and compute the corresponding efficiencies. Consider the efficiency of the generator to be 100 per cent.

First, assume that we generate at 100 volts. To supply 30 K.W. at this voltage, the generator must deliver $\frac{30{,}000}{100}$ or 300 amperes to the line.

Now, to force 300 amperes through a line wire having a resistance of 3 ohms, would require 300×3 or 900 volts; which is 800 more volts than we are even generating. It is clearly impossible to transmit 300 amperes through the line at this pressure.

Let us then assume that we generate at 200 volts. To supply 30 K.W. at this voltage, the generator must deliver only $\frac{30{,}000}{200}$ or 150 amperes to the line.

Here again, to transmit as much as 150 amperes through

3 ohms, requires 3×150 or 450 volts, which is 250 volts more than we assumed generated.

If we generate at 300 volts, only $\frac{30,000}{300}$ or 100 amperes must be transmitted. This would require 100×3 or 300 volts to force it through the line alone. Since we are generating but 300 volts, this leaves no voltage to force the current through the motor *M*. Efficiency of transmission, then, is zero, since all the power is lost in the line.

If we generate at 400 volts, it is necessary to force but $\frac{30,000}{400}$ or 75 amperes, through the line. The volts required for this would be 75×3 or 225, leaving $400-225$ or 175 volts to run the motor.

Watts lost in line $=75^2\times3=16,875$ watts.

Watts left for motor $=30,000-16,875=13,125$ watts.

Efficiency of transmission $=\frac{13,125}{30,000}=43.8$ per cent.

The results of similar computations for a range of voltages may be tabulated as follows:

30 K.W. GENERATED AT *E* VOLTS

Volts *E*	Amperes. *I*	Line Loss in Volts. R=3 ohms *IR*	Line Loss in Watts. I^2R	Volts Left for Motor *V*	Watts Transmitted to Motor. *IV*	Efficiency. Per cent.
100	300	900	Impossible			
200	150	450	"			
300	100	300	30,000	0	0	0
400	75	225	16,875	175	13,125	43.8
500	60	180	10,800	320	19,200	63.3
600	50	150	7,500	450	22,500	75.
800	37.5	112.5	4,220	687.5	25,780	86
1,000	30	90	2,700	910	27,300	91
1,200	25	75	1,875	1,125	28,125	93.8
1,500	20	60	1,200	1,440	28,800	96
2,000	15	45	675	1,955	29,325	97.8
3,000	10	30	300	2,790	29,700	99
5,000	6	18	108	4,994	29,964	99.8
10,000	3	9	27	9,991	29,973	99.9

The curve plotted in Fig. 181 shows the relation of voltage to efficiency of transmission, as brought out by the above problem. Note that the efficiency increases very rapidly with the voltage until about 1200 volts are reached. From there on, the increase is much slower, though the efficiency continues to rise a little with each increase of voltage. If the voltage were the only condition affecting the efficiency, it would be advisable to use indefinitely high voltages. The difficulties in the insulation of the machine and line, limit the voltage at the present time to

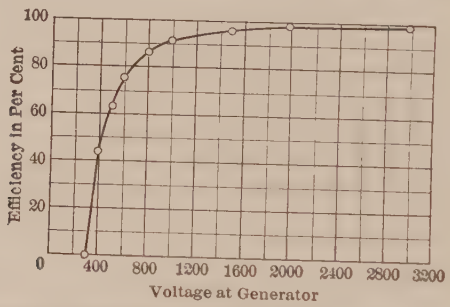

FIG. 181.—Relation of efficiency of transmission to brush potential of generator.

some more moderate value. Yet the fact is evident that as far as the line loss (I^2R) is concerned, the higher the voltage used, the higher the efficiency of transmission.

Problem 1–9. Construct a table as above for 80 K.W. generator and 2 ohm line, starting at 100 volts and gradually increasing it to 10,000 volts. Plot curve between volts and efficiency of transmission.

Problem 2–9. Repeat Problem 1–9, using 8 K.W. generator.

150. Relation of Line Loss to Voltage of Generator Maintaining Constant Output. On inspection of above table, it will become evident that the line loss in watts varies inversely as the square of the voltage of the generator (assuming a generator of constant watt output.)

For instance, the line loss at 500 volts is 10,800 watts, while at 1000 volts it is only $\frac{10,800}{4}$ or 2700 watts. By doubling the voltage we have quartered the line loss.

Problem 3–9. (*a*) If the line loss on a 110-volt system is 4 K.W., what will it become if the voltage of the system is changed to 220 volts? (*b*) To 550 volts? Assume same power delivered to the line.

151. Relation of Size of Conductor to Voltage (Line Loss Constant). In the above paragraphs we have been considering a means of decreasing the operating expenses by decreasing the power lost in the line. We have seen that the power lost decreases as the square of the voltage of transmission. It is also important to consider the effect on the original cost of transmission if we raise the voltage. In analyzing this point, we will assume that we have fixed upon an amount of power which we are willing shall be lost in the line, and that we keep this constant.

By referring again to the above table, we find that we have a loss of 1.2 K.W., when the generator voltage was 1500 volts. Let us assume this to be the line loss we are willing shall be kept constant for economical reasons. By doubling the voltage to 3000 volts, we of course decreased the line loss to $\frac{1.2}{4}$ or .3 K.W. But since we are allowed a line loss of 1.2 K.W., we may now increase our line resistance by using a wire of a diameter enough smaller to make up this line loss.

The resistance of the line has been kept at 3 ohms. At 1500 volts, the line current was 20 amperes; line loss = $20^2 \times 3 = 1.200$ K.W.

At 3,000 volts, line current was 10 amperes; line loss = $10^2 \times 3 = .300$ K.W.

But being allowed 1.2 K.W. line loss, if we use the 3000 volts, and 10 amperes line current, we may use a wire of 12 ohms, instead of 3 ohms. Then the line loss would

become $10^2 \times 12 = 1.2$ K.W., the same value it had at 1500 volts with a 3 ohm line. That is, by doubling the voltage we are able to use a line wire of four times the resistance, and not increase the line loss.

This may be stated in more general terms as follows:

The line resistance for a constant line loss varies directly as the square of the voltage of transmission.

Since the resistance of a wire varies directly as the length and inversely as the cross-section area, the above law may be applied as meaning, that with the same line loss we may either:

(*a*) Use the same size wire and transmit four times as far at double voltage, nine times as far at triple voltage, etc., or

(*b*) Transmit the same distance and use wire of one-quarter cross-section or of one-quarter weight if we double the voltage; one-ninth weight, if we triple the voltage, etc.

It can be seen from (*b*) that in transmitting electric power between any two points at given line loss, a great saving in the cost of copper can be made by transmitting at a high voltage. In fact, since the cost of installed moderate-sized copper wire is almost directly proportional to its weight, we may say that as far as the cost of the line wire is concerned, the initial expense varies inversely with the square of the voltage of transmission.

Problem 4–9. A 110-volt line is to transmit the same power at the same line loss as a 220-volt line of equal length. How will the size of the line wires of the two systems compare?

Problem 5–9. If No. 00 (B. & S.) wire is used for the 110-volt line of Problem 4–9, what size wire would be necessary for the 220 volt line?

In choosing the line wire to be used in the installation of a system, such a size must be used as will maintain a fair balance between the initial cost and the cost of running, i.e., the line loss. The initial cost *increases* as the size

of the wire increases, while the cost of running (line loss) *decreases* as the size increases.[1]

152. Feeders. In order to avoid a heavy line loss in both power and voltage, it is customary in many low voltage systems, i.e., 550 volts or under, to parallel the main conductor with a second conductor called a FEEDER.

Such a feeder is often run alongside of a trolley wire and joined to it at regular intervals.

This arrangement is merely a case of parallel conductors, but it presents some interesting problems in current, voltage, and power distribution.

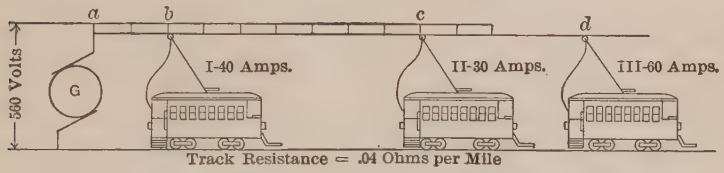

FIG. 182.—Trolley line with feeder. Cars at juncture of feeder and trolley wire.

Example. Trolley in Fig. 182 is No. 0, hard-drawn copper, .530 ohm per mile. Feeder is No. 0000 annealed copper .258 ohm per mile. Track resistance is .04 ohm per mile.

Car No. I is 1 mile from Generator station.
" II is 2 miles from Car I.
" III is 2 miles from Car II.

Feeder extends 4 miles from Station and is tied to trolley every half mile.

Find:

(*a*) Volts lost between Generator and I.
" " " I " II.
" " " II " III.

(*b*) Voltage across each car.

(*c*) Power lost in each section of line.

(*d*) Efficiency of transmission to three cars.

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[1] Lord Kelvin deduced the following rule with regard to the *most economical area of conductor* for transmitting a given amount of power at a given voltage:

The conductor should be of such an area that the value of power lost per year in the line, equals the interest per year on the money invested in the line.

Solution. The feeder and trolley may be considered as two parallel circuits with a combined resistance per mile, found as follows:

Resistance per mile	Conductivity per mile
.530 ohm	1.89 mhos
.258 ohm	3.88 mhos
	5.77 mhos $=\frac{1}{5.77}$, or .173 ohm.

If the trolley and feeder had both been of the same kind of copper, it would have been possible to have considered them as one wire, the cross-section area of which was equal to the sum of the cross-section areas of each. Then the resistance per mile could have been found from the equation $R=\frac{Kl}{A}$ as explained in Chapter V.

The trolley resistance between Gen. and Car I $= 1\times.173=.173$ ohm.
" " " " Car I and Car II $= 2\times.173=.346$ "
" " " " Car II and Car III $=.173+.530=.703$ "
" total " " Gen. and Car I $=.173+.04=.213$ "
" " " " Car I and Car II $=.346+.08=.426$ "
" " " " Car II and Car III $=.703+.08=.783$ "

Current between Car II and Car III $= 60$ amps.
" " " I and " II $=60+30=90$ "
" " Gen. and " I $=90+40=130$ "

Drop in *line* from Gen. to Car I $=130$ amps. $\times .213$ ohms $=27.8$ volts.
" across Car I $=560-27.8=532$ volts.
" in *line* from Car I to Car II $=90$ amps. $\times .426$ ohms $=38.3$ volts.
" across Car II $=532-38.3=494$ volts.
" in *line* from Car II to Car III $=60\times.783=47$ volts.
" across Car III $=494-47=447$ volts.

Power lost between Gen. and Car I $= 130^2 \times .214 = 3620$ watts.
" " " Car I and Car II $= 90^2 \times .426 = 3450$ "
" " " Car II and Car III $= 60^2 \times .783 = 2820$ "

Total power lost in line $= 9890$ "
Power delivered by Gen. $= 560 \times 130 = 72{,}800$ watts.
" " to Cars $= 72{,}800 - 9890 = 62{,}910$ "

or

Power delivered to Car I $= 532 \times 40 = 21{,}280$ watts.
" " " II $= 494 \times 30 = 14{,}820$ "
" " " III $= 447 \times 60 = 26{,}820$ "

" " to Cars $= 62{,}920$ "

Efficiency of transmission $= \frac{62{,}910}{72{,}800} = 86.4$ per cent.

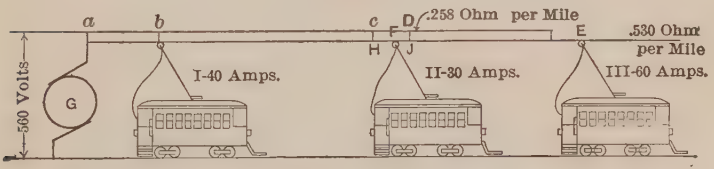

FIG. 183.—Trolley line with feeder cars not at juncture of trolley and feeder.

When the cars do not happen to be exactly at a junction of the feeder and trolley, the problem becomes a little more involved.

Assume, in Fig 183, the hard-drawn trolley wire .530 ohm per mile and feeder .258 ohm per mile, joined every *mile* to the trolley.

Distance between Gen. and Car I = 1 mile.
" " Car I and Car II = $1\frac{1}{2}$ miles.
" " Car II and Car III = $1\frac{1}{2}$ miles.

Track resistance = .04 ohm per mile.

Solution:

As before the combined resistance per mile of trolley = .173 ohm.

As before, the current through section $\overline{AB} = 130$ amperes.

Volts lost in line between Gen. and Car I $= 130 \times (.173 + .04) = 27.8$.

Drop across Car I $= 560 - 27.8 = 532$ volts.

It is necessary now to investigate how the current divides among the sections $\overline{CD}$, $\overline{HF}$, and $\overline{FJ}$.

Let the current through $\overline{CD} = x$ amperes.

Then the current in $\overline{FJ}$, plus the current in $\overline{CD}$ must equal 60 amperes, since these two branches both feed to Car III.

Thus we may write the equation,

$$\text{Current in } \overline{FJ} = (60 - x) \text{ amperes.}$$

The current in $\overline{HF}$ must be 30 amperes more than that in $\overline{FJ}$, since $\overline{HF}$ feeds 30 amperes to Car I before it feeds to $\overline{FJ}$.
Hence:

$$\text{Current in } \overline{HF} = 60 - x + 30 = (90 - x) \text{ amperes.}$$

Resistance of $\overline{CD} = .258$ ohm.
Drop along $\overline{CD} = .258x$ volt.

Resistance of $\overline{HF}$ = resistance $\overline{FJ} = \frac{.530}{2} = .265$ ohm.

Drop along $\overline{HF} = .265 \times (90 - x) = (23.85 - .265x)$ volts.
Drop along $\overline{FJ} = .265 \times (60 - x) = (15.9 - .265x)$ volts.
Drop along $\overline{HJ}$ = drop along $\overline{HF}$ + drop along $\overline{FJ}$.
$= (23.85 - .265x) + (15.9 - .265x)$
$= 39.75 - .53x$ volts.

But since $\overline{CD}$ and $\overline{HJ}$ are in parallel, the drop along each must be the same (assuming ties have no resistance).
Hence:

$$39.75 - .53x = .258x$$
$$.788x = 39.75$$
$$x = \frac{39.75}{.788} = 50.4 \text{ amperes.}$$

Current through $\overline{CD}$ = 50.4 amperes.
" " $\overline{HF} = 90 - 50.4 = 39.6$ "
" " $\overline{FJ} = 60 - 50.4 = 9.6$ "

Trolley drop between Car I and II
= drop along $\overline{BC}$ + drop along $\overline{HF}$
$= (90 \times .173) + (39.6 \times \frac{.530}{2}) = 26$ volts.

Total drop between Car I and II
$= 26 + (90 \times 1.5 \times .04) = 31.4$ volts.

Trolley drop between Car II and III
= drop along $\overline{FJ}$ + drop along $\overline{JE}$.
$= (9.6 \times \frac{.530}{2}) + (60 \times .173) = 12.9$ volts.

Total drop between Car II and III

$$= 12.9 + (60 \times 1.5 \times .04) = 16.5 \text{ volts.}$$

Power lost in line and efficiency of transmission may now be found as in previous example.

Problem 6–9. An eight-mile electric railway has a No. 0 trolley wire of hard-drawn copper. A feeder, No. 0000 soft copper wire, extends from the station 5 miles along the trolley wire and is tied to it every mile. At a certain instant there are 4 cars on the line. Car I is 2 miles from station, taking 40 amperes, Car II is $3\frac{1}{2}$ miles from station, taking 60 amperes. Car III is 6 miles from station, taking 30 amperes. Car IV is $7\frac{1}{2}$ miles from station, taking 50 amperes. Track resistance is .05 ohm per mile. What must terminal voltage of generator be in order that Car IV may have a pressure of 500 volts?

Problem 7–9. If the resistance of Generator in Problem 6 is .08 ohm, what E.M.F. must it generate? Shunt field = 150 ohms.

153. Three-Wire System. We have seen that the amount of wire needed for distribution with given line loss varies inversely as the square of the voltage of transmission.

Whatever may be the voltage of transmission, the usual voltage to be delivered to the customer is from 110 to 115 volts. In order to gain the advantage of transmitting a direct current at a high voltage, and still deliver power at but 110 volts, an ingenious system has been developed called the Three-wire System.

Fig. 184 is a diagram of this scheme. The 110-volt generators G_1 and G_2 are joined in series. Three wires A, B and C are run as shown, C being called the neutral wire. The voltage between A and B will be 220 volts. If we cause all the power to be distributed on the two wires

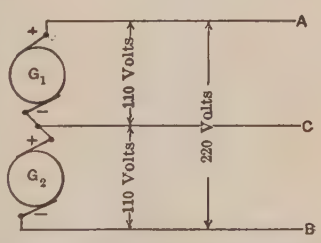

FIG. 184.—Three-wire system using two generators.

A and B, the current flowing out along A and back through B, we will have to use wires but one-quarter as large as though we were transmitting the same power at 110-volts, allowing same line loss.

The system is generally so "balanced", that this is approximately the case. To "balance" a three-wire system, it is necessary to so arrange the loads on each side of the neutral that the neutral carries practically no current.

Fig. 185 shows a balanced three-wire system. Notice that if the lamps are all the same size, no current is taken from

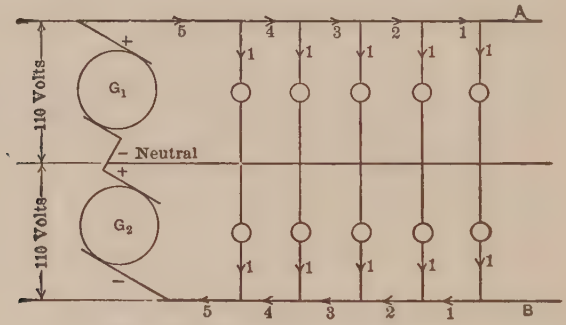

Fig. 185.—Balanced three-wire system.

the generators or returned to them by the neutral. The effect is the same as running the lamps in sets of two in series across 220 volts.

Fig. 186 shows an unbalanced three-wire system. The figures represent the proportional parts of the current carried by each section of the lead wires.

Since B is returning less current than A is supplying, the neutral must help B return the current from the lamps. If B were returning more current than A was supplying to the lamps, then the neutral would help A deliver current to the lamps. Although it is intended that the system be kept balanced, the neutral is generally made of the same size as A and B. As each wire is one-quarter the size of

each wire in a two-wire system of 110 volts, only three-eighths as much copper is used in a three-wire as in a two-wire system in transmitting equal power.

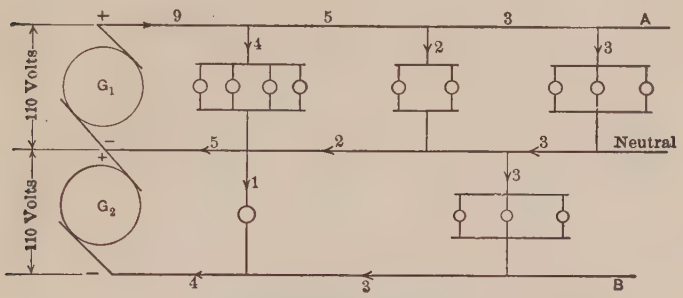

Fig. 186.—Unbalanced three-wire system.

It is costly to install and operate two generators, so a special three-wire generator has been invented which allows this system to be operated by one machine. See advanced text-books for a description of this generator.

Example. Assume that each lamp in Fig. 187 takes 1 ampere.

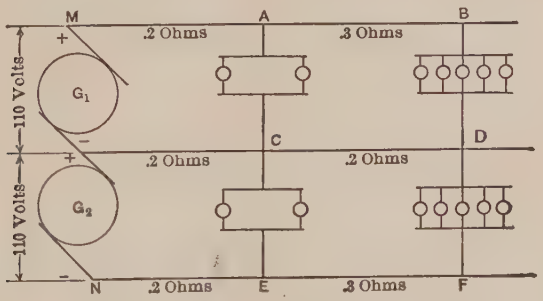

Fig. 187.—Balanced three-wire system.

Find:
(1) Line drop.
(2) Volts across each set of lamps.
(3) Line loss.

Since the system is balanced, no current flows in neutral. The system, then, amounts to a two-wire 220-volt line.

Current from A to $B = 5$ amperes.
Current from F to $E = 5$ amperes.
Line resistance from A to $E = .3 + .3 = .6$ ohm.
Line drop in section $\overline{AB}$ and $\overline{FE} = 5 \times .6 = 3.0$ volts.
Current from M to $A = 7$ amperes.
" E to $N = 7$ amperes.
Resistance of line $\overline{MA}$ and $\overline{EN} = .4$ ohm.
Line drop " " $= .4 \times 7 = 2.8$ volts.
Drop across lamps A to $E = 220 - 2.8 = 217.2$ volts.
" each set of lamps $\overline{AC}$ and $\overline{CE} = 108.6$ volts.
" " " $\overline{BD}$ and $\overline{DF}$

$$= \frac{(217.2 - 3)}{2} = 107.1.$$

Line loss $\overline{MA}$ and $\overline{EN} = 7^2 \times .4 = 19.6$ watts.
" $\overline{AB}$ and $\overline{FE} = 5^2 \times .6 = 15$ watts.

Total line loss $= 34.6$ watts.

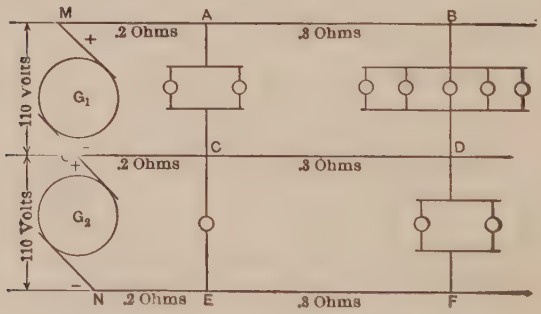

FIG. 188.—Unbalanced three-wire system.

Example. Assume each lamp in Fig. 188 takes 1 ampere. Find:

(1) Line drop.
(2) Line loss.
(3) Voltage across each set of lamps.
(4) Efficiency of transmission.

Since the system is unbalanced, there will be a current flowing in the neutral.

The current in $AB = 5$ amperes
" " $DF = 2$ "
" " $CD = 3$ "
" " $AC = 2$ "
" " $MA = 7$ "
" " $CE = 1$ "
" " $OC = 4$ "
" " $NE = 3$ "

Drop along $MA = 7 \times .2$ ohm $= 1.4$ volts.
" $AB = 5 \times .3$ " $= 1.5$ "
" $DC = 3 \times .3$ " $= .9$ "
" $OC = 4 \times .2$ " $= .8$ "
" $EF = 2 \times .3$ " $= .6$ "
" $NE = 3 \times .2$ " $= .6$ "

Voltage across $AC = 110 - (1.4 + .8) = 107.8$ volts.
" $AE = 220 - (1.4 + .6) = 218$ "
" $CE = 218 - 107.8 = 110.2$ "
" $BD = 107.8 - (1.5 + .9) = 105.4$ "
" $BF = 218 - (1.5 + .6) = 215.9$ "
" $DF = 215.9 - 105.4 = 110.5$ "

Line loss in $MA = 7^2 \times .2 = 9.8$ watts.
" $AB = 5^2 \times .3 = 7.5$ "
" $CD = 3^2 \times .3 = 2.7$ "
" $OC = 4^2 \times .2 = 3.2$ "
" $EF = 2^2 \times .3 = 1.2$ "
" $NE = 3^2 \times .2 = 1.8$ "

Total line loss $= 26.2$ watts.
Power delivered by $G_1 = 110 \times 7 = 770$ watts.
" " $G_2 = 110 \times 3 = 330$ "
Total power delivered $= 1100$ "
Power used by lamps $= 1100 - 26.2 = 1073.8$ watts.

$$\text{Efficiency of transmission} = \frac{1073.8}{1100} = 97.5 \text{ per cent.}$$

Problem 8–9. Assume each lamp in Fig. 189 takes 4 amperes.
Find:
(*a*) Current in *AB*, *CD*, *EF*.
(*b*) Line drop in *AB*, *CD*, *EF*.
(*c*) Line loss.
(*d*) Voltage across *BD* and *DE*.
(*e*) Efficiency of transmission.
(*f*) Resistance of each set of lamps.

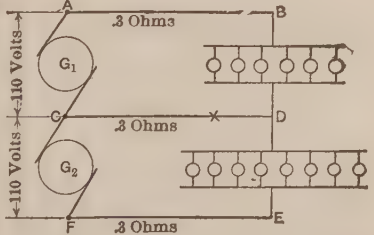

FIG. 189.—Unbalanced three-wire system.

Problem 9–9. Suppose neutral in Problem 8 broke at x, what voltage would there be across BD and DE? (Assume resistance of lamps constant).

154. More Difficult Applications of Ohm's Law. Kirchhoff's Laws. Several further laws have been deduced from Ohm's Law and called Kirchhoff's Laws, in honor of the man who first stated them. They are merely an extension of Ohm's Law and need not be learned separately. In fact, we unconsciously have been applying them to most of the problems dealing with the relation of current, resistance and voltage, in the more complicated systems.

It is well, however, to make a definite statement of the principles under the proper heading of KIRCHHOFF'S LAWS, in order to apply them more directly to certain complex problems in distribution.

Kirchhoff stated:

First Law. That at any point in a circuit, there is as much current flowing *away* from the point as there is flowing *to* it.

Second Law. That the sum of the several IR drops *around any one path* of an electric circuit equals the sum of the E.M.F.'s impressed on that same path. Care must be taken to get the algebraic signs of the E.M.F.'s correct. If there is no source of E.M.F. in any given circuit, then the sum of the IR drops in one direction equals the sum of IR drops in the other direction.

First Law. This can best be explained by referring to diagram in Fig. 190. Assume each lamp to take 1 ampere and consider the point B.

Since there are 8 amperes flowing away from B, there must be 8 amperes flowing to B along AB.

Consider point E.

Since there are 8 amperes flowing to point E, there must be 8 amperes flowing away from E.

We have unconsciously used this law again and again,

considering that it was too obvious, from the very nature of an electric current, to need demonstration.

Second Law. This law is as obvious as the first, but more difficult to state clearly. Referring again to Fig. 190, the E.M.F. of the generator G must equal the IR drop in $AB+BE+EF+$the IR drop of the generator itself.

Or considering another complete circuit on the diagram; the E.M.F. of generator must equal the IR drop in $AB+BC+CD+ED+FE+$generator IR drop. Or again, considering the circuit $BCDE$, which contains no source of E.M.F., we can say that the IR drop of BE must equal the IR drop of $BC+CD+DE$, since the IR drop of BE

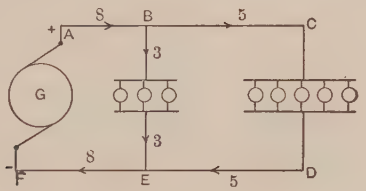

FIG. 190.

is counter clockwise in the circuit $BCDE$ and the IR drop of $BC+CD+DE$ is clockwise.

We have been accustomed to use this same principle under the two separate rules (1st) that the IR drop around any path in an electric circuit equals the sum of the impressed E.M.F.'s, and (2d) that the voltage drop along parallel paths between two points is the same.

Thus we would have said that the voltage across BE equals the voltage across $BC+CD+DE$ because both the path BE and the path $BC+CD+DE$ are in parallel between the points B and E.

There are, therefore, no new facts to be learned from Kirchhoff's laws, but slightly different viewpoints from which to regard familiar facts. The following example will make this clear:

Example. Assume trolley line, Fig. 191, to be fed by two generators, G_1 of 560 volts, and G_2 of 555 volts. Two cars are on the line; Car I taking 300 amperes, Car II, 200 amperes. The resistance of trolley and track as marked.

Find:

(1) Voltage across each car.

(2) Efficiency of transmission.

Solution. Assume current to flow in direction indicated in different sections.

NOTE.—If on solution any current values come out negative, it merely means, as in solving force diagrams in mechanics, that the *direction* of the current should be the reverse of that indicated; the numerical result will be correct.

Let x = current in section AB.

Then x = current in section HK.

" $x-300$ = " " BC and KF.

" $500-x$ = " " CD and EF.

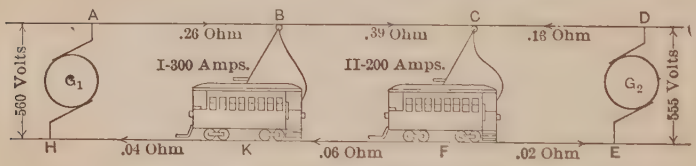

FIG. 191.—Trolley line with generators at ends of line.

The above assumptions are made in accordance with the common-sense law that the amount of current which flows away from a point must be the same as that which flows to it. This, as we have seen, is called Kirchhoff's " First Law."

Voltage across Car II equals

(1) $555-.15(500-x)$.

Voltage across Car II also equals

(2) $560-.30x-.45(x-300)$.

Therefore

(3) $555-.15(500-x)=560-30x-.45(x-300)$.

These assumptions are made in accordance with the principle stated in Chapter III, that the voltage across a parallel combination is the same as that across any and all of the parallel paths between the same two points. This, as we have seen, is part of " Kirchhoff's Second Law." Thus the voltage across Car II must be the same whether computed from the voltage of G_2 minus line

drop, or from the voltage of G_1 minus the line drop from that Generator.

Solving (3) for x,

$x = 239$ amperes = current in AB and HK.

Voltage across Car I $= 560 - 239 \times .30 = 488.3$ volts.

Current in CD $= 500 - 239 = 261$ amperes.

Voltage across Car II $= 555 - 261 \times .15 = 515.8$ volts.

Current in $BC = 239 - 300 = -61$ amperes (the minus sign means it must be flowing in the direction opposite to that marked).

Power delivered by $\text{Gen}_1 = 560 \times 239 = 134{,}000$ watts.

" " $\text{Gen}_2 = 555 \times 261 = 145{,}000$ "

Total power = 279.000 K.W.

Power used by Car I $= 488.3 \times 300 = 146{,}500$ watts.

" " II $= 515.8 \times 200 = 103{,}200$ "

Total power used by cars = 249.700 K.W.

Efficiency of transmission $= \frac{250}{279} = 89.7$ per cent.

By means of the principles illustrated above, all problems involving current and voltage relations in " mesh-work " may be solved.

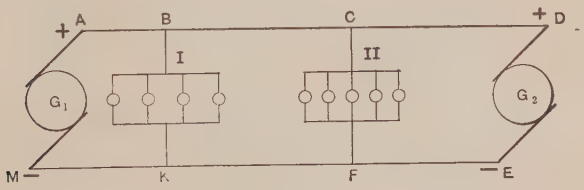

FIG. 192.

Problem 10–9. In Fig. 192, each lamp takes 12 amperes. Resistance of $AB = BC = .04$ ohm. Resistance of $CD = MK = .03$ ohm. Resistance of $EF = KF = .02$ ohm. What is the voltage across Group I and Group II, if terminal voltage of $G_1 = 120$ volts and of $G_2 = 125$ volts.

Problem 11–9. What is the efficiency of transmission of system in Problem 10.

154. (a) Parallel Combinations of Unlike Generators or Battery Cells. When battery cells, of either PRIMARY or

STORAGE type, are used in series and parallel combination, they are considered to have the same internal resistance and E.M.F. This is approximately true, and the process of finding resulting voltage and current is a matter of simple addition. Even if the cells are unlike and are joined in series, the E.M.F.'s and resistances merely add together. This case need not be considered.

But it is interesting to see what will happen if we have two or more cells of unlike E.M.F. and resistance, joined in parallel and feeding a line. Such combinations represent actual conditions in many telegraph and telephone circuits, and afford excellent practice in the application of Ohm's Law as extended by Kirchhoff.

Example. Consider Fig. 193. Cell E_1 has 2 volts E.M.F. and .5 ohm internal resistance. Cell E_2 has 1.4 volts E.M.F. and .8 ohm internal resistance. When they are joined in parallel to feed a line of 1.7 ohms resistance:

Find:

(*a*) Amount and direction of current through each cell and through the line.

(*b*) Terminal voltage (across AB).

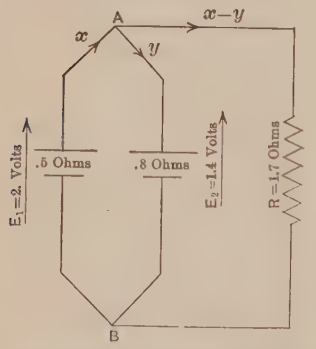

FIG. 193.—Unlike cells in parallel feeding line.

Solution. Assume current flows as marked. There is probably a reversed current through E_2 on account of the higher E.M.F. of cell E_1. We will assume this, and if the current value comes out a negative quantity, we have merely to reverse the arrow head.

Let x = current through E_1.

Let y = " E_2.

Then $x-y$ = current in line.

The voltage drop across the points AB will be equal along the three paths.

Thus voltage across $AB = 2-.5x$

$1.4+.8y$

$1.7(x-y)$

Therefore

$$2-.5x=1.4+.8y,$$

$$y=\frac{.6-.5x}{.8}, \quad \ldots\ldots\ldots \quad (1)$$

and

$$2-.5x=1.7x-1.7y,$$

$$y=\frac{2.2x-2}{1.7}. \quad \ldots\ldots\ldots \quad (2)$$

Since (1) = (2)

$$\frac{.6-.5x}{.8}=\frac{2.2x-2}{1.7};$$

$$1.02-.85x=1.76x-1.6;$$

$$x=1.00 \text{ ampere};$$

$$y=\frac{.6-.5x}{.8};$$

$$=\frac{.6-.5}{.8};$$

$$=.125 \text{ ampere}.$$

$$x-y=1.00-.125$$

$$=.875 \text{ ampere}.$$

Thus current through $E_1=1.00$ ampere.
" $E_2=.125$ "
" $R=.875$ "

Voltage across $AB=2-.5x$;
$=2-.5$;
$=1.5$ volts.

Check

Voltage across AB = (current through R) × (resistance of R);
$=.875\times1.7$;
$=1.49$ volts.

From the results of our computation we see that the current was backing up through cell E_2 on account of its low E.M.F. This shows what is likely to happen in a battery of storage cells when one cell or set of cells, joined in parallel with others, becomes worn out before the rest.

It is common practice, as we have seen, to operate shunt generators in parallel and compound generators in parallel. The necessity for using machines of approximately equal E.M.F. and resistance is apparent from an inspection of this example. The

distribution of current in line and generators so used, is similar to that in the case of battery cells. Thus, if, in above example, *Generators* be substituted for *Battery Cells*, the method, computation, and results would be the same.

Problem 12–9. Assume the following values for Fig. 193 and find current through cells and line.

E_1 has 2.2 volts E.M.F. and .4 ohm internal resistance.
E_2 has 1.8 " " .6 " "
R has .2 ohm resistance.

Problem 13–9. A series set of 5 cells each having 1.4 volts E.M.F. and .25 ohm internal resistance, is joined in parallel to a series set of 6 cells of 1.3 volts E.M.F. and an internal resistance of .2 ohm each. The parallel combination feeds a line having a resistance of .5 ohm.

Find:

(*a*) Current through .5 ohm resistance.
(*b*) Voltage across .5 ohm resistance.

Problem 14–9. A shunt generator of 120 volts E.M.F. and .2 ohm armature resistance is joined in parallel to another shunt generator of 115 volts E.M.F. and .4 ohm armature resistance. The parallel combination is arranged to feed a line having 10 ohms resistance.

Find:

(*a*) Current through line and through each generator.
(*b*) Voltage across line.

155. Efficiency of Direct Current Machines. Stray Power. The efficiency of any machine or any combination of machines has been defined as the ratio of the OUTPUT to the INPUT. As an equation it is written as follows:

$$\text{Efficiency} = \frac{\text{output}}{\text{input}}.$$

In *generators* it is difficult to measure the input. So the output and losses are measured. The *input equals the output plus the losses.* The equation for efficiency of a generator then becomes:

$$\text{Efficiency} = \frac{\text{output}}{\text{input}} = \frac{\text{output}}{\text{output} + \text{losses}}.$$

In *motors* it is easier to measure the input and the losses. *The output equals the input minus the losses.* The equation for the efficiency of a motor then becomes:

$$\text{Efficiency} = \frac{\text{output}}{\text{input}} = \frac{\text{input} - \text{losses}}{\text{input}}.$$

The *losses* in a machine, whether running as a generator or a motor may be divided into three parts:

(1) Copper loss = (I^2R) in armature and field windings.

(2) Iron losses = (hysteresis and eddy currents in armature core);

(3) Mechanical losses = bearing friction, brush friction, and windage.

The 2d and 3d parts, i.e., iron losses and mechanical losses, are difficult to measure separately. For this reason they are generally included in one term and spoken of as the STRAY POWER LOSS.

A machine, then, may be said to have but two losses.

(1) Copper loss.

(2) Stray power loss.

It will be noted that all the losses, of which the STRAY POWER is composed, depend chiefly upon the speed of the machine, and the strength of the magnetic field.

It can be shown also that, within narrow limits, the stray power of any machine is nearly proportional to the voltage induced in the armature (which, we have seen, depends upon the speed and magnetic field). Thus if the field is maintained at a constant strength and the speed of rotation not changed, the stray power of either a motor or a generator is practically constant. Since the field of a shunt machine can be kept constant at all loads, the stray power can usually be measured at no load and assumed to remain constant when the machine is loaded, provided the speed does not change.

If either, or both the field strength and speed change with the load, the stray power at any load can be found by a direct proportion between the stray power and induced E.M.F. at the different loads. This is approximately true only when speed change is slight, say under 10 or 12 per cent.

156. Method of Finding Stray Power Loss in Shunt Dynamo. Suppose a shunt machine is run unloaded, as a separately excited motor, at any desired speed. Since the machine is doing no work, *all the power* taken by the armature *is lost.* This power then takes in both the

(1) Copper loss and

(2) Stray power loss.

The copper loss can be computed by getting the product of the armature resistance by the square of the armature current: If now this *computed copper loss be subtracted* from the *total power* received by the armature the remainder is the *stray power loss.* This may be stated by an equation as follows:

$$P_s = EI_a - I_a^2 R_a,$$

when P_s = Stray power loss in *watts.*

E = Impressed voltage across the armature.

I_a = Current through armature.

R_a = Resistance of armature.

Example (1). A shunt dynamo is run unloaded as a motor with the field separately excited. The pressure across the armature is 120 volts, the current through armature is 3.5 amperes. Armature resistance is .2 ohm. What is Stray Power Loss at this speed?

The total power, and, in this case, the total loss,

$= EI_a$

$= 120 \times 3.5$

$= 420$ watts.

Copper loss $= I_a^2 R_a$

$= 3.5^2 \times .2$

$= 2.45$ watts.

Stray power loss = Total loss − copper loss
= 420 − 2.5
= 417.5 watts.

Example (2). Assume that the above dynamo, when run as a shunt generator at same speed and field strength, delivers 50 amperes at 110 volts. Field resistance = 55 ohms. What are the total losses?

Solution. Since same speed and field are maintained, the induced E.M.F. in armature must be the same. Thus the stray power is constant.

Total losses = stray power + copper loss.

Stray power = 417.5 watts.

Copper loss in field = $I_f^2 R_f$

$$I_f = \frac{110}{55} = 2 \text{ amperes.}$$

$$I_f^2 R_f = 2^2 \times 55$$
$$= 220 \text{ watts.}$$

Copper loss in armature = $I_a^2 R_a$
= $52^2 \times .2$
= 541 watts.

Total loss = 541 + 220 + 418

= 1179 watts
= 1.18 K.W.

Example (3). If the generator ran at sufficient speed to supply 50 amperes at 114 volts, what would total losses be?

Solution:

Current in field = $\frac{114}{55}$ = 2.07 amperes.

Current in armature = 50 + 2.07 = 52.1 amperes.

Stray power losses in this and previous example are proportional to the values of the induced E.M.F. since the variation is not great.

Induced E.M.F. of a generator equals terminal voltage + voltage required to force current through the armature.

Induced E.M.F. in example (2) = $E + I_a R_a$;
= 110 + 52 × .2
= 120.4

Induced E.M.F. in example (3) $= E + I_a R_a$
$= 114 + 52.1 \times .2$
$= 124.4.$

$$\frac{P_s \text{ (at 114 volts)}}{P_s \text{ (at 110 volts)}} = \frac{P_s \text{ (at 114 volts)}}{417.5} = \frac{124.4}{120.4},$$

$$P_s = \frac{124.4}{120.4} \times 417.5$$
$$= 430 \text{ watts.}$$

Copper loss in field $= I_f^2 R_f$
$= 2.07^2 \times 55$
$= 237$ watts.

Copper loss in armature $= I_a^2 R_a$
$= 52.1^2 \times .2$
$= 542.$

Total loss = copper loss in armature + copper loss in field + stray power loss
$= 430 + 237 + 542$
$= 1209$ watts
$= 1.21$ K.W.

When it is possible, the usual method of obtaining the stray power loss of a shunt generator is to run the machine unloaded as a separately excited shunt motor.

The voltage impressed across the armature is made equal to the E.M.F. which the machine generates when supplying full load current, i.e., the terminal voltage as generator at full load plus the voltage lost in armature at full load. The field strength is then adjusted until the machine runs at full load speed. The total input into the armature is then equal to the stray power at full load plus the small $I_a^2 R_a$ loss of no load current.

Problem 15–9. A shunt generator is run as a separately excited motor at no load with speed equal to full load generator speed, and impressed voltage equal to full load generator E.M.F.

Current taken by armature = 4.5 amperes.
Impressed voltage across armature = 115 volts.
Armature resistance = .08 ohm.
What is Stray power loss?

Problem 16–9. The field of generator in Problem 15 has a resistance of 100 ohms. What is total loss at full load of 62.5 amperes at 110 volts?

Problem 17–9. If generator in Problem 15 were run at such a speed that it delivered 75 amperes at 110 volts, what would stray power loss be?

157. Generator Efficiency; Commercial and Electrical. The efficiency of a generator may be stated in two ways: either as the COMMERCIAL EFFICIENCY or as the ELECTRICAL EFFICIENCY. The COMMERCIAL EFFICIENCY is the true efficiency, since it is the ratio of the *total output* to the *total input*. That is

$$\text{Commercial efficiency} = \frac{\text{Output}}{\text{Output} + \text{Total losses}} = \frac{\text{Output}}{\text{Output} + \text{Copper loss} + \text{Stray power loss}}.$$

The commercial efficiency of generator in example (2), page 255, then equals

$$\frac{\text{Output}}{\text{Output} + \text{Copper loss} + \text{Stray power loss}} = \frac{(50 \times 110)}{(50 \times 110) + 1180} = \frac{5500}{6680} = 82.4 \text{ per cent.}$$

The ELECTRICAL EFFICIENCY is the ratio of the *electrical output* to the *electrical power generated*. The electrical power generated equals the electrical power delivered to line, plus electrical power lost in armature and field coils; that is, plus the copper loss. Thus the

$$\text{Electrical efficiency} = \frac{\text{Output}}{\text{Output} + \text{Copper losses}}.$$

The electrical efficiency of generator in example (2), page 255, then equals:

$$\frac{\text{Output}}{\text{Output} + \text{Copper losses}} = \frac{50 \times 110}{(50 \times 110) + (220 + 541)} = \frac{5500}{6261} = 88 \text{ per cent.}$$

Unless otherwise stated, the COMMERCIAL EFFICIENCY of a generator is understood by a statement of its efficiency.

Problem 18–9. What is the commercial efficiency and the electrical efficiency of generator in Problem 16?

Problem 19–9. What is the commercial and the electrical efficiency of generator in Problem 17?

Problem 20–9. What is the electrical efficiency of generator in Problem 7?

Problem 21–9. A 50 K.W. generator has a stray power loss at full load of 1500 watts, and a copper loss of 1800 watts. What is the electrical and the commercial efficiency?

158. Motor Efficiency. Commercial and Mechanical. The efficiency of a motor may be stated in two ways: either as the COMMERCIAL EFFICIENCY, or as the MECHANICAL EFFICIENCY. The commercial efficiency is the true efficiency, since it is the ratio of the *total output* to the *total input*. That is, of a motor, the

$$\text{Commercial efficiency} = \frac{\text{Input} - \text{Total losses}}{\text{Input}}.$$

Example. The stray power of a shunt motor at full load speed and voltage = 400 watts. The armature resistance = .02 ohm. The field resistance = 55 ohms. When running at full load the motor takes 62 amperes at 110 volts. What is the commercial efficiency?

$$\text{Commercial efficiency} = \frac{\text{Input} - \text{Total losses}}{\text{Input}}.$$

$$\text{Input} = 62 \times 110 = 6820 \text{ watts.}$$

Copper losses:

$$\text{In Armature} = 60^2 \times .02$$
$$= 72 \text{ watts.}$$
$$\text{In Field} = \frac{110^2}{55}$$
$$= 220 \text{ watts.}$$

Total copper loss = 292 watts.
Stray power " = 400 "
Total losses = 692 "

$$\text{Commercial efficiency} = \frac{6820 - 692}{6820}$$
$$= \frac{6128}{6820}$$
$$= 89.8\%.$$

The MECHANICAL EFFICIENCY is ratio of the *Mechanical Output* to the *Mechanical Power Developed* in armature. The Mechanical Power Developed equals all the power not used in overcoming electrical resistance of field and armature.

That is, total power developed in armature equals

Input − Copper losses, or

Output + Stray power losses.

$$\text{Mechanical efficiency} = \frac{\text{Input} - \text{Total losses}}{\text{Input} - \text{Copper losses}}$$

or

$$= \frac{\text{Output}}{\text{Output} + \text{Stray power}}.$$

Example. What is the Mechanical Efficiency of motor in above example?

$$\text{Mechanical efficiency} = \frac{\text{Input} - \text{Total losses}}{\text{Input} - \text{Copper losses}}$$
$$= \frac{(62 \times 110) - 692}{(62 \times 110) - 292}$$
$$= \frac{6128}{6528}$$
$$= 93.8\%;$$

or

$$\frac{\text{Output}}{\text{Output} + \text{Stray power}}$$
$$= \frac{(62 \times 110) - 692}{(62 \times 110) - 692 + 400}$$
$$= \frac{6128}{6528}$$
$$= 93.8\%.$$

Unless otherwise stated, the COMMERCIAL EFFICIENCY is meant by the term efficiency when applied to a motor.

Problem 22–9. (*a*) What is the commercial efficiency of motor in Problem 34–8 if the stray power loss is 600 watts? (*b*) What is the mechanical efficiency?

Problem 23–9. Assume the stray power loss in motor of Problem 40–8 to be 700 watts. What is the mechanical and the commercial efficiency?

Problem 24–9. Mechanical efficiency of motor in Problem 41–8 is 92 per cent. What is the stray power loss?

Problem 25–9. The commercial efficiency of motor in Problem 47–8 is 90 per cent. (*a*) What is stray power loss? (*b*) What is mechanical efficiency?

SUMMARY OF CHAPTER IX

ELECTRICAL TRANSMISSION. Is convenient and efficient over long distances.

LINE LOSS. Varies inversely as the square of the voltage of transmission. For a given line loss, the size of the wire, either by cross section or by weight, varies inversely as the square of the voltage.

FEEDERS are used in low voltage systems to increase size of conductors. Power, voltage and current distribution solved by Ohm's Law.

THREE-WIRE SYSTEM. Neutral wire intended to carry very little current.

Requires about three-eighths as much wire as a two-wire system of same capacity, and of same voltage as one leg.

Affords two standard voltages.

KIRCHHOFF'S LAWS. (An extension of Ohm's Law.) (1st) Sum of currents flowing away from a point in a circuit equals sum of currents flowing to the point. (2d) In any circuit, algebraic sum of E.M.F.'s equals sum of IR drops. In a circuit containing no source of E.M.F., sum of IR drops clockwise around the circuit, equals sum of IR drops counter clockwise.

Are applied to find current distribution in "mesh work" and in systems of generators or battery cells operated in parallel.

POWER LOSS IN DYNAMOS. There are three sources of loss in electrical machines, whether run as generators or motors:

(1) Copper loss, (I^2R in Armature and Field.)
(2) Iron loss (Hyteresis and Eddy Currents.)
(3) Mechanical losses (Windage, Friction, etc.)

Items (2) and (3), i.e., iron loss and mechanical losses, are generally grouped as the stray power loss.

EFFICIENCY OF GENERATORS.

$$\text{Commercial Efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{\text{Output}}{\text{Output} + \text{Total Losses}}.$$

$$\text{Electrical Efficiency} = \frac{\text{Output}}{\text{Electrical Power Developed}} = \frac{\text{Output}}{\text{Output} + \text{Copper Losses}}.$$

EFFICIENCY OF MOTORS.

$$\text{Commercial Efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{\text{Input} - \text{Total Losses}}{\text{Input}}.$$

$$\text{Mechanical Efficiency} = \frac{\text{Output}}{\text{Mechanical Power Developed}}$$

$$= \frac{\text{Input} - \text{Total Losses}}{\text{Input} - \text{Copper Losses}} = \frac{\text{Output}}{\text{Output} + \text{Stray Power}}.$$

PROBLEMS ON CHAPTER IX

26–9. A 500 K.W. generator is to be selected to transmit power 6 miles over a No. 0 copper wire. Plot a curve showing relation of voltage to efficiency of transmission for above power, ranging from 0 per cent efficiency to 99 per cent. Have at least 12 points about equally spaced along the curve.

27–9. State what voltage you would select to transmit power in Problem 26. Using 3 times this voltage, what size wire could be used with the same line loss?

28–9. Allowing 5 per cent of the power generated to be lost in the line, plot a curve between size of the copper wire (in circular mils) and voltage of transmission in Problem 26. Plot at least 8 points.

29–9. What size aluminum wire could be used in Problem 27, with same line loss?

30–9. What size copper wire is required to deliver current at 110 volts to a 25 H.P. motor of 90 per cent efficiency, if the motor is 1500 feet from the 115-volt generator?

31–9. Generator armature of Problem 30 has a resistance of .02 ohm. (*a*) What is its E.M.F.? (*b*) What is its electrical efficiency? (*c*) What is the efficiency of transmission? Neglect field resistance.

32–9. An electric railway is in the form of a rectangle 6 miles by $2\frac{1}{2}$ miles. The generator station is in the middle of one of the long sides. The trolley wire is No. 0 hard-drawn copper. The track resistance is .01 ohm per mile. There is one No. 0000 feeder running from generator station directly across rectangle to the other side, where it is joined to the trolley wire. There are 2 cars on line at a given instant, one in middle of each short

side of rectangle; one is taking 100 amperes, the other 60 amperes. What is the voltage across each car, if the brush potential of the generator is 570 volts?

33–9. If there were 4 cars on the line in Problem 32, one at each corner of the rectangle, each taking 60 amperes, what would voltage be across each car?

34–9. Suppose feeder in Problem 32 broke, what would be the drop across each car?

35–9. A 9-mile trolley line with generator station at one end has two feeders. One of No. 0000 soft copper wire extends from generator 6 miles along trolley and is tied to trolley every 2 miles. The other of hard-drawn copper, 800,000 C.M. in cross-section, extends from generator along the trolley for 3 miles and is tied to trolley at the end only. There are 3 cars on the line, distributed as follows: Car I, 3 miles from generator station, takes 100 amperes. Car II, 5 miles from station, takes 50 amperes. Car III, 7 miles from station, takes 40 amperes. Trolley wire is No. 0 hard-drawn copper. Track resistance is .03 ohm per mile. Generator voltage is 580 volts. (*a*) What is voltage across each car? (*b*) What is efficiency of transmission?

36–9. If trolley wire in Problem 35 breaks between generator and place where No. 0000 feeder is tied to it, what would voltage across each car become?

37–9. The shunt motor of a motor-generator set takes 60 amperes at 110 volts. The stray power loss of motor is 430 watts. Field resistance is 100 ohms. Armature resistance is .08 ohm. The stray power loss in generator is 420 watts; armature resistance = .0008 ohm. Shunt field resistance, 10 ohms. Electrical efficiency of generator 93 per cent. What is terminal voltage of generator, if it is supplying 600 amperes?

38–9. (*a*) What is the commercial and the mechanical efficiency of motor in Problem 37? (*b*) What is the commercial efficiency of the generator? (*c*) What is the efficiency of the set?

39–9. A shunt generator, the armature of which has a resistance of .5 ohm is charging a set of 40 storage cells in series. E.M.F. of generator = 120 volts. Each cell has a resistance of .002 ohm and an E.M.F. of 2.5 volts. Allow .15 ohm for lead wire, connections, etc. Neglect current through field coils. (*a*) What current is generator delivering? (*b*) What is terminal voltage of generator? (*c*) What is voltage across each cell?

40–9. Each of two shunt generators has an E.M.F. of 120 volts and an armature resistance of .08 ohm. If the two generators are joined in series across a line of 10 ohms resistance, what current will flow? Neglect field currents.

41–9. If generators of Problem 40 are joined in parallel across same line, what current will flow? Neglect field currents.

42–9. Assume the field resistance of each generator of Problem 40 to be 60 ohms. (*a*) What is the brush potential of each when joined as in Problem 40? (*b*) When joined as in Problem 41?

43–9. Assume one of the generators in Problem 40 had an E.M.F. of 115 volts and an armature resistance of .06 ohm, and other remained as in Problem 40. What current would flow through line if joined as in Problem 40? Neglect field currents.

44–9. If generators of Problem 43 were joined as in Problem 41, what current would flow? Neglect field currents.

45–9. An electric arc between carbons has a counter E.M.F. of 33 volts. When 42 volts are impressed across the arc, and 9.5 amperes flow, what resistance has arc?

46–9. It is desired to light a house with 300 incandescent lamps, in parallel. The lamps have a hot resistance of 200 ohms and operate on 110 volts. A battery of storage cells is to be used, each of which has an E.M.F. of 2.2 volts, an internal resistance of .004 ohm, and normal discharge rate is 30 amperes. How many cells are required and how should they be arranged?

47–9. A motor being tested by a Prony brake, runs at 1200 R.P.M., and takes 67 amperes at 112 volts. Brake arm is 16 inches long, balance registers 28 lbs. What is input, output, and efficiency of motor?

48–9. Assume that each lamp in Fig. 194 takes 2 amperes, and that resistance of the lamps remains constant. Brush potential of each generator is 110 volts. Find:

(*a*) Line drop in each section.
(*b*) Voltage across each set of lamps.
(*c*) Power delivered by G_1 and G_2.
(*d*) Efficiency of transmission.

49–9. If a break occurs in neutral between O and S, what would be the values of (*a*), (*b*), (*c*) and (*d*), Problem 48–9?

50–9. If a break occurs in the neutral between S and V what would be the values of (*a*), (*b*), (*c*) and (*d*), Problem 48–9?

51–9. A building is supplied by a two-wire system, the wiring diagram of which is as per Fig. 195. Values on lines represent the resistance of that section of line wire. Motor *M* takes 40 amperes. Each lamp takes 4 amperes, and resistance of each

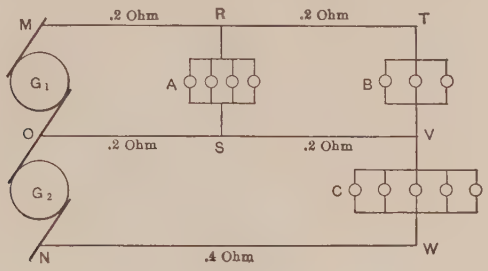

Fig. 194.

is constant. (*a*) Find voltage across each set of lamps. (*b*) Draw diagram indicating amount and direction of current in each section of line.

Note. The *IR* drop across *NMR* must equal *IR* drop across *NSR*, since these are parallel circuits between the same two points *R* and *N*. Similarly treat points *P* and *T*.

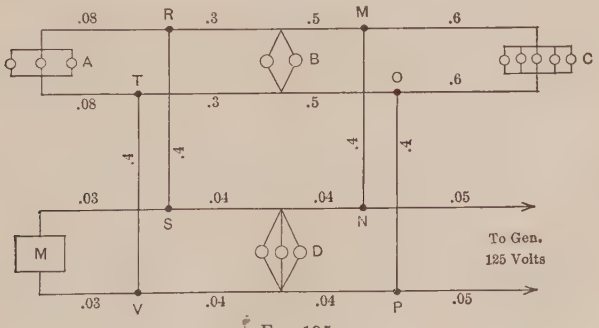

Fig. 195.

52–9. Find the voltage across the various sets of lamps of Problem 51–9, when the motor is not running.

53–9. Repeat Problem 52–9, leaving out jumpers *MN* and *OP*.

54–9. For running a coil of 2 ohms resistance, 4 storage cells in parallel are used. Each had, when new, an E.M.F. of 2.4 volts

and internal resistance of .04 ohm. What current flows through the line in this case, and what is terminal voltage of the battery?

55–9. One of the cells of Problem 54 was damaged. Its E.M.F. fell to 2.0 volts and its internal resistance rose to .08 ohm. The four cells are still used as in Problem 54. (*a*) What is current in line? (*b*) Terminal voltage of battery?

56–9. A shunt motor when delivering full load takes 40 amperes at 112 volts. When running with no load at same speed it takes 3 amperes at 106 volts. Field resistance is 100 ohms and armature resistance is .125 ohm. (*a*) What power does it deliver at full load? (*b*) What are the commercial and the mechanical efficiencies?

57–9. In system arranged as in Fig. 192, each lamp takes 10 amperes. The resistance of the different sections of the line is as follows:

AB = .05 ohm. CD = .06 ohm.
BC = .04 " EF = .03 "
KF = .05 " G_1 = .02 "
MK = .03 " G_2 = .025 "
E.M.F. of G_1 = 130 volts. E.M.F. of G_2 = 135 volts.

Find:

(*a*) Voltage across group I and across group II.
(*b*) Terminal voltage of G_1 and G_2.

58–9. Find efficiency of transmission in Problem 57–9.

59–9. The generators in Problem 57 are of the shunt type. The field of G_1 has 58 ohms resistance, of G_2, 62 ohms. What is the electrical efficiency of each?

60–9. What would the total losses be in generator of Problem 15–9 at full load with terminal voltage of 112 volts? Field = 100 ohms.

61–9. A 5-K.W. 110-volt compound generator, of long shunt type, has stray power loss at full load of 600 watts. Resistance of series field = .025 ohm, of armature = .015 ohm, of shunt field, 55 ohms. Find the electrical and the commercial efficiencies.

62–9. If same data as in Problem 61–9 held for a short shunt generator, what would the two efficiencies be?

63–9. If machine in Problem 61 is used as a motor and takes 48 amperes at 112 volts at full load, what will the commercial and the mechanical efficiencies be?

64–9. If data of Problem 63 held for a short shunt motor, what would the two efficiencies be?

65–9. In Fig. 106 the terminal voltage of the battery cell is 2 volts. Resistance $R_1=5$ ohms, $R_2=8$ ohms, $R_3=10$ ohms, $R=12$ ohms. Galvanometer line is assumed closed, the resistance of which is 25 ohms. What current is flowing in each section of unbalanced bridge?

66–9. According to Kelvin's Rule for most economical size conductor, what size copper wire, B. & S. gauge, should be used under the following conditions?

Cost of installed copper wire per lb., \$.20.
Cost of generating power \$.01 per K.W. hr.
Probable rate of interest, 5 per cent.
Power to be transmitted, 1000 K.W.
Voltage of transmission, 550 volts.
Power to be used 10 hrs. per day throughout year.
Copper weighs .3195 lb. per cu.in.

67–9. Compute size of aluminum wire that should be used under conditions of Problem 66, assuming that aluminum costs twice as much per lb. installed as copper. Aluminum weighs .0963 lb. per cu.in.

68–9. If double the voltage of Problem 66–9 is used what size copper conductor is the most economical?

CHAPTER X

INDUCTANCE

Mutual Inductance—Cause of; Effect of; Lenz's Law—Induction Coils; Jump Spark; Ruhmkorff—Transformers—Self Inductance; Cause and Effect of—Induction Coil; Make and Break—Inductance, a Property of the Circuit; Unit of Inductance, the Henry—Computation of Self Inductance; of Mutual Inductance; of Inductance in Transmission Lines—Effect of Inductance in Alternating Current Circuits.

The exact nature of magnetism and electricity is still unknown, but much has been discovered concerning the relations existing between them.

We have seen that wherever there is a current of electricity flowing, there is always a magnetic field present. The strength of this magnetic field is proportional to the strength of the current, and the direction of it always holds the same relation to direction of the current.

We have also seen that when an electric conductor cuts the lines of force in a magnetic field, an electric pressure is induced in the wire tending to cause a current to flow. The strength of this induced voltage is proportional to the *rate* of cutting. The direction of it has a definite relation to the direction of the field and to the direction of the motion.

We have further seen that when a steady current is flowing through an electric circuit, a certain definite pressure is required to keep this current flowing. This pressure depends upon the resistance of the circuit and upon the work to be done by the current. That is, the resistance of the conductors must be overcome, and the counter

E.M.F. of any revolving motor armature, etc.; just as in a machine, the friction of the bearings must be overcome and the opposing force offered by any work that is being done. As long as the current in an electric circuit is steady, we have seen by Ohm's Law, that a definite steady electromotive force is required to maintain it, just as a definite steady mechanical force is required to keep a machine turning against a steady resistance. So far, we have been dealing with steady currents, only. Nothing has been said about the force required in starting and stopping a flow of electricity.

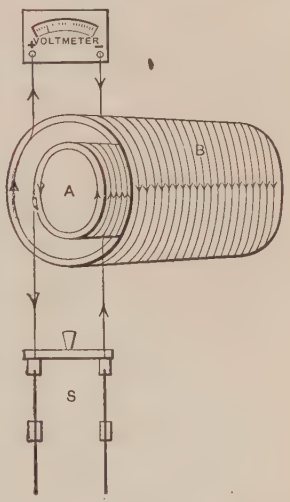

FIG. 196.—Induction coils. Mutual induction.

159. Inductance. Induced E.M.F. Suppose we have two coils of wire, A and B, Fig. 196. Coil A is placed within coil B, but has *no electrical connection to it.* Across the terminals of B a voltmeter is placed. Now when switch S is thrown to power, a current rushes into coil A and flows around the coil counter-clockwise, as marked. But also, strangely enough, a *momentary* current flows around in coil B, through the voltmeter, *only in the opposite direction.* This momentary current in B lasts but for an instant, and dies out.

If now, we suddenly open the switch S, in order to stop the current in A, we notice that another *momentary* current is set up in B. This time the *momentary current in* B is in the same direction as the current we are stopping in A. This momentary current, like the first one, dies out almost instantly. Thus when we closed the switch and started a current in A, we noticed a *momentary* current set up in

B, opposite in direction to the current we were starting in A. On the other hand, while we were stopping the current in A, we noticed a momentary current set up in B, in the same direction as the one we were stopping. These momentary currents set up in B, whenever there was any change in the current in A, are called INDUCED CURRENTS. The E.M.F. tending to cause them to flow is called an INDUCED E.M.F. and the two electric circuits are said to possess the property of MUTUAL INDUCTANCE.

Note that, though in one case the induced current was in the opposite direction to the current in A, and in the other case in the same direction, in both cases it *opposed the change taking place* in the current in A. It *opposed* the *setting up* of a current in A, when we threw the switch, by setting up one in the opposite direction. When we pulled the switch, it *opposed* the *stopping* of the current in A, by setting up one in the same direction as the one we were stopping. In each case as soon as the *change* in the current in A was over, the induced current died out.

Let us consider also the magnetic field set up by these induced currents in B. When we sent the current into A, we were setting up a field within the coil making this end a north pole. Note that the induced current in B *opposed this setting up* of a north pole by setting up a field with a south pole at this end, in order to neutralize it. But after the field was once set up, the current in B ceased trying to neutralize it and stopped flowing. As soon, however, as we began to destroy this field by stopping the current in A, a current was induced in B, which *opposed the dying out of the field*, by setting up a field of its own in the same direction as the one we were destroying. In each case the field of the induced current *opposed the change* that was taking place in the field within the coils.

All these phenomena are merely manifestations of a great law first stated by Lenz, and called Lenz's Law, part of which is:

That an induced current is always in such a direction that its field *opposes any change* in the existing field.

It is part of the law of "conservation of energy," as applied to the electric circuit.

It remains to be shown how it is really the magnetic field set up by an electric current which causes these phenomena of induction; how the magnetic field contains the kinetic energy of a circuit, just as a fly wheel and other moving parts contain the kinetic energy of a machine in motion.

In getting a machine up to speed, we must overcome not only the force of friction but also the INERTIA reaction during the process. In setting up a current in a circuit, we must overcome not only the resistance (friction) of the circuit, but also the INDUCTANCE (inertia) reaction of the magnetic field. In trying to stop a machine which has a large fly wheel, we feel conscious of some reaction tending to keep the machine turning, i.e., an opposing force, which increases enormously if an effort is made to stop the wheel suddenly. This is the reaction due to the inertia of the machine. Similarly, we feel conscious that there is a reaction which tends to oppose the sudden stopping of an electric current which has a strong magnetic field, i.e., there is a force which tends to keep such a current flowing. This is the reaction due to the INDUCTANCE of the circuit. The larger the MOMENT OF INERTIA of the fly wheel, the larger this force opposing any effort to stop it in a given time. The larger the INDUCTANCE of the electric circuit, the larger the force which opposes the stopping of the current flowing through it in a given time.

Just as the reaction due to inertia opposes the increasing or decreasing the speed of a fly wheel, so the reaction due to inductance opposes the increasing or decreasing of the current in an electric circuit. Let us investigate the causes and the practical results of this property of a magnetic field.

160. Induction Coils. Mutual Inductance. In Fig. 196, the inner coil A is composed of few turns of coarse wire. If a current were sent into coil A, let us consider what would happen while the current was building up to its maximum value. Fig. 197 (b), represents a vertical cross section along the axis of the coil.

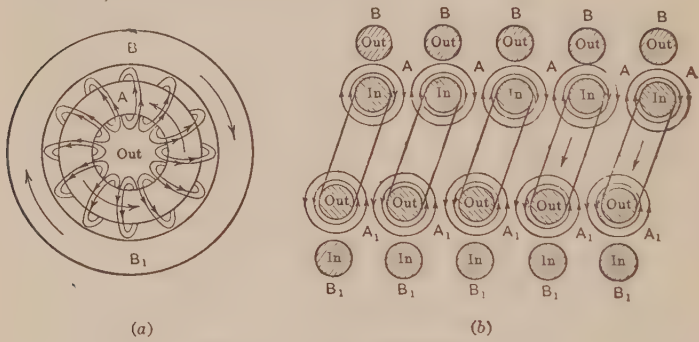

Fig. 197.—Section of induction coil.

Fig. 197 (a), represents an ideal end view. Assume the current to enter coil A at right-hand end and to flow *in* at the top of the loop A and *out* at the bottom of the loop A. This would cause a clockwise field to grow around the top wires of coil A and a counter-clockwise field around the bottom wires. This field spreads out in ever widening rings as the current is increased and cuts across the sides of the coil B. This is shown in Fig. 198 (1), (2), (3), where the wire A represents the end of a wire in coil A, and B represents the end of a wire in coil B. It shows that as the current in A grows, the magnetic field caused by it, spreads out and cuts across the wires of coil B. The wires

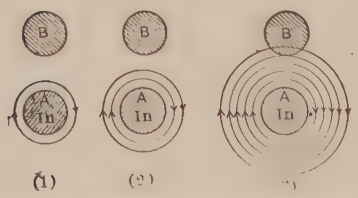

Fig. 198.—Effect of growing field in wires of induction coil.

on the top of coil B are cut by the lines as they move upward. This is equivalent to the wires of B moving downward and cutting the lines as shown in Fig. 199. By applying the right-hand rule, we see that there would be a voltage induced in B tending to send the current OUT of B. *This is in the opposite direction to the inducing current in A.*

If we apply this to the lower sides of the coil, Fig. 197, we find that the inducing current in coil A is *out* in the wires (A_1) at the bottom, and that the wires B_1 of coil B are cut by the growing field in such a way as to induce a voltage which tends to send a current *in* at B_1. Here again we see that the *induced voltage* in coil B *is opposed*

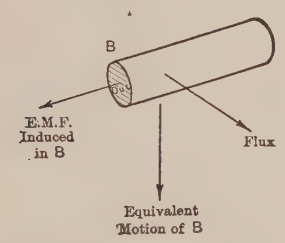

FIG. 199.—Direction of E.M.F. induced in secondary coil.

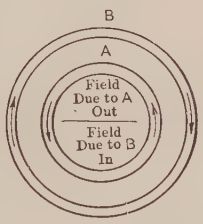

FIG. 200.—End view of induction coil, primary current growing.

in direction *to the increasing current* in coil A. If we look at the end of the coil, Fig. 200, we see that the inducing electric current in A is in a counter-clockwise direction. As long as it is increasing, it is causing an induced current to flow in B in a clockwise direction.

The magnetic field in the air core, due to the current in B, is opposed to the building up of the field due to the current in A.

The current in A is trying to build up a field OUT (Fig. 200), while the current induced in B tends to oppose this building up of a field OUT by neutralizing it with a field IN.

There is no electrical connection between coil A and B. They are very carefully insulated from each other. Yet

all the time that a *current is increasing* in one direction in *A*, it causes a current to build up in *B*. The field of this induced current in *B opposes the building up* of the field by the current in *A*. As soon as the current in *A* reaches its normal value, the lines of force around the wires no longer will be spreading out and there will be no lines cutting the sides of *B*, and thus the induced current in *B* will die out.

The induced current thus lasts *only as long as the "primary" current* (the current in *A*) *is growing*. While the current in *A* was growing, it had to overcome both the resistance of the wires composing the coil, and also the opposition of the induced current in *B*, to the building up of the field. As soon as the current in *A* reached its normal value, the induced current in *B* died out, and there was left the resistance only of the wires of *A* to be overcome. The opposition to the building up of the field is due to the INDUCTANCE of the coil. Since this opposition exists in a coil not connected electrically with first coil, the two coils are said to possess MUTUAL INDUCTANCE.

As long as a force is trying to speed up a machine, it has to overcome both the friction (resistance) and a certain opposition to the building up of speed. As soon as the normal speed is reached, the opposition to building up ceases, and there is only the friction left to be overcome. This opposition to building up the speed is due to the MOMENT OF INERTIA of the moving parts, as we have stated before.

By comparing these two statements, it can be seen that there is a close analogy between *inertia* and *inductance*, in that they both oppose an *increase*.

But inertia reaction not only opposes any increase in speed, but also opposes any *decrease* in speed. Let us now consider how inductance reaction opposes the *decrease* of the current. Fig. 201 (1), (2), (3), represents the current in *A* dying out. The field around each wire now contracts and in so doing, again cuts the wires of coil

B. But this time the cutting is in the opposite direction to what it was when the field was spreading out. As the field was spreading out, there was a current of OUT induced in the coil *B*, which was *opposite* to the current of IN in *A*. Now, by applying the right-hand rule, we find that when the field is contracting, there is a current of IN induced in *B*, which is in the *same direction* as the dying current IN in *A*.

If we again look at the end view of the coil (Fig. 202) as the current is dying out in *A*, instead of increasing, we see that the field due to the dying current in *A* is OUT and the field due to the induced current in *B* is also OUT.

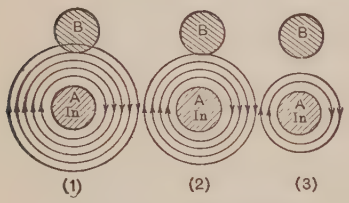

FIG. 201.—Effect of dying field in induction coil.

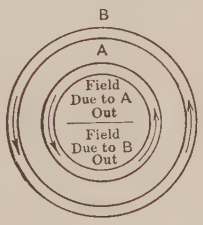

FIG. 202.—End view of coil, current in *A* dying.

This shows that the field due to the induced current *B* *opposes the decreasing* of the magnetic field of current in *A* by building one up in the same direction as the dying field.

This is as we should expect, since we have seen that inductance is analogous to inertia. *Inertia reactions oppose any increase or decrease in the existing speed. Induced currents oppose any increase or decrease in the existing magnetic field.*

161. Direction of E.M.F. Due to Inductance. Lenz's Law. THE INDUCED E.M.F. TENDS TO SET UP A CURRENT WHOSE MAGNETIC FIELD ALWAYS OPPOSES ANY CHANGE IN THE EXISTING FIELD. The important word in this law

is the word CHANGE. The field of the induced current does not always oppose the existing field, nor does it always aid it, but it always opposes any *change* in it. Thus if the existing field is zero, and we send a current through the primary coil to set up a field, there will be induced in the secondary coil a current whose field will *be opposite* to the growing field, and thus will tend to keep the condition of the field as near zero as possible. But if there is a field already within the coil, and we try to weaken it, say by decreasing the current in the primary, there will be induced in the secondary coil a current whose field will *aid* the existing field and tend to keep it at the same strength. The induced current thus is always such that its field *opposes any change* in the existing magnetic field. This can be very easily proved by thrusting a magnet into a coil of wire which has a galvanometer in the circuit and noting that there is a current induced in the coil, the field of which opposes the direction of the magnetic field, thus tending to neutralize it and keep the condition within the coil neutral. When the magnet is withdrawn there is a current induced, the field of which is in the same direction as the magnet field, thus tending to keep up the existing condition of the field.

162. Induction Coils: Jump Spark. One type of induction coil often used for igniting the charge in a gas engine depends upon this induced current in the secondary coil.

The primary has comparatively few turns of heavy wire; the secondary has many turns of fine wire. A core of soft iron is put within the coils to strengthen the magnetic field. See Fig. 203. The secondary circuit B has an air gap in it. This gap is between two points (R and R_1) which are inside the cylinder of the gas engine. The primary circuit A is connected to a set of battery cells. The cam first closes the primary circuit and causes a field to be set up in the coil. Then, as the cam opens the primary

circuit, the field is suddenly diminished as the current in the primary dies out. This causes a high induced E.M.F. to be set up in the secondary, because the field on dying out cuts the large number of turns in the secondary at a rapid rate. This induced E.M.F. is high enough to cause a spark to jump between the points R and R_1, and ignite the charge of gas in the cylinder. There is a small spark when the circuit in the primary is made, because the *growing* field cuts the wires of the secondary coil. But the field *grows* more slowly than it *dies out* and thus the induced E.M.F. is not as great and the spark is much more feeble.

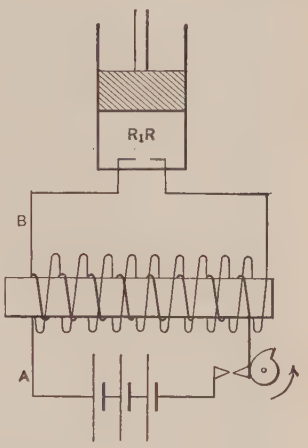

FIG. 203.—Jump spark coil.

163. Ruhmkorff Coils. A Ruhmkorff coil is built on the same plan as the jump spark coil. The number of turns in the secondary is many times the number in the primary, and thus the voltage across the secondary is many times that across the primary. The voltage of the secondary should be in about the same ratio to the voltage of the primary as the number of turns in the secondary is to the number of turns in the primary. Of course what is gained in voltage is lost in amperage.

The Ruhmkorff coil has an interrupter in the primary, which works very rapidly, sometimes as high as 200 times a second. This will cause a continuous discharge of sparks to pass across the spark gap in the secondary when the current is broken. The current for the primary is usually supplied by battery cells. This type of coil is in general use with a gas engine, in which case, when the cam in Fig. 203 closes the primary circuit, the interrupter makes

and breaks it very rapidly, causing a series of sparks across the gap in the secondary.

164. Transformers. Transformers are mutual induction coils used on *alternating current* lines for raising or lowering the voltage. Each transformer consists of a primary coil and a secondary coil wound on a core of soft iron or annealed steel.

We have seen that it is much more efficient to transmit power at high voltages. Accordingly, the coil of few turns is connected to the generator and the coil of many turns to the line. The voltage of the line is then approximately as many times higher than the voltage of the generator as the number of turns in the line coil is greater than those in the coil connected to the generator. A transformer so connected is called a "STEP-UP" transformer. Since it is difficult to use this high voltage for driving motors, etc., it is "stepped down" wherever it is to be used. This is done by connecting in the same kind of a transformer, putting the high voltage side (the one with the large number of turns) in the line, and connecting the low tension side to the motor. Thus, by means of two transformers of the same kind, alternating current power may be generated and used at low voltage, yet transmitted at high voltage. The transformers of the present date being remarkably efficient, the loss in transmission is thus reduced to a minimum.

Transformers are made in two general types: (*a*) the CORE-TYPE, Fig. 204, in which the coils are wound around the iron cores. (*b*) The SHELL-TYPE, in which the iron core is built around the coils.

Figs. 205 to 208 present a good idea of the appearance and construction of a shell type transformer. Fig 205 illustrates the appearance of the transformer in its iron case. The leads or "taps" coming through the case are the ends of the low voltage coils. Note, by inspecting the diagrams, that both the primary and secondary coils are divided into two parts. This gives a possibility of two

voltages being taken from the transformer, by permitting the coils to be joined either in series or parallel. Note also

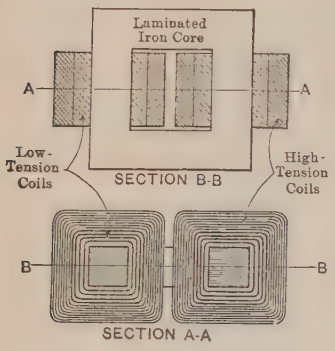

FIG. 204.—Core type of transformer.

FIG. 205.—Fort Wayne transformer, shell type.

FIG. 206.—View of core and coils.

that the core is laminated to reduce the eddy currents to a minimum.

FIG. 207.—Another view of core and coils.

FIG. 208.—Arrangement of coils in two sections.

A transformer is usually represented by a diagram as in Fig. 209. Sometimes a diagram as in Fig. 210 is used.

The action of a transformer follows the same principle as the action of a jump spark coil except that the current

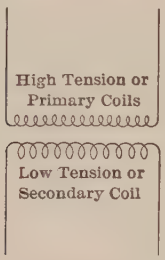

FIG. 209.—Conventional diagram of transformer.

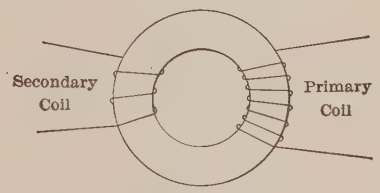

FIG. 210.—Another way of representing a transformer.

in the low tension side instead of being periodically *broken*, is periodically *reversed* by being connected to a source of alternating current.

The current in the high tension side on a step-up connection thus opposes not only the dying out of a magnetic field in the core, but also an actual setting up of a field in the opposite direction. Since current in the low tension

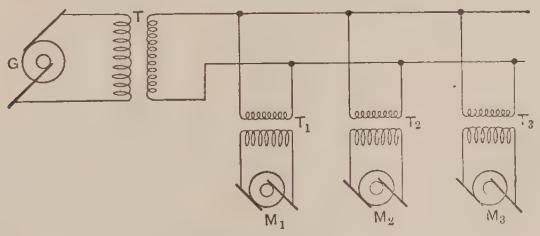

FIG. 211.—High tension transmission by means of transformers.

coils alternates, the field in the core continually changes from a maximum in one direction to a maximum in the other, and there is thus induced in the high tension coils an alternating E.M.F. which tends to oppose this *continuous change*, according to Lenz's law.

The method of transmission by step-up and step-down transformers is shown in Fig. 211. Notice the peculiar circumstance that the alternating current motors M_1, M_2 and M_3, have no electrical connection with the generator G, yet they draw all their power from it. Note also that the transformers T_1, T_2 and T_3 are connected in parallel.

165. Self Induction. Suppose we have a piece of wire 1000 ft. long and of 11 ohms resistance. If we stretch this wire out straight, "line and return" for transmitting power 500 ft., and impress 110 volts across its terminals, a current of 10 amperes will flow, according to Ohm's law.

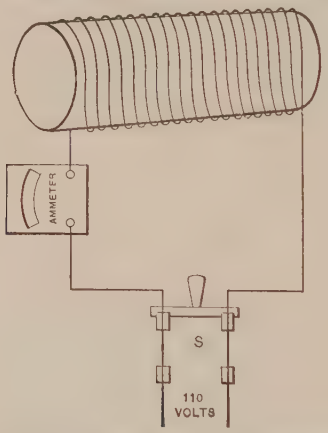

FIG. 212.—Induction-coil. Self induction.

The instant we close the switch the current rises to a steady value of 10 amperes and remains so until we break the circuit. On breaking the circuit, a very slight spark, if any, is noticed.

Now suppose we wind this same wire into a single coil on an iron core as in Fig. 212. The coil still has 11 ohms resistance. When a pressure of 110 volts is now put across its terminals, the ammeter in series with the coil will register 10 amperes, in obedience to Ohm's law. But it will not register the 10 amperes *the instant* that the switch S is thrown. It will require a considerable interval of time

to acquire the steady value of 10 amperes. If we should take data concerning the current and the length of time since the switch was closed, we would secure some such results as the following:

Time Elapsed since Switch was Closed, in Seconds	Current in Amperes.
0	0
.1	4.2
.2	6.6
.3	8.0
.4	9.0
.5	9.3
.6	9.6
.7	9.8
.8	9.9
.9	10.0
1.0	10.0

Plotting these data, for current strength and time, we would get a curve as in Fig. 213.

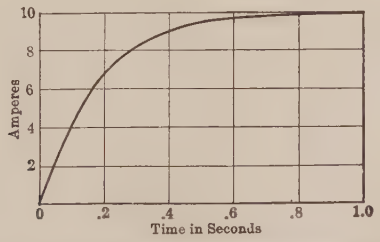

FIG. 213.—Relation of current in coil to time elapsed since voltage was impressed across terminals.

Note that it required .9 second for the current to rise to its full value, after the pressure of 110 volts was thrown across the coil. But once the current reached the value 10 amperes, which it should have according to Ohm's law, it remained constant. After the voltage had been on for

.2 second, the current had risen to only 6.6 amperes. There must be some property in a single coil wound in this way which was not present in a straight wire and which now opposes any rise of the current.

If, as before, we pull the switch S, we find that the current has a tendency to keep flowing, as is evidenced by the spark at the breaking points of the switch. There is evidently some property in the circuit in this form which opposes the dying out of the current. This property in a single circuit which opposes both the building up and the dying out of the current is called SELF INDUCTANCE. A momentary E.M.F. which is set up in the opposite direction to the change that is taking place is called the INDUCED E.M.F. It is this E.M.F. induced in the circuit which keeps the current down. The current in any circuit is always proportional to the algebraic sum of the impressed E.M.F. and induced E.M.F. When a circuit possesses high self inductance, which we shall see it does when it possesses a strong magnetic field, this induced E.M.F. is great if we try to suddenly build up a current, or destroy a current in the circuit.

During the time the current was increasing, the induced E.M.F. was opposing the impressed E.M.F. and the difference between the two voltages determined the current. Thus, at the end of .2 second, since the current was only 6.6 amperes and the impressed voltage was 110 volts, there must have been an induced E.M.F. which opposed the impressed E.M.F. and cut the current down. For a growing current we might express this in the form of an equation:

$$i = \frac{E_x - e}{R},$$

where i = momentary value of growing current;
e = " " induced E.M.F.;
R = resistance of circuit.

The induced E.M.F. at the end of .2 second can then be found as follows:

$$6.6 = \frac{110 - e}{11},$$

$$e = 110 - 66;$$

$$= 44 \text{ volts.}$$

At the end of .4 second, the induced E.M.F. would be much less:

$$9 = \frac{110 - e}{11};$$

$$e = 110 - 99$$

$$= 11 \text{ volts.}$$

By the curve being steeper at the .2 second point, we see that the current is changing more rapidly then than at .4 second point.

The induced E.M.F. is then greatest when the current is changing at the greatest rate, and dies out entirely when the current becomes steady.

As the current is dying out, the self inductance of the circuit tends to keep it flowing, by setting up an induced E.M.F. to cause a current to flow in the same direction. This induced E.M.F. is also proportional to the rate of change in the current. And since a current has a tendency to die out very rapidly as the circuit is broken, the induced E.M.F. may be very great and cause a heavy momentary surge of current. This is noted in the arcing at the break, when a circuit of high self inductance is broken.

Thus we have seen that even a single circuit possesses the property of inductance if it has a strong magnetic field. The inductance sets up an E.M.F. which opposes any change in the existing current of the circuit. This property is called SELF INDUCTANCE, when the momentary E.M.F. is induced in the *same* circuit in which the current change takes place. It is exactly like the induced E.M.F.

which the property of MUTUAL INDUCTANCE sets up in another coil. The induced E.M.F. in each case obeys Lenz's law and tends to set up a current which shall *oppose any change* in the existing magnetic field.

166. Cause of Self Inductance. Make and Break Spark Coil. Consider Fig. 214. A current is made to enter at M. It first reaches the wire A_1. A field in growing around A_1 cuts A_3, and by the right-hand rule, induces an E.M.F. which tends to send a current OUT. But a current of IN is being sent through A_3. This growing current of IN in A_3 is thus opposed by the E.M.F. of OUT induced by the growing field around A_1. Similarly, the current in

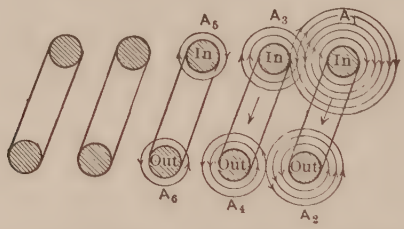

FIG. 214.—Self induction. Effect of growing current.

A_5 is opposed by an E.M.F. induced by the growing field around A_3, and the current in A_4 by the growing field of current in A_2, etc.

Also, if we consider the entire field within the coil, we see that an E.M.F. is induced in the coil, which tends to set up a current opposing the building up of a field within it. Thus a growing current in such a coil is said to be "choked" back, and takes some measurable length of time to come up to the full value as expressed by Ohm's law.

In a like manner, when the circuit is broken and the field around the wires and within the coil starts to die out, the wires are again cut by the field, though this time in a direction which sets up an E.M.F. tending to maintain the field and current as it is.

If the magnetic field is sufficiently strong and the current is broken suddenly, the voltage induced by the dying field cutting the wires of the coil may be many times the original impressed voltage. Use is made of this fact in the MAKE AND BREAK SPARK COIL.

A single coil of many turns, as in Fig. 215, is wound on core of soft iron wires. A moving point P_2, makes contact with a stationary point P_1. Both are inside the cylinder of a gas engine. This causes a current to flow through the coil and set up a strong magnetic field. When the point P_2 leaves P_1 there is a sudden breaking down of this magnetic field which sets up enough induced E.M.F. to cause a spark to jump across the gap now existing between P_1 and P_2.

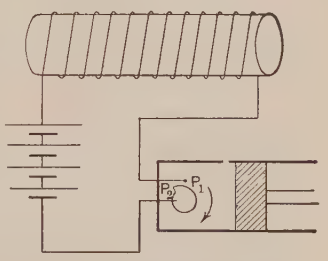

FIG. 215.—Induction coil. Make and break spark coil.

It may be stated as a general principle that whenever an inductive circuit (one having a strong magnetic field) is suddenly broken, the induced E.M.F. is likely to set up an arc at the breaking point.

167. Inductance a Property of the Circuit. The property of inductance is a *property of the circuit*, not of the electric current or voltage. It depends entirely upon the shape and size of the circuit and upon the magnetic permeability of the surrounding medium. If the medium consists of iron, which has a high permeability, and the circuit consists of a coil of wire, the inductance of the circuit is great. On the other hand, if the circuit consists of a straight wire strung in the air, which has low permeability, the inductance of the circuit will be extremely small. The current, voltage, power, etc., have nothing to do with the amount of inductance of the circuit, though these are often affected by the inductance.

INDUCTANCE, then, is not a *material thing*, but merely a *term* which expresses the result of a certain *arrangement* of wires, iron, air, etc., in an electric circuit. It has been likened to MOMENT OF INERTIA, which is not a *material thing*, but merely a term which expresses the result of a certain arrangement of weights, etc., about an axis. Moment of inertia is not a property of matter, force, or energy, but of the machine which conveys the energy; that is, of the *energy circuit.*

And just as the moment of inertia is represented by an algebraic equation, so is the inductance also. In fact, INDUCTANCE can be defined as the *name of a certain algebraic expression* relating to the shape, size, etc., of an electric circuit.

Now that we are familiar with the effect of this property of inductance, we may study the algebraic expression by which it is represented and measured.

168. Computation of Self Inductance. We have seen that the effect of inductance in an electric circuit is analogous to the effect of inertia in a moving machine. Inertia produces a *reactive force* which opposes any *change of speed* in the machine. The amount of this reaction is measured by the product of the mass of the machine times the rate of change produced in the velocity. That is, we write the equation:

$$f = \text{mass} \times \text{rate of change of speed},$$

or

$$f = ma,$$

where f = reacting force due to *inertia;*
m = *mass* of body;
a = *acceleration* (rate of change of speed).

If we use "v" (velocity) instead of "a" (acceleration), and consider the body to start from rest ("0" velocity) and to get up "v" ft. per sec. in "t" seconds, we could express the "rate of change of speed" by the quantity $\frac{v}{t}$ instead of by a.

We might then write the equation as,

$$f = m \times \frac{v}{t};$$

Similarly, *inductance* produces a reacting force (an electromotive force) which opposes the *change* of the *current.*

We may therefore write the equation,

E (force) $= L$ (inductance) $\times$ rate of change of current.

If we consider the current to start at "0" and rise to "I" amperes in "t" seconds, we can represent the "rate of change of current" by the quantity $\frac{I}{t}$.

Our equation then becomes

$$E = L \times \frac{I}{t}. \tag{1}$$

where E = *induced electromotive force* in *volts*;
L = inductance in *henrys*;
I = current in *amperes*;
t = time in *seconds.*

A HENRY may then be defined as the Inductance of a circuit, in which a change of ONE AMPERE PER SECOND produces an Induced E.M.F. of one VOLT.

This equation is similar to the mechanics equation, $f = m \times \frac{v}{t}$, in which we might define the unit m as the *mass* of a body, in which a *change of velocity* of one ft. per sec. in a second produces a reaction of one pound. It will be remembered that we do not measure mass as a quantity in itself, but always express the value of m by the algebraic quantity $\left(\frac{w}{g}\right)$, which contains quantities which we can measure. In the same way, we have to find the value of L from an algebraic expression containing quantities which we can measure.

This algebraic expression for the value of L for a coil can be worked out from the definition and from the above equation.

We have seen that L is the value of the inductance when E volts are produced by a change of I amperes in t seconds.

We know that one volt is produced by one wire cutting 10^8 lines of force per sec., or

$$E = \frac{\phi}{10^8 t}$$

where E = induced E.M.F.;
ϕ = number of lines cut by each wire;
t = time of cutting.

Now if there are N turns of wire in a coil which cut the lines, then the E.M.F. induced in the coil becomes:

(2) $$E = \frac{\phi N}{10^8 t}.$$

But we have seen by Eq. (1) that $E = L \times \frac{I}{t}$.

Therefore (1) = (2), or

$$\frac{LI}{t} = \frac{\phi N}{10^8 t}.$$

Multiplying each side by t

(3) $$LI = \frac{\phi N}{10^8}.$$

We have seen in Chapter VI that

(4) $$\phi = \frac{M}{\mathfrak{R}},$$

and

(5) $$M = 1.26\ NI,$$

and

(6) $$\mathfrak{R} = \frac{l}{\mu A}.$$

Then by substituting (5) and (6) in (4) we get,

$$\phi = \frac{1.26NI\mu A}{l}. \tag{7}$$

Substituting this value of ϕ in Eq. (3) we obtain,

$$LI = \frac{1.26N^2I\mu A}{10^8 l},$$

and dividing through by I,

$$L = \frac{1.26N^2\mu A}{10^8 l}, \text{ sometimes written } \frac{4\pi^2N^2r^2\mu}{10^9 l}, \tag{8}$$

where L = inductance in *henrys;*
N = number of turns;
μ = permeability of core;
A = section area of core in *sq.cms.*;
l = length of core in *centimeters.*

This is the usual equation which is used whenever it is desired to find the inductance of a given coil, wound on a given core.

If the coil is a solenoid with mean radius r, and length l, with an air core, the equation may be changed as follows:

$$1.26 = \frac{4\pi}{10};$$
$$A = \pi r^2;$$
$$\mu = 1.$$

Thus the equation:

$$L = \frac{1.26N^2\mu A}{10^8 l}, \tag{8}$$

becomes,

$$L = \frac{4\pi^2N^2r^2}{10^9 l}. \tag{9}$$

Inasmuch as we are generally dealing with the inductance of transformer coils and electric appliances containing more or less iron in the circuit, Eq. (8) is of the

greatest practical importance. The two may be used interchangeably without any very great error.

Example (1). What is the inductance of the primary coil of a transformer having 400 turns, if the iron core is 60 centimeters long and 300 sq.cms. section area? Assume $\mu = 1500$.

$$L = \frac{1.26N^2\mu A}{10^8 l};$$

$N = 400$ turns;

$\mu = 1500$;

$A = 300$ sq.cms;

$l = 60$;

$$L = \frac{1.26 \times 400 \times 400 \times 1500 \times 300}{10^8 \times 60}$$

$= 15.1$ henrys.

Example (2). Suppose the current in the above coil to change from 26 to 2 amperes in 3 seconds, what average voltage would be induced?

$$E = L \times \frac{I}{t};$$

$$\frac{I}{t} = \frac{26-2}{3} = 8 \text{ amps. per sec.};$$

$E = L \times 8$

$= 15.1 \times 8$

$= 120.8$ volts.

Example (3). If the above change had taken place in .004 of a second, what average voltage would have been induced in the coil?

$$\frac{I}{t} = \frac{26-2}{.004} = 6000 \text{ amps. per sec.};$$

$E = L \times 6000$

$= 15.1 \times 6000$

$= 90{,}600$ volts.

Problem 1–10. A coil of 800 turns is wound on a wrought iron ring, the mean diameter of which is 15 cms., cross-section area 20 sq.cms. Permeability of iron = 1200. What is the inductance of the coil?

Problem 2–10. A current of 30 amperes is flowing in coil of Problem 1. If the current is reduced to 18 amperes, the decrease

taking place in one-quarter second, what average voltage will be induced?

Problem 3–10. Compute the inductance of a coil 75 cms. long, 4 cms. in diameter, containing 2000 turns. Air core.

Problem 4–10. The core of a transformer has two coils wound on it. The primary consists of 120 turns, the secondary of 1200 turns. The core is annealed sheet steel, average permeability of 2000, 72 cms. long, 140 cms. cross-section area. What is the inductance of primary and secondary coils?

Problem 5–10. A "choke" coil is formed by winding 200 turns on a ring consisting of annealed sheet iron, permeability 1800, 60 cms. long and 200 sq.cms. cross-section area. What is the inductance of circuit?

169. Computation of Mutual Inductance. When two coils are placed together in such a way that an increase or decrease in the current in the turns on one coil causes a cutting of lines of force by the turns of the other coil, they are said to have a MUTUAL INDUCTANCE, as we have seen. The symbol for mutual inductance is L_m, and is equal to unity, or one henry, when a change of one ampere per second in one coil causes an induced E.M.F. of one volt in the other.

Jump spark induction coils, Ruhmkorff coils and transformers depend upon this principle of mutual induction for their action, as explained at the beginning of the chapter.

The equation for the value of the mutual inductance of two coils becomes very simple if the two are wound on the same magnetic core and have about the same diameter.

$$L_m = \frac{4\pi^2 N_1 N_2 r^2 \mu}{10^9 l},$$

where N_1 = number of turns on one coil;
N_2 = number of turns on the other coil;
r = radius of the *inner* coil in *centimeters*.

Note that the equation is the same as that for the self-inductance of the two coils wound as one on a core, *except* that instead of squaring the total number of turns in both coils, the number of turns on one coil is multiplied by the number of turns on the other.

In fact, this equation for mutual inductance can easily be derived in the same way as that for self-inductance by taking into account the fact that the changing current in the coil with N_1 turns causes the conductors of the coil with N_2 turns to be cut by the changing flux, and induces voltage in them.

Problem 6–10. What is the mutual inductance of two coils wound on a circular wooden core 4 cms. in diameter and 60 cms. long? One coil contains 200 turns, and the other 400 turns.

Problem 7–10. If the core of Problem 6–10 were of iron ($\mu = 1600$), what would the mutual inductance be?

Problem 8–10. (*a*) If the current in coil containing 200 turns Problem 6–10, were changed at the rate of 100 amperes per sec., what voltage would be induced in the other coil? (*b*) Answer same question for coils in Problem 7–10.

Problem 9–10. If the current in coil containing 400 turns, Problem 7–10, be changed at the rate of 100 amperes per sec., what voltage will be induced in the other coil?

170. Inductance of Transmission Lines. It is necessary at times to compute inductance of straight parallel conductors, especially when used in telephony, telegraphy, or long power transmission. When twin non-magnetic wires are strung in this way in the air, and used as "line and return," the inductance may be computed by means of the following equation:

$$L = \frac{l\left(9.21 \log_{10} \frac{S}{r} + 1\right)}{10^9},$$

where l = length of *one wire* in centimeters;
S = space between centers of wires in centimeters;
r = radius of wire in centimeters.

Problem 10–10. Compute the inductance in a 100-mile aerial line consisting of two copper wires 20 inches apart, each No. 6 B. & S.

Problem 11–10. What voltage would be induced across the terminals of the line in Problem 10-10 if the current changed from $+20$ amperes to -20 amperes in $\frac{1}{50}$ of a second.

171. Effect of Self Inductance in an Alternating Current Circuit. Suppose the coil in Fig. 212 were placed in an alternating current circuit of 110 volts. As soon as the voltage was thrown across its terminals the current would start to rise as in curve 213. But long before the current had time to reach a value of 10 amperes, the voltage would

be reversed by the generator. The current would continue to rise for a little while, although the voltage was falling, but would not have time to reach the value of 10 amperes before it, too, started to fall. In the meantime, the voltage has actually reversed and is now tending to force a current through the coil in the opposite direction. Under such conditions the current falls very rapidly, and begins to flow in the opposite direction. But here, too, long before it reaches a value of 10 amperes, the voltage that started it flowing in this direction has fallen to zero and is urging it to flow in the direction in which it started. Thus the effect of inductance in an alternating current circuit is to keep the current value down, or to "choke" it back, by making it lag so far behind the impressed voltage that it does not have time to get up to the value it would reach on a direct current circuit. The higher the frequency, i.e., the faster the voltage is reversed, the smaller the current in the circuit, because the shorter the time in which it has to grow in either direction, and the further it lags behind the voltage; the changes in current taking place considerably later than the corresponding changes in the voltage.

SUMMARY OF CHAPTER X

The kinetic energy of an electric circuit lies in its magnetic field. The strength of the magnetic field, other things being equal, depends upon the shape of the electric circuit and the medium in which the field lies. It is convenient to express this effect of "shape of circuit and medium of field" by means of an algebraic equation.

INDUCTANCE is the term applied to this equation, as described above. It may be likened to the algebraic expression used in mechanics and known by the term Moment of Inertia. As long as a current is at a constant value in a circuit, the property of Inductance does not make itself known, just as the moment of inertia of a fly wheel does not affect the running of a machine as long as the speed is constant.

LENZ'S LAW. By means of this property of inductance there is an induced E.M.F. set up, whenever the strength of any magnetic field is changed, tending to cause an electric current to flow in such a direction as to oppose any change taking place in the strength of the field.

SELF INDUCTANCE is the property which causes this induced E.M.F. to be set up in the same electric circuit in which a change in current, and consequently a change in magnetic field, is taking place.

MUTUAL INDUCTANCE is the property which causes an induced E.M.F. to be set up in a second circuit, by a change of current in the first.

HENRY; UNIT OF INDUCTANCE. When a change of one ampere per sec. sets up an induced E.M.F. of one volt, the circuit is said to possess an Inductance of one henry. Symbol (L).

EQUATIONS FOR INDUCTANCE.

Self Inductance, $$L = \frac{1.26N^2\mu A}{10^8 l},$$

or

$$\frac{4\pi^2 N^2 r^2}{10^9 l}.$$

Mutual Inductance, $$L_m = \frac{4\pi^2 N_1 N_2 r^2}{10^9 l}.$$

For straight transmission lines

$$L = l \frac{\left[9.21 \log_{10} \frac{S}{r} + 1 \right]}{10^9}.$$

INDUCTION COILS. JUMP SPARK: TWO COIL. By suddenly breaking the current in the primary coil, which is of few turns wound on an iron core, a high E.M.F. is induced by mutual Induction, across the terminal of the secondary coil which has many turns. If these terminals are separated by a small space, a spark will jump across this gap when the primary circuit is broken. Accordingly, these secondary terminals are often located within the cylinder of an explosion engine for the purpose of firing the explosive mixture, by means of the spark thus produced.

RUHMKORFF: TWO COIL. An automatic device in an arrangement like the above, opens and closes the primary circuit very rapidly, thus causing a flow of sparks across the terminals of the secondary. In general use with gas engines.

MAKE AND BREAK SPARK: SINGLE COIL. When even a small current is broken in a single coil of many turns wound on iron core, the self induction causes a spark to jump across the break. The break may be made to occur in the cylinder of a gas engine and explode the mixture.

TRANSFORMERS consist of primary and secondary coils wound on a ring of laminated annealed steel. An alternating current in one coil causes a continual change in the magnetic field. This induces an E.M.F. in the other coil. The ratio of the E.M.F.'s across the two coils approximately equals the ratio of the number of turns in the two coils. Transformers can therefore be used to raise or lower the voltage in an A.C. circuit. Efficiency, very high.

EFFECT OF SELF INDUCTANCE IN A.C. CIRCUITS. (1) Reduces the value of the current. (2) Causes current to "lag" behind the voltage.

PROBLEMS ON CHAPTER X

12–10. The current in a circuit is changed from 0 to 80 amperes in 2 seconds. The induced voltage is 16 volts. What is the inductance of the circuit?

13–10. What voltage is required to *reverse* a current of 80 amperes in 2 seconds in circuit of Problem 12–10?

14–10. How often per second will an alternating E.M.F. of 220 volts reverse a current of 4 amperes in a circuit of .05 henry inductance?

15–10. Find the inductance of a coil 14 cms. long, 8 cms. diameter, having 200 turns of wire, (*a*) with air core; (*b*) with iron core of 2000 permeability.

16–10. Find the inductance of a primary coil of a transformer having 1000 turns. Length of magnetic circuit is 50 cms., area 300 sq.cms. Permeability of iron = 1600.

17–10. If secondary coil of above transformer has 100 turns, what is the inductance of its circuit?

18–10. An alternating pressure of 2000 volts, maximum value, which alternated 120 times (i.e., went from maximum value in one direction to maximum value in the other direction in one second) was placed across the primary coil of transformer in Problem 16. What was the maximum current? (Assume zero resistance for primary and no current in secondary).

19–10. How much voltage would be induced across the secondary coil in Problem 17, if the conditions of Problems 16, 17, and 18 held true?

20–10. An iron ring ($\mu = 1000$) has a cross section of 40 sq.cms., and an average length of 90 cms. How many turns of wire must be wound on it to produce an inductance of 12 henrys?

21–10. When 500 turns of wire are wound on an iron ring, 20 sq.cms. section and average length of 75 cms., the inductance is 1.2 henrys. What value does this give for the permeability of iron?

22–10. If 1000 turns were wound on the iron ring of Problem 21, what would the inductance be?

23–10. A transformer core of annealed steel is 40 cms. long and 120 sq.cms. cross-section area. The primary coil has 2000

turns of wire and the secondary, 200 turns. Permeability of the core is 2100. What is the mutual inductance of the coils?

24–10. What is the self inductance of each of the two coils in Problem 23?

25–10. What maximum current would be sent through primary coil by an alternating E.M.F., maximum value of 1100 volts, which made 50 alternations per sec.? Assume zero resistance and no current in secondary. Use data of Prob. 23–10.

26–10. What voltage would be induced across terminals of secondary in Problem 25? Compute this by means of mutual inductance and rate of current change in primary.

27–10. What is the inductance of a line 200 miles long, consisting of No. 2 copper hung 2.5 ft. apart in the air.

28–10. If coils on core in Problem 23 are joined in series, what will be the self inductance of combination?

29–10. How many turns of wire must be wound on core in Problem 23 to produce 2.8 henrys inductance?

30–10. What voltage would be induced in coil of Problem 21 by a current changing from 15 amperes to zero in $\frac{1}{240}$ sec.?

CHAPTER XI

CAPACITY

Capacity: The Elasticity of an Electric Circuit—Farad and Microfarad, the Unit of Capacity—Relation of Charge, Voltage and Capacity—Positive and Negative Electricity—Bound and Free Charges—Condensers: Equation for Capacity of—Dielectric Power—Dielectric Strength—Capacity of Cables; Equation for—Measurement of Capacity; by Direct Deflection of Ballistic Galvanometer; Bridge Method—Locating a Break in a Cable—Capacity of Condensers Joined in Parallel and in Series.

172. Capacity; Farad. Inductance has been likened to inertia. Capacity may be likened to elasticity. Thus if the walls of the water pipes in Fig. 309 were elastic, either along the whole length or only in places, they would expand with each forward stroke of the pump and contract with each back stroke and thus tend to equalize the pressure throughout the entire stroke by acting as a source of pressure. Similarly, capacity in an electric circuit gives a certain elasticity to the circuit. Should the impressed voltage momentarily fall, if there is considerable capacity in the circuit, it will tend to equalize the pressure by acting as a source of E.M.F.

Inductance makes its presence known when the *current* in a circuit changes. Capacity makes its presence known when the *voltage* across a circuit changes. The capacity of a condenser, for instance, is measured by the amount of electricity that can be added to it, by *raising* the voltage *one volt.*

It is like the capacity of a tank for holding compressed gas. The number of cubic feet of gas that would have to

be put into the tank to raise the pressure one pound per sq.in. might be called the capacity of the tank. The capacity of the tank would *not be all the gas it could hold without bursting.*

In the same way the number of coulombs of electricity that must be added to a condenser to raise the voltage one volt is called the capacity of the condenser. The capacity is *not the number of coulombs it can hold without rupture.*

When the addition of ONE COULOMB raises the voltage across the condenser ONE VOLT, we say that the condenser has a capacity of ONE FARAD. This unit is so large that one-millionth part of it has been adopted as the practical unit, and is called a MICROFARAD.

The capacity of a condenser, then, is stated in terms of the quantity it will hold *per volt pressure.* When it holds one coulomb for every volt pressure across its terminals, we have said that the capacity is called a FARAD. If it holds 2 coulombs per volt pressure, its capacity is 2 farads, etc.

The capacity being stated in farads, i.e., in coulombs per volt, we may write the equation:

$$C(\text{farad}) = \frac{Q(\text{coulombs})}{E(\text{volts})};$$

where C = capacity in *farads;*
Q = quantity in *coulombs;*
E = pressure in *volts.*

Example 1. How many coulombs of electricity will a condenser of 15 microfarads capacity hold, when the pressure between its terminals is 200 volts?

$$C = \frac{Q}{E}$$
$$Q = CE$$
$$= .000015 \times 200$$
$$= .003 \text{ coulomb.}$$

Example 2. If the .003 coulomb in above example takes $\frac{1}{500}$ of a second to flow into the condenser, what is the average current?

$$I = \frac{Q}{t};$$

$$= \frac{.003}{.002}$$

$$= 1.5 \text{ amperes.}$$

Problem 1–11. What is the capacity of a condenser that holds .0012 coulomb under a pressure of 110 volts?

Problem 2–11. How many volts would be required to put .007 coulomb into the condenser of Problem 1?

Problem 3–11. The capacity of a condenser is 20 mfs. How many coulombs will it hold when the pressure is 220 volts?

Problem 4–11. What is the average charging current if it takes .08 sec. for condenser in Problem 3 to become fully charged on a 110-volt circuit?

Problem 5–11. If condenser in Problem 4 be placed on a 220-volt circuit and it took same length of time to charge, what would be average current?

Problem 6–11. If condenser in Problem 4 be charged on 220-volt circuit and then disconnected and an 8-ohm wire be placed across its terminals, what average power would be used in heating the wire? $\left(\text{Average voltage of discharge} = \frac{220}{2}.\right)$

Problem 7–11. How much work (in watt-sec.) would be done on wire in Problem 6?

Problem 8–11. How long would discharge last in Problem 6? (Assume average rate of current.)

Problem 9–11. If a 16-ohm wire were used in Problem 6, what would be average current, average power, and total work done on wire?

Problem 10–11. If a 16-ohm wire be used to discharge condenser of Problem 2, what would be average current, average power, and total work done on wire?

Problem 11–11. How long would it take to discharge condenser in Problem 10? (Assume average rate of discharge.)

Problem 12–11. (*a*) How much work would be done on a wire of any resistance in discharging condenser of Problem 4? (*b*) In discharging condenser of Problem 5?

173. Two Kinds of Electricity. According to the theory which best explains the various electrical phenomena, there are two kinds of electricity—a positive and a negative. Thus a battery or a generator sends out two streams of electricity, one flowing around the circuit in one direction, the other in the other direction. In Fig. 216, the battery cell may be thought of as sending out a stream of positive charges to the plate *A* and of negative charges to the plate *B*, since *A* is connected to the positive terminal and *B* to the negative terminal of the battery. According to this theory one-half of the work in an electric circuit is done by the positive charges flowing in one direction, and the other half by the negative charges flowing in the other direction. This part of the theory need not confuse us, since the work done is the *same* whether it is all done by the negative or all by the positive or is divided between the two.

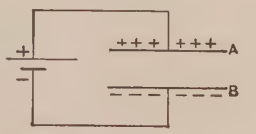

FIG. 216.—"Bound" charges.

The part of the theory that particularly interests us is that which deals with these two kinds of electricity, negative and positive, in their effects upon one another.

It has been found by experiment that the laws for bodies so charged are similar to the laws for bodies magnetized. It must not be inferred from this statement that *electricity* and *magnetism* are the same. As has been stated, they are *not* the same, though possessing peculiar interrelations and analogies.

Electrified bodies resemble magnetized bodies in that bodies charged with the same kind of electricity repel one another, while bodies charged with unlike kinds attract one another. This is usually illustrated as follows:

If we bring two objects near together which have unlike charges, they attract each other with considerable force: and the charge on one seems to attract the charge on the other.

Thus A in Fig. 217 represents an isolated body carrying a positive charge of electricity. The charge is practically uniformly distributed around the body. If a body B similarly charged be brought near A, the charges would appear to repel one another and the distribution would be somewhat as in Fig. 218. If, however, a body C negatively charged were brought near A, the charges would appear to attract each other and the distribution would be as in Fig. 219. The two charges in this case are said to *bind* each other.

FIG. 217.—Isolated charged body.

Now suppose body A were connected to a source of supply of positive electricity as, for example, to the positive terminal of a battery cell or a generator, and body C were connected to a source of supply of negative electricity, for example, the negative terminal of a cell or a generator. Fig. 216 represents such a case. It is easy to imagine now, how

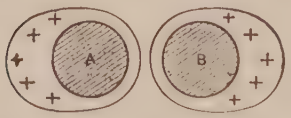

FIG. 218.—Distribution of charge on two bodies carrying like charges.

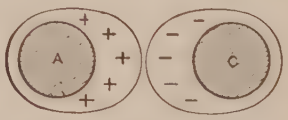

FIG. 219.—Two bodies carrying unlike charges.

the positive charge on A draws a much larger negative charge from the battery cell out to B than if A were not near B. Also the negative charge on B, in turn, attracts a much larger charge of positive electricity from the battery cell out to A. The nearer A and B are together, the greater this attraction will be, and the greater the charge each will have. In this way the binding effect of one charge

on another is very evident. The result is that two plates will hold a larger charge, with the same voltage across them, if they are near together, than if they are separated by a large distance. They will thus have a greater capacity according to our definition of capacity. Of course the larger the plates, the greater the capacity also.

The capacity of a piece of electrical apparatus, then, depends upon the area of parts charged with unlike charges, and upon the distance between them. Experiments show that the capacity of two plates such as A and B in Fig. 216 is *directly* proportional to their area and inversely proportional to the distance between them. This may be expressed algebraically as follows:

$$C \propto \frac{A}{d},$$

or

$$C = K\frac{A}{d};$$

where C = capacity;
A = inside area of plates;
d = distance between plates;
K = constant whose value is determined as explained later.

174. The Condenser. According to the above proportion, if we wish to construct a piece of apparatus with a large capacity, we must place large plates very near to one another. This is done by putting thin strips of mica or oiled paper between sheets of tin or lead foil. The sheets of foil compose the plates and the mica acts as an insulator. In such a case it is customary to call the mica the DIELECTRIC. A piece of apparatus so constructed is called a condenser. Fig. 220 shows the conventional way of representing such a condenser. Note that every other plate is joined to one side of the line, and the rest of the plates to the other side. This gives large negative and positive

plate areas which are very near each other. A high capacity is therefore produced by the BINDING effect of the two unlike charges. The dielectric is not represented.

Such a condenser placed in a circuit acts like an air chamber on a circuit with a pump. It tends to counteract the effect of Inductance, and to steady the voltage, just as an air chamber counteracts the effect of the inertia of the water, and steadies the pressure of water. It gives a kind of elasticity to the circuit, acting as a reservoir for any surplus quantity of electricity. For this reason, it has a very important part in alternating current circuits and telephone lines, where the voltage and current are constantly changing.

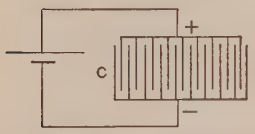

FIG. 220.—Conventional way of representing a condenser and battery

In constructing a condenser several points have to be considered:

(1) To have the capacity high,

(*a*) The plates must have a large total area;

(*b*) They must be as near together as possible;

(*c*) The dielectric must have the *power* of conveying the *influence* of the charges through it, in order that they may bind each other.

(2) To have the unlike plates well insulated from each other,

(*a*) The dielectric must have a *strength* such that the voltage across it will not rupture it and cause a charge to pass through it in the form of an arc.

The term "dielectric power" must not be confused with "strength of the dielectric."

When we speak of the "strength of the dielectric" we mean merely its insulation qualities or ability to resist rupture or leakage of current. The expression "Dielectric Power," however, is a technical term which needs further explanation.

175. Dielectric Power. A condenser built up of certain size plates, and of a certain number of plates, placed a certain distance apart, with *mica sheets* as the dielectric, will have about eight times the capacity of one built with the same dimensions, but the *air spaces* between the plates acting as the dielectric. Mica seems to possess the POWER of conveying the binding influence of one charge on the other to a much greater extent than does air.

This *power* of conveying the binding effect of one charge on another is called DIELECTRIC POWER, and is possessed to greatly varying degrees by different substances.

In equations for calculating the capacity of electrical apparatus, this dielectric power is usually designated by the letter K. Below is a table of the value of K for substances commonly used as dielectrics.

DIELECTRIC POWER OF VARIOUS SUBSTANCES

Material.	K
Air, at ordinary pressure, standard	1.0000
Manila paper	1.50
Paraffin	1.68 to 2.30
Beeswax	1.86
Paraffin, solid	1.9936 to 2.32
Resin	1.77 to 2.55
Ebonite	2.05 to 3.15
India rubber, pure	2.22 to 2.495
Gutta percha	2.45 to 4.20
Shellac	2.74 to 3.60
Glass	3.013 to 3.258
Mica	4.00 to 8
Porcelain	4.38
Flint glass, light	6.85
Flint glass, double extra dense	10.10

INDUCTIVITY and SPECIFIC INDUCTIVE CAPACITY are other names for this DIELECTRIC POWER. Air is generally taken as the standard for Dielectric Power and all other substances are compared with it.

176. Capacity of Plate Condensers. The capacity of a condenser constructed as described above may be computed from the general equation:

$$C = \frac{KA}{4\pi d \times 9 \times 10^5},$$

or

$$\frac{885AK}{10^{10}d};$$

where C = capacity in *microfarads;*
A = area (one side) of all the dielectric actually between plates in *sq.cms.;*
d = average thickness of dielectric in *cms.;*
K = dielectric power.

Example. What is the capacity of a condenser which has 2000 plates? Dielectric consists of sheets of paraffined paper, .005 cm. thick. The part of each sheet actually between plates has an area of 16×20 cms.

$$C = \frac{885 \times 2.1 \times 16 \times 20 \times 2000}{10^{10} \times .005}$$

$$= 24 \text{ microfarads.}$$

Problem 13–11. What is the capacity of a condenser consisting of 200 plates of lead foil? The dielectric consists of 10×15 cms. mica sheets, .1 mm. in thickness.

Problem 14–11. What charge will condenser in Problem 13–11 hold under a pressure of 220 volts?

177. Capacity of Telephone and Telegraph Cables, etc. One of the greatest hindrances to sending telegraphic messages across the ocean is the large capacity of the submarine cable. The insulating material around the wire acts as a dielectric between two conductors, the wire and the water. The great length of the cable gives a sufficiently large area to the dielectric to produce a very high capacity. In transmitting a message, then, it is necessary to alternately fill up and empty a piece of apparatus of great capacity. The time necessary for these

operations puts many difficulties in the way of efficient and rapid transmission.

Telephone cables being laid in pairs, and often in conduits, also possess considerable capacity, which would make it almost impossible to operate long lines, were it not for the possibility of "loading" a line with inductance coils. As has been explained, CAPACITY and INDUCTANCE have practically the opposite effect on a circuit; inductance tending to choke back the flow of electricity, capacity tending to increase the flow. Under the proper conditions, one may be made to neutralize the other. Thus the inductance of the "loading" coils on a telephone line, practically neutralizes the capacity effect of the long insulated cables.

Sometimes condensers are used in a similar way, to counteract the effect of inductance. Examples of this are frequently found in alternating current practice. See Chapter XV for a discussion of inductance and capacity in alternating current circuits.

178. Capacity of Cables. The capacity of submarine cables, and of cables laid in metal sheaths is an important factor in telegraphy and telephony.

The capacity of such lines may be computed by means of the same equation as for a plate condenser. In which case, (d) is the thickness of the insulation in centimeters, and A is *average* area in sq.cms., found as follows:

In Fig. 221:

Let

D = outside diameter of insulation.
d = inside " "
D' = average " "

Then,

$$D' = \frac{D+d}{2}.$$

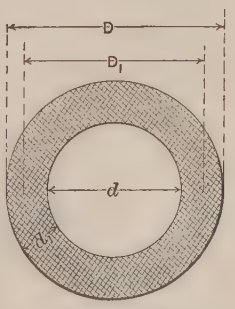

FIG. 221.—Cross-section of insulated cable.

The average area A of the insulation would then be the area of a cylinder whose diameter $= D'$ and whose length, l, equals the length of the cable.

$$A = \pi D' l \quad \text{or} \quad \pi l \frac{(D+d)}{2}.$$

A more common equation, however, for the capacity of a cable is the following:

$$C = \frac{2.413 K l}{10^7 \log_{10} \frac{D}{d}}$$

where C = capacity of cable in *microfarads;*
l = length of cable in *centimeters;*
D = outside diameter of dielectric (insulation) in cms.;
d = inside diameter of dielectric in *cms.;*
K = dielectric power of insulation.

Problem 15–11. A No. 6 (B. & S.) copper wire covered with .05 inch of gutta percha insulation is held in a lead sheath. Find capacity per mile of this cable and grounded sheath.

Problem 16–11. A submarine telegraph cable is of copper .4 inch in diameter, and covered with .6 inch of gutta percha insulation. What is capacity between cable and water of 50 miles of such cable?

The Capacity of an Aerial Line of twin wires may be found by means of the following equation:

$$C = \frac{12.5 l}{\log_{10} \frac{S}{r} \times 10^8},$$

where C = capacity in *microfarads;*
l = length of *one wire* in *centimeters;*
S = space between wires in *centimeters;*
r = radius of each wire in *centimeters.*

Problem 17–11. What is the capacity per mile of a line consisting of two wires, .16 inch in diameter and separated by a space of 20 inches?

Problem 18–11. What is capacity of a 20-mile line consisting of two No. 4 (B. & S.) copper wires hung 18 inches apart?

179. Measurement of Capacity. Direct Deflection of Ballistic Galvanometer. In measuring capacity we make use of the fact, that in order to charge a circuit with electricity, the charge must flow from the source along a conductor. If a ballistic galvanometer is inserted in series in the line, this flow of the charge can be made to give a momentary deflection, or THROW, proportional to the amount of the *charge.*

Thus in Fig. 222, when the switch S is closed, a positive charge flowing along the upper conductor to charge plate A, must pass through galvanometer G. This will cause

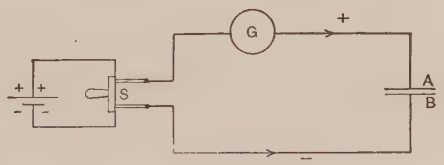

FIG. 222.—Ballistic method of measuring charge flowing into condenser.

the galvanometer to "throw." Since the galvanometer is constructed so that the needle swings, or "throws," to a reading proportional to the moving force and immediately returns to zero, it is said to be a BALLISTIC galvanometer. This term distinguishes it from one which requires a steady current to cause its deflections to be proportional to the moving force.

A more common way of connecting the galvanometer is shown in Fig. 223. The switch S is thrown down to charge the condenser plates A and B, and then quickly thrown up and allowed to discharge through the galvanom-

eter. A special charge and discharge switch is required for accurate results.

To measure the capacity of any device, such as a given length of telephone cable, it is first charged for one minute, then suddenly discharged through the galvanometer and the throw noted. A condenser of known capacity is then inserted in the place of the unknown and charged for a minute, and the throw of the galvanometer noted as it is discharged. The capacities of the two pieces are then in the same ratio as the galvanometer throws, *providing the throws have not too great a difference.* Since the known capacity is usually the variable standard condenser, shown in Figs. 231 and 232, its capacity can be adjusted until the galvanometer throw on discharge is very nearly equal to the throw caused by the unknown capacity. In this way a considerable degree of precision can be obtained.

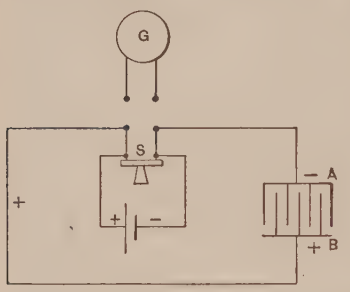

FIG. 223.—More precise ballistic method for measuring charge in condensers.

Problem 19–11. In a circuit arranged as per Fig. 223, a cable is charged to a given potential. On being discharged through a ballistic galvanometer, it causes the instrument to deflect 12.6 scale divisions. A standard condenser of .5 mfs. capacity after being charged to the same potential, on discharge causes the same galvanometer to deflect 11.8 scale divisions. What is capacity of cable as tested?

180. Capacity Measurement. Bridge Method. Another method for measuring capacity is shown in Fig. 224. It resembles a Wheatstone bridge, with the exception that condensers C_1 and C_2, one known and the other unknown, replace two of the resistances. A source of alternating current supply G takes the place of a battery cell, and a

telephone receiver T replaces the galvanometer. G is usually a small hand magneto. A balance is obtained, as in a Wheatstone bridge, by adjusting R_1 and R_2 until there is no sound in receiver T when the magneto G is running.

The following equation is then used to compute the capacity of the unknown, say C_2:

$$\frac{R_1}{R_2}=\frac{C_2}{C_1}.$$

Note that the ratio of the capacities is the *inverted* ratio of the resistances. In this respect it differs radically

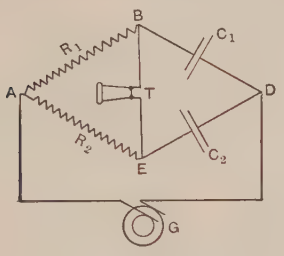

FIG. 224.—Bridge Method of measuring capacity.

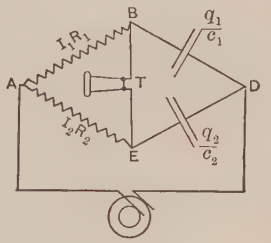

FIG. 225.

from the Wheatstone bridge equation for resistance measurements.

The proof of the above equation is as follows:

When there is no sound in receiver T (Fig. 225), B and E must be at the same potential. The voltage from A to B (I_1R_1) must equal the voltage from A to E (I_2R_2), or

(1) $$I_1R_1=I_2R_2.$$

Also the voltage from B to D must equal the voltage from E to D. The voltage across a condenser equals the charge in it divided by its capacity. (See paragraph 172.)

If q_1 be the charge on C_1, and q_2 the charge on C_2 at any instant, then

$$\text{Voltage } (B \text{ to } D)=\frac{q_1}{C_1};$$

$$\text{Voltage } (E \text{ to } D)=\frac{q_2}{C_2};$$

Therefore,

$$\frac{q_1}{C_1}=\frac{q_2}{C_2},$$

or,

(2) $$C_2q_1=C_1q_2.$$

But in order that the quantity (q_1) get to the condenser (C_1) it must flow through (R_1) as there is no flow along EB.

In like manner the quantity q_2 must flow through R_2. Thus we have equation,

(3*a*) $$I_1 \text{ (current in } R_1)=\frac{q_2}{t};$$

and

(3*b*) $$I_2 \text{ (current in } R_2)=\frac{q_2}{t};$$

where t = the time in seconds in which the charges flow through the resistances. This time must be the same in each resistance since the two resistances are in parallel across the same alternator.

Then, multiplying (3*a*) by R_1 and (3*b*) by R_2, we have,

$$I_1R_1=\frac{q_1}{t}R_1;$$

and

$$I_2R_2=\frac{q_2}{t}R_2;$$

substituting these values in (1),

$$\frac{q_1R_1}{t}=\frac{q_2R_2}{t};$$

or

(4) $$q_1R_1=q_2R_2;$$

Dividing equation (4) by (2) we have,

$$\frac{q_1R_1}{q_1C_2}=\frac{q_2R_2}{q_2C_1};$$

or

$$\frac{R_1}{C_2}=\frac{R_2}{C_1};$$

This may be transposed to read

$$\frac{R_1}{R_2}=\frac{C_2}{C_1}.$$

181. Locating a Break in a Telephone Cable. The above method is often used in locating a break in one of a pair of telephone cables. In Fig. 226, C_1 represents a pair of good wires, C_2 represents a pair in which one of the wires is broken at F.

When bridge is balanced, $\frac{R_1}{R_2}=\frac{C_2}{C_1}$, where C_2 represents the capacity of the faulty pair.

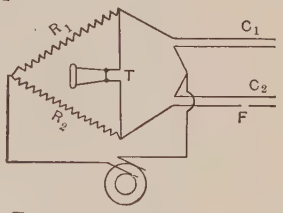

FIG. 226.—Capacity test for locating break in cable.

Now, if the same conditions exist with regard to the wires (except that one wire of one pair is broken) the capacity of the faulty pair as far as the break will have the same relation to the capacity of the good pair, that the distance out to the break has to the length of good pair.

Thus:

$$\frac{l_2}{l_1}=\frac{C_2}{C_1},$$

where l_2 = distance to break;
l_1 = length of good pair;
C_2 = capacity to break;
C_1 = capacity of good pair.

Therefore

$$\frac{R_1}{R_2}=\frac{l_2}{l_1}.$$

If the length of the good pair is known, the distance l_2 out to the break can easily be computed.

Problem 22-11. In a cable test for a break, arranged as in Fig. 226, the length of the pair of good wires (C_1) is 2500 ft. A balance is obtained when $R_1=14$ ohms and $R_2=30$ ohms. How far out is the break in the faulty cable (C_2)?

Problem 23–11. If the capacity of the good pair in Problem 22 is 2.83 mfs., what is the capacity of the faulty pair as far out as the break?

182. Condensers in Parallel. Consider two condensers I and II, Fig. 227, joined in parallel across the mains, the voltage of which is E. The capacity of condenser I is C_1; of condenser II, C_2. Find the combined capacity of the two when so joined.

Let C be the combined capacity;
" Q_1 = quantity of charge in condenser I;
" Q_2 = " " " II.
Then total quantity in both condensers $= Q_1 + Q_2$.

(1) $$Q_1 + Q_2 = CE.$$

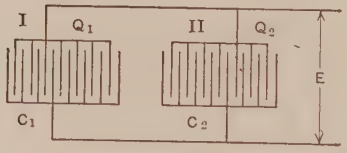

FIG. 227.—Condensers in parallel.

(Quantity of total charge equals total voltage times total capacity.)

But $$Q_1 = C_1E$$

and

$$Q_2 = C_2E.$$

Therefore (2) $Q_1 + Q_2 = (C_1 + C_2)E.$

From (1) and (2) we have

$$CE = (C_1 + C_2)E,$$

or

$$C = C_1 + C_2.$$

Thus the capacity of condensers joined in parallel equals the sum of the capacities of the separate condensers. Joining condensers in parallel is merely adding the plate area of one to that of the other.

Example. What is the capacity of 4 condensers of 3, .2, 7, and 2.5 microfarads respectively, when joined in parallel?

$$C = C_1 + C_2 + \ldots .$$
$$C = 3 + .2 + 7 + 2.5 = 12.7 \text{ microfarads.}$$

Problem 24–11. What charge is sent into condenser I, Fig. 227, when $E = 110$ volts? Capacity of I = 6 mfs.

Problem 25–11. (*a*) What charge is sent into condenser II, Fig. 227, when $E = 110$ volts? (*b*) What charge goes to combination? Capacity of II = 4 mfs.

183. Condensers in Series. Condensers in series present a peculiar phenomenon. Let condenser I of capacity C_1, Fig. 228, be joined in series with condenser II of capacity

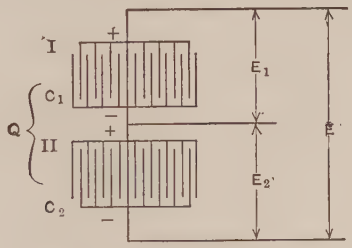

FIG. 228.—Condensers in series.

C_2. Let E be voltage across combination, E_1 across condenser I, and E_2 across condenser II.

The charge Q is sent into the condensers under the action of the voltage E. Since the two condensers are in series the same charge must be sent into each, just as the same current is sent through resistances in series. Thus the charge sent into *each* is Q, and the charge sent into *the combination* is also the same Q.

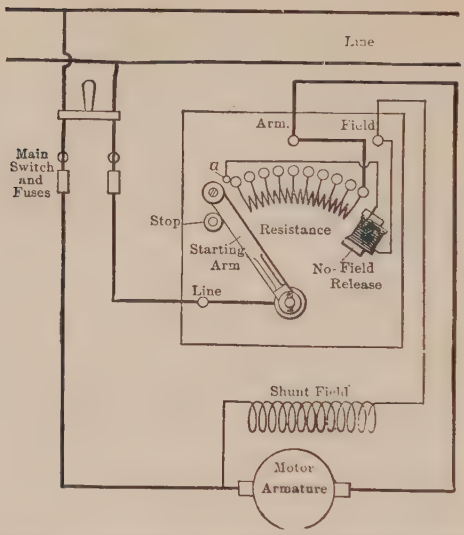

FIG. 229.—Diagram of connections for a starting box with "no-field" release.

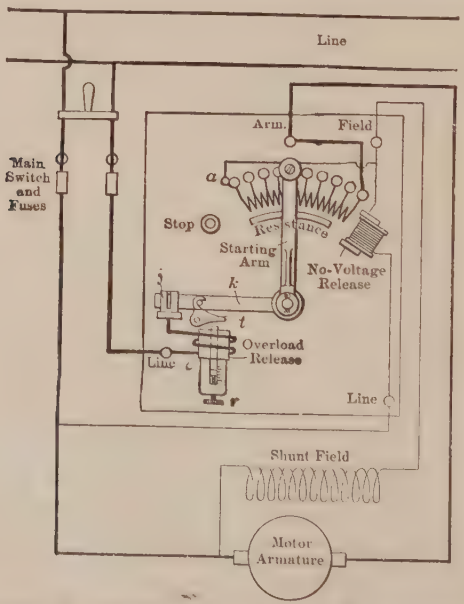

FIG. 230.—Diagram of connections to starting box with "no-voltage" and "overload" release

Let C = combined capacity of C_1 and C_2.

(1) $$E = \frac{Q}{C};$$

$$E_1 = \frac{Q}{C_1};$$

$$E_2 = \frac{Q}{C_2};$$

But

(2) $$E = E_1 + E_2.$$

Therefore, from (1) and (2),

$$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2};$$

and

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}.$$

Thus the reciprocal of the combined capacity of condensers in series equals the sum of the reciprocals of the capacities of the separate condensers.

Example. If the capacity of Condenser I in Fig. 228 is 2 mfs. and that of condenser II, 5 mfs., what is the capacity of the two condensers joined in series?

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2};$$

$$\frac{1}{C} = \frac{1}{2} + \frac{1}{5} = \frac{7}{10};$$

$$C = \frac{10}{7} = 1.43 \text{ mfs.}$$

Problem 26–11. What charge is sent into condensers, of above example, if the voltage across the combination is 110 volts?

Problem 27–11. What is the voltage across condenser I and across condenser II, Fig. 228, if voltage across the two in series is 110 volts? Capacity of I = 8 mfs., of II = 3 mfs.

SUMMARY OF CHAPTER XI

CAPACITY: The property which gives an electric circuit elasticity and tends to steady the pressure.

FARAD, UNIT OF CAPACITY: A piece of apparatus has a farad capacity when it holds one coulomb of electricity for every volt pressure across its terminals. A microfarad, the practical unit, is one-millionth of a farad.

THE RELATION BETWEEN CHARGE, VOLTAGE AND CAPACITY is expressed by the equation

$$C = \frac{Q}{E}.$$

BOUND CHARGES. The two kinds of electricity, positive and negative, attract each other, like two unlike magnetic poles. Two oppositely charged plates near each other have a greater capacity on account of this attraction than when widely separated. The charges are then said to be bound.

PLATE CONDENSERS. EQUATION FOR CAPACITY OF: Consist of thin sheets of lead or tin foil, separated by thin sheets of insulating material called the dielectric. The large number of the metal plates affords a large area on which to collect the charge, and the oppositely charged plates being separated by the thickness of the dielectric only, bind the charges on one another.

The equation for the capacity of C condenser is

$$C = \frac{885KA}{10^{10}d}.$$

The value of K in the equation depends upon the material of the dielectric and is called its DIELECTRIC POWER. It represents the power of the dielectric, as compared with air, to convey the binding effect of one charge on the other.

CAPACITY OF CABLES. All cables have some capacity. Submarine cables have a large capacity due to water forming one plate, separated but a small distance by the insulation from the cable, which forms the other plate.

Usual equation for capacity of a submarine cable is;

$$C = \frac{2.413Kl}{10^7 \log_{10} \frac{D}{d}};$$

of an aerial line:

$$C = \frac{12.5l}{\log_{10}\frac{s}{r} \times 10^8}.$$

Capacity may be measured: (1) By direct deflection method, using standard condenser. Deflections of ballistic galvanometer on discharge of standard and unknown, charged to same potential, are proportional to the respective capacities. (2) By bridge, using source of A.C. power, standard variable resistances, standard condenser, and telephone receiver. Equation of balanced bridge is

$$\frac{R_1}{R_2} = \frac{C_2}{C_1},$$

which is not the Wheatstone bridge equation.

LOCATING BREAK IN CABLE. Apparatus arranged as in bridge measurement of capacity; using good pair of twin wires of known length exactly like the faulty pair. Equation is

$$\frac{l_1}{l_2} = \frac{R_2}{R_1}.$$

CAPACITY OF CONDENSERS IN PARALLEL. Equals sum of their separate capacities.

CAPACITY OF CONDENSERS IN SERIES. Equals the reciprocal of the sum of the reciprocals of their separate capacities.

PROBLEMS ON CHAPTER XI

28–11. What is the capacity of a condenser made up of 500 plates of lead foil with a dielectric of paraffined paper 10×9 cms. and .003 cm. thick?

29–11. What is the capacity of a condenser having 500 plates of lead foil, if the dielectric consists of mica sheets 10×9 cms. .003 cm. thick?

30–11. If condensers of Problem 28 and Problem 29 were joined in parallel, what would be their joint capacity?

31–11. If condensers of Problems 28 and 29 were joined in series what would their joint capacity be?

32–11. What charge would condensers joined as in Problem 30 hold, if a voltage of 110 volts were placed across the terminals?

33–11. If the voltage across the condensers connected as in Problem 31, were changed from 115 to 15 volts, in .05 sec., what average current would flow from the condensers?

34–11. In a testing outfit arranged as in Fig. 226, the telephone receiver gives no sound, when $R_1 = 40$ ohms and $R_2 = 76$ ohms. If the length of the good pair of cables, C_1 is 4940 ft., how far out is the break in the faulted pair, C_2?

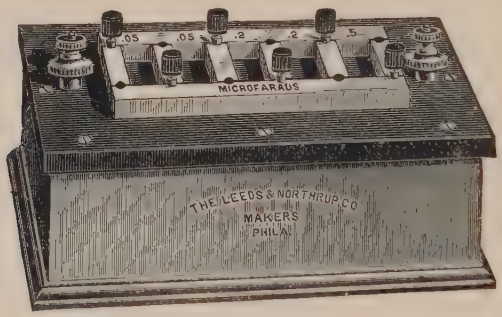

Fig. 231.—Standard condenser.

35–11. It is desired to build up a capacity of 2 microfarads. There are at hand 3 condensers of 4, 1, and 3 mfs. respectively. Show by diagram what arrangement of these condensers will produce a capacity nearest to 2 mfs.? Compute exact capacity of such an arrangement.

36–11. A condenser of 4.2 mfs. capacity charged at 110 volts is suddenly discharged through a short wire. What amount of energy in watt-secs. is used in heating the wire? Note.—Average voltage on discharge $= \frac{E}{2}$.

37–11. If wire in Problem 36 has a resistance of 5 ohms, in what time will condenser be discharged through it at average rate?

38–11. If 70 volts are used to send messages through cable in Problem 16, how long does it take for each charge and discharge of cable at average rate? Assume return of zero resistance.

39–11. If the same size wire and same voltage as in Problem 16 had been used in aerial 50-mile line, how long would each charge and discharge take at average rate? Line to consist of two wires hung 18 in. apart.

40–11. A 10 microfarad condenser is charged to a potential of 110 volts and a 15-microfarad condenser to a potential of 220 volts. They are then joined together, positive to positive and negative to negative. What is voltage across the parallel combination?

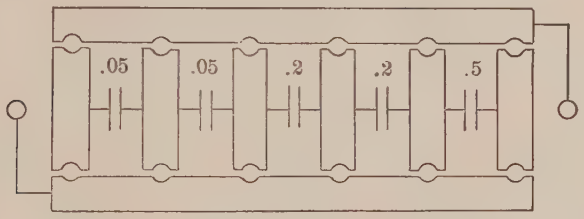

FIG. 232.—Diagram of electrical connections in a standard condenser.

41–11. If the condensers in Problem 40 had been joined in series after being charged, what would voltage across the combination become?

42–11. What charge would there be on each condenser in Problem 40 after being joined in parallel?

43–11. What charge would there be on condensers in Problem 40 after being joined in series?

CHAPTER XII

ELECTROCHEMISTRY

Primary and Secondary Cells—E.M.F. Dependent on Materials—Chemical Action of Simple Cell—Electrolysis—Electrochemical Equivalents—Electroplating and Electrotyping—Storage Cells; Theory, Construction of Planté, Faure, and Edison Types—Use and Maintenance—Advantages and Disadvantages.

184. Electromagnetic Induction. In Chapter VII., it has been shown that whenever an electrical conductor cuts magnetic lines of force, an electromotive force is set up between the ends of the conductor. A high electromotive force is secured by joining a large number of these conductors in series (in an armature) and causing them to cut rapidly a field of great intensity. Such a machine, as described in Chapter VII., is the ordinary generator. It may be said to transform the mechanical energy supplied to the pulley into electrical energy, which it delivers from the armature terminals. This is the usual method of obtaining electric pressure for power purposes.

185. Electric Battery. Primary and Secondary Cells. But it is possible to transform not only mechanical energy but also chemical energy into electrical energy. A device for developing electric pressure by means of chemical action is called an ELECTRIC BATTERY.

It has been found by experiment that when two electrical conductors of different materials are placed in a liquid so that they do not touch each other, and the liquid acts on one of them chemically, then an electric pressure is set up between the conductors. The liquid is called an ELECTROLYTE.

The conductor of the highest potential is called the POSITIVE PLATE; the other, the NEGATIVE PLATE. If the two plates are then joined by a wire, an electric current will flow through it from the positive to the negative plate. The voltage, or pressure, between the plates depends entirely upon what the electrolyte is, and upon what the materials of the two plates are. The size, distance apart, etc., have nothing to do with the voltage of a battery, any more than the size of the conductors cutting lines of force affects the pressure across the terminals of a generator. The number of *lines cut per second* determines the voltage of a generator, and the *materials of the plates and electrolyte* determine the voltage of a battery cell.

When the liquid is dilute sulphuric acid, one plate is zinc and the other copper, the electric pressure set up is always about one volt. The chemical action takes place between the zinc and the acid. When one plate is zinc, the other carbon, and the electrolyte chromic acid, the E.M.F. is always about 2 volts, regardless of size of plates. The chemical action in this case takes place between the zinc and the chromic acid.

The important thing to remember is that, in order to have any voltage set up between the plates of a cell, the plates must be of different materials, and one of them must be acted upon chemically by the electrolyte. Even a dry cell is not really *dry*. The liquid, however, is retained in blotting paper or in some filling, placed between the plates and in close contact with them. As long as the cell is delivering current, one of the plates (the negative) is being consumed, and the character of the electrolyte is changing. If this plate is replaced by another of the same kind when it is used up, and the electrolyte renewed by replacing it with fresh liquid, the cell is then called a PRIMARY CELL.

If an electric current from an outside source be sent through the cell in the opposite direction, it will often cause the electrolyte to deposit the consumed material

back on the plate. When this method of renewing the materials is used, the cell is said to be a SECONDARY or STORAGE CELL.

186. Action of a Primary Cell. Since the action of both Primary and Secondary cells is the same when delivering current, it is well to study the primary cell, it being the simpler, a little more in detail.

Perhaps the very simplest cell is one composed of zinc, copper and sulphuric acid diluted with water. Fig. 233 represents such a cell. Cu is the chemical symbol for copper and Zn for zinc.

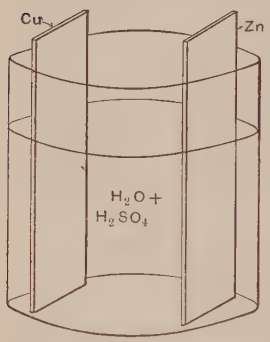

FIG. 233.—Simple primary battery cell.

Water is composed of two parts hydrogen and one part oxygen, and accordingly is represented chemically by the symbol H_2O.

Sulphuric acid is composed of two parts hydrogen, one part sulphur and four parts oxygen, and is represented by the symbol H_2SO_4.

When the acid H_2SO_4 acts chemically upon the zinc, Zn, it is broken up into two parts, H_2 and SO_4. *Wherever there is chemical action there is also electrical activity.* In this case, the two parts of the acid are charged with opposite kinds of electricity on being separated. The SO_4 part is negatively charged and clings to the zinc plate, giving up its negative charge to that plate, and taking to itself some of the zinc plate, Zn. This small amount of zinc now takes the place of the H_2 and makes a new compound, $ZnSO_4$, zinc sulphate, which is dissolved by the water in the cell, just as ordinary salt is dissolved in water. Note that during this process the zinc plate becomes negatively charged.

The other part, H_2, of the acid is charged positively and instead of clinging to the zinc plate, it goes over to the copper

plate, where it gives up its positive charge, and being a gas, rises in bubbles to the surface of the acid. The copper plate thus becomes positively charged.

Fig. 234 represents this action of the acid breaking up into two oppositely charged parts and charging the plates. Note that the positive charged part of the electrolyte goes *with the current* through the cell, while the negative part goes *against the current.* The fact that the charges on the plates are of opposite kinds of electricity causes a difference of potential of approximately 1 volt between the plates. Therefore if the copper plate is joined to the zinc by a wire, a current will flow through the wire from copper to zinc, and the chemical action will continue to go on. Thus the energy of the chemical action is transformed into the electrical energy of an electric current. If the plates are not joined by an electrical conductor, the chemical action goes on until the plates are charged and then stops.

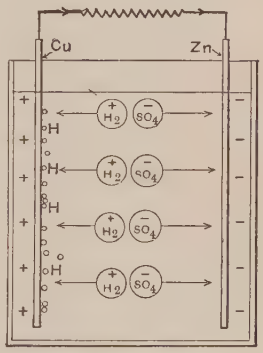

FIG. 234.—Action within a simple cell.

The action of all cells is similar to this. The electrolyte is always broken up into two oppositely charged parts. One part gives up its charge to one plate, the other part gives up its charge to the other plate. The difference of potential thus set up depends entirely upon the composition of the plates and electrolyte.

In a well-designed cell, the chemical action takes place only when the cell is delivering energy. The rate at which electrical energy is delivered by the cell, then, determines the rate at which the zinc is consumed; just as the rate at which steam energy is delivered by a boiler determines the rate of the coal consumption. Zinc may, therefore, be said to be a fuel, the chemical energy of which is turned

into electrical energy by means of a battery cell, just as coal is a fuel, the chemical energy of which is converted into heat by means of oxygen, a furnace, water, and a boiler.

If energy could be obtained as cheaply by the consumption of zinc as by the consumption of coal, primary cells and electric motors would be in as great evidence as boilers and steam engines.

187. Polarization. Depolarizers. When the hydrogen H_2 gives up its positive charge to the positive plate, it is likely to cling to the plate and form a layer of hydrogen gas around it. This gathering of hydrogen gas on the positive plate is called POLARIZATION. As hydrogen gas is a very poor conductor of electricity, this makes the internal resistance of the cell high and a larger part of the pressure set up by the chemical action must be used to force the current through the cell. This leaves a smaller amount of pressure to force the current through any outside circuit. The E.M.F. is also reduced, because there is an E.M.F. set up between hydrogen and copper, which opposes the E.M.F. set up between the zinc and sulphuric acid.

Thus, for these two reasons, the terminal voltage falls as the cell becomes POLARIZED. Accordingly, some method must be devised to clear the positive plate of these hydrogen bubbles. The various means employed to thus DEPOLARIZE a cell give rise to various forms of cells. The most common method is to introduce into the electrolyte some chemical, called a depolarizer, which has a surplus of oxygen in it. This oxygen then very easily unites with the hydrogen and forms water, H_2O; two parts hydrogen to one part oxygen.

When the depolarizer acts quickly and gets rid of the hydrogen bubbles as fast as they are formed on the plate, the cell can be run continuously without polarizing, and accordingly without much decrease in the terminal voltage. Such a cell is called a CLOSED CIRCUIT CELL.

When the depolarizer acts more slowly and the hydrogen

gradually gathers on the plate in spite of the depolarizer, the cell can be used only intermittently, in order to allow the polarizer time in which to act on the hydrogen and clear up the positive plate. Such a cell is called an OPEN CIRCUIT CELL. If an open circuit cell is run continuously, the terminal voltage falls as explained above. This explains the action of a dry cell on closed circuits. See Fig. 236 for the results of such a test on an open-circuit cell. If a closed circuit cell is allowed to stand on open circuit, the depolarizer generally ruins the cell by causing certain chemical changes in the electrolyte. It is essential, then, to use the different types of cells on the circuits for which they are intended. See any hand-book for a discussion of the various forms of each type.

188. Local Action. Besides polarization, which takes place on the positive plate, there is also the danger of LOCAL ACTION, which usually takes place on the negative plate. It is generally caused by some impurity in the materials. Suppose, for instance, that there is a small piece of carbon in the zinc plate. When the zinc alone is put into the acid, the necessary requirements for a battery are all present, two different conductors and a liquid. Now, referring to Fig. 235, when the acid is broken up into its two parts H_2 and SO_4, the positively charged hydrogen, instead of going across the cell to the copper and charging that plate, merely goes to the little piece of carbon, and gives up its charge to the carbon. Thus a small battery is set in action which consumes the zinc plate but produces no terminal voltage, inasmuch as the carbon impurity, being in contact

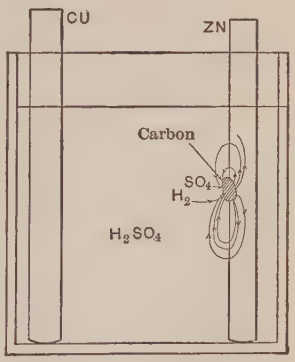

FIG. 235.—Local action.

with the zinc, causes a short circuit current to flow from the carbon spot to the zinc. To avoid this, it is necessary to have strictly pure materials, or at least to amalgamate the zinc plate with mercury in order to cover up the impurities.

189. Tests on Primary Batteries. In Fig. 236 are shown the results of tests made by students at Pratt Institute on an open circuit cell. The curves show clearly

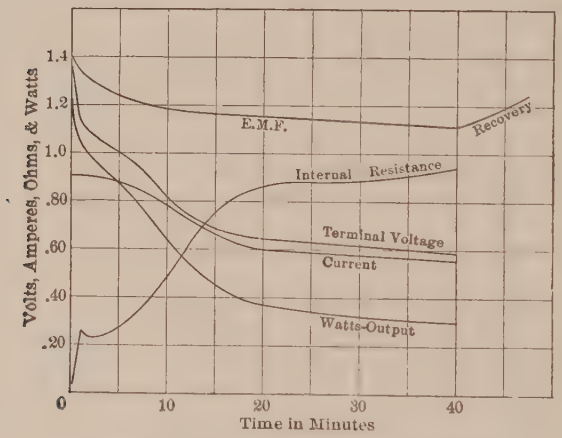

FIG. 236.—Test of primary battery (dry cell).

the effect on an open circuit cell when it is kept running for a considerable length of time.

The external resistance was kept constant and readings of the terminal voltage, current, and E.M.F., were taken every few minutes for one hour.

The *output* in watts was computed from the product of the terminal voltage times the amperes.

The *internal resistance* was computed by dividing the difference between the E.M.F. and terminal voltage by the current. E.M.F. − terminal voltage = volts required to force current through the cell.

Notice the rapid decrease in the value of the E.M.F., terminal voltage, current, and output. Also note the continued increase in the internal resistance. All of these effects are due to polarization, as explained above. The cell was then allowed to stand on open circuit for 15 minutes. Note the fact that the E.M.F. gradually recovers. The depolarizer is gradually clearing up the positive plate.

190. Electrochemical Equivalent. Electrolysis. The rate at which the negative plate, generally made of zinc, is consumed, depends entirely upon how much current is being supplied by the battery. When no current is being supplied, all chemical action ceases if the metal is pure, and thus no material is consumed. The negative plate thus acts as fuel just as coal acts as fuel in the firebox under a boiler. The rate at which coal is consumed depends upon how much heat energy must be converted into mechanical energy per hour. Similarly the rate at which the negative plate is consumed depends upon how much chemical energy must be converted into electrical energy per hour. When one ampere is drawn from the cell it consumes a definite quantity of zinc per hour. If two amperes are drawn from the cell, twice as much zinc is consumed per hour. Now this process is reversible. If an ampere of current were forced back through the cell in the opposite direction, this same amount of zinc per hour would be recovered from the electrolyte and deposited back on the negative plate again. If two amperes were forced through in this opposite direction, twice the amount of zinc would be deposited per hour on the negative plate. This reversed process, in which the liquid is broken up and the metal deposited, is called ELECTROLYSIS.

The definite quantity of a substance which is consumed per hour when one ampere is drawn from a cell, or which is deposited per hour by electrolysis when one ampere is caused to flow back through a cell, is called the ELECTROCHEMICAL EQUIVALENT of the substance. The following

table for the common elements is taken from the "Standard" Handbook.

ELECTROCHEMICAL EQUIVALENTS OF ELEMENTS.

Element.	Symbol	Grams per Ampere Hour.	Ampere Hour per Gram.
Aluminum	Al	0.3369	2.969
Copper	Cu	1.186	0.843
Gold	Au	3.677	0.2720
Hydrogen	H	0.03759	26.60
Iron	Fe	0.695 1.042	1.439 0.959
Lead	Pb	3.858	0.2592
Nickel	Ni	1.094	0.914
Nitrogen	N	0.261	3.828
Oxygen	O	0.2983	3.352
Silver	Ag	4.025	0.2485
Zinc	Zn	1.219	0.820

By means of this table we can determine the amount of zinc, or of any other metal, necessary to produce a given current for a given length of time.

Example 1. How many ounces of zinc would be consumed in 8 hours by a battery delivering 20 amperes? 1 oz. = 28.4 gms.

From table, 1.219 gms. of zinc are consumed in 1 hour by a current of 1 ampere.

$8 \times 20 \times 1.22$ gms. = 195 gms., or 6.87 oz. consumed in 8 hours by 20 amperes.

Also by means of this table, it is possible to find the amount of copper or other metal that would be deposited in a given time by a given current from a salt solution of the metal.

Example 2. How much copper would be deposited from a solution of copper sulphate in 12 hours by a current of 2 amperes?

From table

1.19 = gms. deposited in 1 hour by 1 ampere.

$2 \times 12 \times 1.19 = 28.6$ gms. deposited in 12 hours by 2 amperes.

Problem 1–12. How many grams of silver are deposited in 8 hours from a solution of silver nitrate by a current of 4.6 amperes?

Problem 2–12. The zinc plate of a cell weighs 4 oz. How long will it last if the battery is required to deliver current of 2 amperes?

Problem 3–12. If it is desired to deposit 1.2 lbs. of copper in 24 hours, what current must be used?

191. Electroplating. In the process called electro-plating, advantage is taken of the above fact that if a current is passed through a solution containing the salt of a metal, the metal will be deposited on the negative plate. The positive plate of a vat used for this purpose is called the ANODE; the negative plate, the CATHODE. The action is as follows:

It is desired to copper plate a piece of iron. See Fig. 237. The iron *B* is placed in a solution of copper sulphate, $CuSO_4$, together with a piece of copper *A*. The current is now forced through the cell from *A* to *B*, and in passing, it breaks up the copper sulphate into a copper part, Cu, and an acid part, SO_4. The copper part, Cu, being positively charged, goes *with* the current, to the negative plate *B* and is deposited on it, thus giving it a coating of copper.

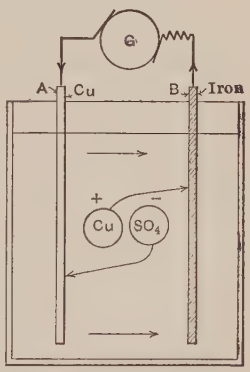

FIG. 237.—Action within an electroplating vat. Copper plating iron.

The acid part, SO_4, being negatively charged, as explained in the action of a single cell, goes *against* the current, to the positive plate *A* and combines chemically with it, forming more copper sulphate, $CuSO_4$. Thus the copper plate is gradually dissolved and conveyed to the iron.

If it is desired to nickel plate the iron, a piece of nickel is used for the ANODE, or positive plate *A*, instead of copper, and nickel ammonium sulphate for the electrolyte, instead of copper sulphate.

192. Electrotyping. The object of electrotyping is to reproduce the printers' set-up type, engravings, etc. A wax impression is first taken of the set-up type. Since wax is a non-conductor of electricity, a thin coating of graphite is now given the wax mold. The whole mold with its coating of graphite is immersed in copper sulphate together with a copper bar. An electric current is now sent through from the copper to the graphite. This causes copper to be deposited on the graphite just as it was deposited on the iron in the cell of the previous paragraph. The current is allowed to run long enough to deposit a plate of sufficient thickness to be handled safely. Then the wax mold is removed and the remaining copper plate is an exact reproduction of the type.

193. Refining of Metals. Since the metal, which is deposited on the negative plate, is remarkably pure, this method is often used to separate the impurities from a mass of metal. The impure mass is made the positive plate. When an electric current is passed through the cell, the pure metal is gradually dissolved by the electrolyte and carried over to the negative plate. At the end of the process, the negative plate is found to consist of very pure metal, since practically all the impurities remain at the positive plate.

For exact data and methods of procedure in the above processes, see "Electrochemical and Metallurgical Industry" and "Transactions of American Electrochemical Society."

Problem 4–12. How long will it take to refine 100 lbs. copper if a current of 150 amperes can be used?

Problem 5–12. An iron casting is to be copper plated and then nickel plated. The current in each case is to be 4 amperes. How long must it remain in each vat in order to have 12 oz. of each metal deposited on it?

Problem 6–12. A copper voltameter, used to calibrate an ammeter, has 1.5 grams of copper deposited on the cathode in 1 hour and 5 minutes. What value should the ammeter indicate during the process?

Problem 7–12. An ammeter being calibrated by a silver voltameter, registers 4.38 amperes for a current which deposits 39.2 grams of silver in 2 hours 15 minutes. What is percentage error of the ammeter at this reading?

Problem 8–12. Two electroplating vats are arranged in series, one for gold plating and the other for silver plating. If a current flowing through the vats deposits 1 oz. of silver in a given time, how much gold is deposited at the same time?

194. Definition of Ampere. Voltameter. Since one ampere flowing for one hour through an electrolytic vat deposits a definite quantity of metal on the negative plate, we can determine very accurately the value of an unknown current by weighing the plate before and after an accurately timed run. On account of this precise method of determining the current strength, the following definition of ampere has been established:

An Ampere is the Rate of Flow of a Steady Current which Deposits 4.025 Grams of Silver per Hour from a Solution of Silver Nitrate Diluted to a Standard Density with Distilled Water.

This measurement is made with a device called a Voltameter, which consists of a glass jar containing the silver plates and the silver nitrate solution. Copper voltameters are also used, consisting of two plates of copper immersed in a copper sulphate solution. By means of a voltameter it is possible to accurately calibrate galvanometers and ammeters, although the method has the disadvantage of requiring too long a time, if many parts of the scale are to be calibrated.

195. Electrolytic Destruction of Metal Water Mains, etc. It is customary in electric railways to use the track as the return circuit. The rails, not being insulated from the ground, allow the current to leak into the ground and follow any low-resistance path it can find, such as a water or a gas main, back to generator, which of course, is also grounded.

Fig. 238 is a diagram of this action. Where the current enters the pipe at *A* no harm is done. But at the point *B*, where the current leaves the pipe, generally near the generator station, there is all the action that takes place at the positive plate in an electroplating vat. The pipe, being in moist ground, is in contact with water containing more or less salt, which causes it to act as an electrolyte. Thus, as the electric current leaves the pipe, there is a chemical action set up between the pipe and salt water, by which the iron of the pipe is eaten away in places, and carried by the electric current to some outside substance, which for the instant is acting as the negative plate.

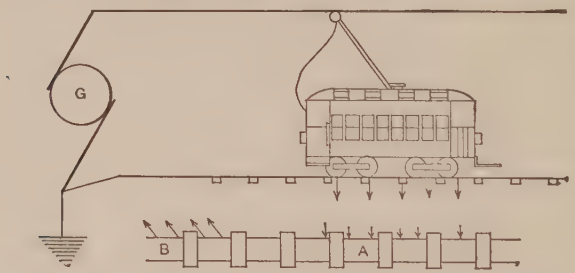

Fig. 238.—Electrolytic destruction of water mains.

In a short time heavy iron pipes have been eaten through by this action. Among the various precautions taken to avoid this destruction by electrolysis are the following: (1) The heavy bonding of the rails to procure a return of low resistance. (2) The use of a second trolley wire for the return. (3) Tapping a return line on to the water mains, to conduct back to the generator whatever current they may be carrying, without allowing it to flow through the water and produce electrolytic action.

Method No. 2 is the most efficient, but also the most expensive and is very seldom used.

196. Storage Batteries. Efficiency. In the ordinary primary cell, when the negative plate is nearly consumed,

it is customary to replace it by another. If, instead of replacing the worn-out negative plate, we send a current from an outside source through the cell in the reverse direction, and deposit the metal back on the negative plate, the cell is then called a STORAGE BATTERY. When such a cell is delivering current it is said to be *discharging;* when receiving current, it is said to be *charging.* A storage cell differs fundamentally in no respect from a primary cell. Any primary cell could be used as a storage cell and have its negative plate restored by electrolysis as an ordinary storage battery does. It is not commercially economical, however, to do this in the case of any but a few cells, especially constructed for this purpose.

Remember that a storage cell does NOT store electricity. It stores nothing but chemical energy. In *charging,* electrical energy is transformed into chemical energy and stored in the cell; in *discharging,* this chemical energy is changed back again into electrical energy.

Of course there are losses in both transformations which prevent an efficiency of 100 per cent. For one thing, the terminal voltage *applied to* the cell on charge must be higher than that *supplied by* the cell on discharge at same current rate, partly because, in each case, the internal resistance of the cell must be overcome.

Charging voltage = E.M.F. of cell + IR drop of cell.

Discharging voltage = E.M.F. of cell − IR drop of cell.

In charging a cell, the current must be sent in *against* the E.M.F. and the resistance of the cell, while on discharge, part of the E.M.F. of the cell must be used to overcome this internal resistance in forcing a current to flow through it in the other direction; so that only *part* of the E.M.F. is available as terminal voltage. In this respect it is merely like any other electrical machine. This difference between the terminal voltage on charge and discharge is further increased by the fact that the E.M.F. itself is less on discharge than on charge, due to the electrolyte changing in density.

Accordingly, good storage cells average only about 75 per cent efficiency at normal load.

197. Planté and Faure Types. Construction. The most common type of storage cell in use at the present time is the lead cell. In any cell, there must be two plates of *different* substances in an electrolyte. In the lead cell, the negative plate is pure spongy lead, Pb; the positive plate is lead peroxide, PbO_2; one part lead to two parts oxygen.

The electrolyte is sulphuric acid, H_2SO_4, diluted with water, H_2O.

The lead peroxide and spongy lead are poor conductors of electricity, and are not sufficiently hard to be made into plates; so it is necessary to attach them to frames of some harder material which is a good conductor of electricity. The material of this framework must also have the property of *not acting as a third plate*, otherwise the acid will produce LOCAL ACTION between the framework and the spongy lead or between the framework and lead peroxide, wherever it touches them. The material usually chosen for this framework is an alloy of lead and antimony, which is mechanically strong and produces no local action in the presence of the other substances in the cell. The lead peroxide and the spongy lead are called the ACTIVE MATERIALS, to distinguish them from the framework, and they form the real plates of the cell. The combination is called a GRID, either positive or negative.

There are two methods of attaching the active materials to the framework, which give rise to two general types of lead cells: 1st, the Planté; 2d, the Faure.

In the PLANTÉ TYPE, Fig. 239, the surface of the framework itself is roughened and then changed into the proper active material by a chemical process; that is, the surface of the negative grid is made into spongy lead; the surface of the positive grid, into lead peroxide.

In the FAURE TYPE, holes are punched in the framework. Lead peroxide is then forced into the holes of the

positive grid, and spongy lead into the holes of the negative. The Gould cell is an example of the Planté type, and the "Chloride Accumulator" of the Faure type. The Planté cell is heavier for the same capacity, but it is maintained that this method of attaching the active material to the framework holds it there more securely than the other.

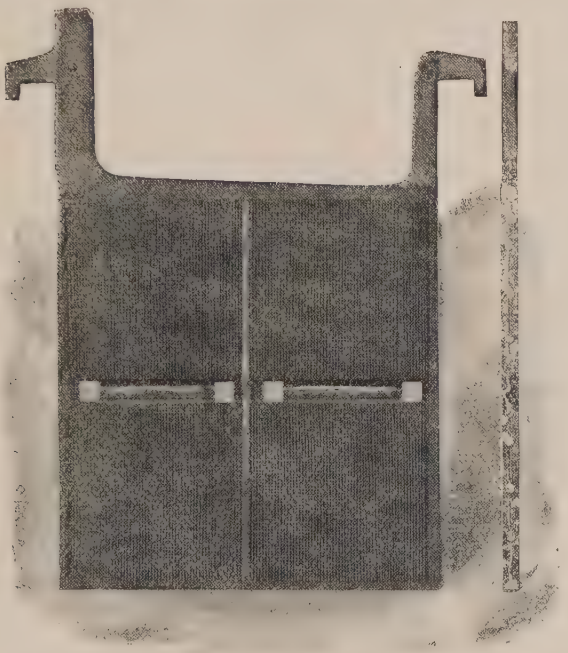

FIG. 239.—Plate of storage battery. Planté type.

198. Chemical Action on Discharge. Consider Fig. 240. The sulphuric acid, H_2SO_4, acts chemically upon the lead plate, Pb, and is broken up into positively charged H_2 and negatively charged SO_4. The SO_4 unites with the lead plate, forming lead sulphate, $PbSO_4$, and gives up its negative charge to it. The hydrogen carries its positive charge to the lead peroxide plate, where it gives it up, and

unites with the oxygen of the lead peroxide, forming water H_2O. The sulphuric acid in contact with the peroxide plate is also broken up into H_2 and SO_4. The H_2 of this portion of the acid also unites with the oxygen of the lead peroxide and forms more water. The SO_4 part of the acid instead of going over to the negative plate unites with the lead, Pb, of the lead peroxide plate and forms lead sulphate, $PbSO_4$, on the positive plate. Thus both plates

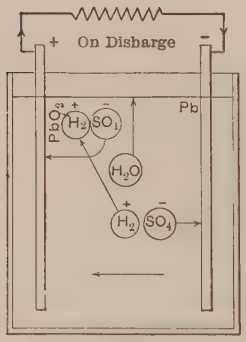

FIG. 240.—Chemical action of lead storage cell on discharge.

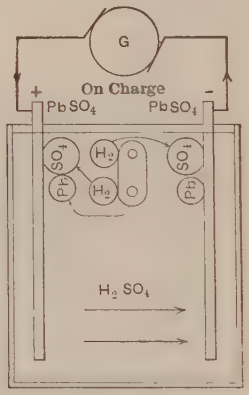

FIG. 241.—Chemical action of lead storage cell on charge.

are being reduced to lead sulphate $PbSO_4$. The cell continues to deliver current until the plates are entirely reduced to lead sulphate, when of course, all action will cease, since there would be but one kind of material present, and a battery requires two kinds. The practical limit of discharge, however, is reached long before both plates are completely reduced to the same material.

Note *two things* which are taking place when a cell is discharging:

1st. The acid is continually growing weaker. This results in a lower E.M.F.

2d. The active materials, lead and lead peroxide, are being replaced by lead sulphate, which has a much higher

resistance and is more bulky than the active materials. It therefore takes up more space in the holes of the grids and tends to buckle them; especially if the cell is discharged very rapidly and the lead sulphate forms quickly.

199. Chemical Action in Charging. Refer to Fig. 241. Assume both plates to consist of lead sulphate, $PbSO_4$. When a current from an outside source G is sent through the cell, it breaks up the water which has been formed during DISCHARGE into positively charged hydrogen H_2 and negatively charged oxygen O. Part of the positively charged hydrogen is now attracted to the negative plate and unites with the SO_4 of the lead sulphate, forming sulphuric acid, H_2SO_4, and leaving pure spongy lead at the negative plate. The negatively charged oxygen O flowing against the current is attracted to the positive plate. Here it unites with the lead Pb of the lead sulphate $PbSO_4$ plate and forms lead peroxide, PbO_2. The SO_4 part of the positive plate is finally united to the rest of the hydrogen liberated when the electric current broke up the water H_2O into H_2 and O. This action forms still more sulphuric acid, and a positive plate of lead peroxide. When all the lead sulphate has been changed over to lead peroxide and pure lead, the battery is restored to the state it was in before it was discharged, and is now ready to furnish current again.

Note that the acid has been growing denser during charge, therefore the E.M.F. must have been increasing as the charging continued.

The chemical equations for the different actions on discharge and charge may be written as follows:

DISCHARGE

$$PbO_2 + Pb + 2H_2SO_4 = 2PbSO_4 + 2H_2O$$

CHARGE

or

DISCHARGING.

Positive Plate		Negative Plate
PbO_2	$\longleftarrow$	Pb
+	$\longleftarrow$	+
H_2	$\longleftarrow$	SO_4
+		
H_2SO_4		

Produces

$PbSO_4$ $PbSO_4$

\+

$2H_2O$

CHARGING

Positive Plate		Negative Plate
$PbSO_4$	$\longrightarrow$	$PbSO_4$
+	$\longrightarrow$	+
O	$\longrightarrow$	H_2
+		
H_2O		

Produces

PbO_2 Pb

\+

$2H_2SO_4$

200. Care of Lead Storage Cells. From a study of the chemical action of a storage cell and the physical results of this action, we can determine what treatment such a cell should receive in order to give the most efficient service.

In the first place, it should be said that storage batteries should have the care of a conscientious skilled attendant. They are costly in the first place and easily ruined. Without proper care they deteriorate at an alarming rate. The main points to be avoided are the following:

(1) Too rapid charging or discharging.

(2) Use of impure electrolyte.

(3) Use of too dense or too light electrolyte.

(4) Over-charging and over-discharging.

The harm that may result from disregarding these points is evident on inspection of the chemical actions.

(1) *Too Rapid Charging and Discharging.* In constructing a storage cell, manufacturers usually allow a capacity of from 50 to 60 ampere hours per sq.ft. of positive plate area, counting both sides of the plates. This capacity is computed on an 8-hour charge and discharge rate, which means that the NORMAL rate of charge or discharge should be $\frac{50 \text{ to } 60}{8} = 6$ or 7 amperes per sq.ft. of positive

plate area. The NORMAL rate of charge and discharge may then be defined as that rate which will satisfactorily charge or discharge the cell in eight hours, and is about 6 or 7 amperes per sq.ft. of positive plate area. In all problems in this book 6.5 amperes per sq.ft. is to be considered normal current.

In order to have as large plate area as possible, both the positive and negative plates usually consist of several grids joined in parallel. To distinguish the positive from the negative, it is merely necessary to remember that the positive plate has one less grid than the negative. The color of the positive plate is reddish brown, of the negative plate, a dark gray.

If charge or discharge is carried on more rapidly than this, the ampere hour capacity of the cell is lowered. When a too rapid charge takes place, the chemical action is violent, and the active elements are not properly deposited on the plates and are likely to form a deposit in the bottom of the jar, which may eventually rise high enough to reach the plates and short-circuit them, if it is not removed.

On too rapid discharge, lead sulphate often forms in such great abundance in the crevices of the plates, as to cause them to buckle, as previously explained.

On the other hand, the lead sulphate may form a layer over the active materials and raise the internal resistance, besides greatly decreasing the actual amount of active materials available.

Problem 9–12. A lead storage cell has a positive plate area of 4.2 sq.ft. (*a*) What is its normal discharge rate? (*b*) What is the ampere-hour capacity?

Problem 10–12. A storage battery is required to deliver 10 amperes. (*a*) What should be the area of the positive plate? (*b*) How many ampere-hours would such a cell have?

Problem 11–12. What should be the area of the positive plates in a lead cell of 120 ampere-hours capacity?

(2) *Use of Impure Electrolyte.* Nothing but the purest sulphuric acid and distilled water should be used for the electrolyte.

Any metallic impurity in the acid or water is deposited on the plates as a pure metal and there causes local action, which we have seen consumes the material of the cell without producing any available energy.

Any acid impurity which will dissolve the lead of the framework is objectionable for very apparent reasons.

Too much care, therefore, cannot be exercised in selecting the materials which go to make up the electrolyte.

(3) *Too Light or Too Heavy Electrolyte.* The specific gravity at which a cell should work depends somewhat upon the use to which it is to be put. If the cell is to be used continuously, it can work at a higher density than if it is to stand unused for any length of time.

It is of advantage to have as high a density as is practicable, because of the *decreased internal resistance* and the *increased* E.M.F. which the cell has when the acid is of high density, as previously explained. But if the cell is to stand inactive, the strong acid tends to change the active material into lead sulphate more rapidly than if weaker acid were used.

When studying the chemical action of the cell we saw that the acid grew lighter as the cell was discharged, and heavier as the cell was recharged. One method, then, of telling what condition a cell is in, with regard to charge or discharge, is to test its specific gravity with a hydrometer. When fully charged it should have a specific gravity of from 1.20 to 1.24, according to the class of work it is to do, as explained above. On discharge the specific gravity should never fall below from 1.185 to 1.195.

For example, suppose a cell had a density of 1.22 when fully charged. If at a certain time it showed a density of 1.189, it would not be safe to discharge it any further.

(4) *Over-charging and Over-discharging.* When all the active material has been changed into lead peroxide and

spongy lead, there is nothing gained in continuing the charging, and a waste of electric power is going on.

There are three ways in which one may tell when a cell is fully charged:

(1) The plates begin to liberate gas in large quantities. The cell is said to "boil."

(2) The specific gravity rises to the value which it should have when fully charged, between 1.20 and 1.24 (see previous paragraph).

(3) The impressed voltage AT NORMAL CHARGING rate of current rises to about 2.5 volts. This is because a fully charged cell has a high E.M.F. which has to be overcome in order to send a charging current through the cell.

NOTE. The voltage of a cell on open circuit shows ABSOLUTELY NOTHING about its condition as to charge or discharge.

The E.M.F. (voltage on open circuit) between lead and lead peroxide in sulphuric acid of 1.2 density is 2.2 volts (approximately). Since the E.M.F. of a cell does not depend upon the area of the plates, as long as there is *any* lead left on one plate and lead peroxide on the other, the E.M.F. will be about 2.2 volts. Therefore always take the voltage of a cell when it is either charging or discharging at its NORMAL rate.

OVER-DISCHARGE is the one great source of trouble in a storage cell.

IT produces an excess of lead sulphate, which causes the plates to buckle. Buckling may either short-circuit the cell, which ruins it; or cause the active materials to fall off, which decreases the capacity.

IT weakens the acid and forms lead sulphate in excess, thus raising the internal resistance so that the terminal voltage becomes very low.

IT weakens the acid so that the E.M.F. is lowered, under working conditions, to a marked extent.

There are two ways in which one may tell when a cell has been discharged as much as is practicable:

(1) The specific gravity falls to 1.185–1.195. (See previous paragraph).
(2) The voltage, ON DISCHARGE AT NORMAL RATE, falls to 1.8 volts.

CAUTION. NEVER CONTINUE DISCHARGING A LEAD CELL AFTER THE VOLTAGE FALLS TO 1.8 VOLTS AT NORMAL DISCHARGE CURRENT RATE.

201. Curves of Charge and Discharge. The changes in the physical properties of a lead storage cell are best

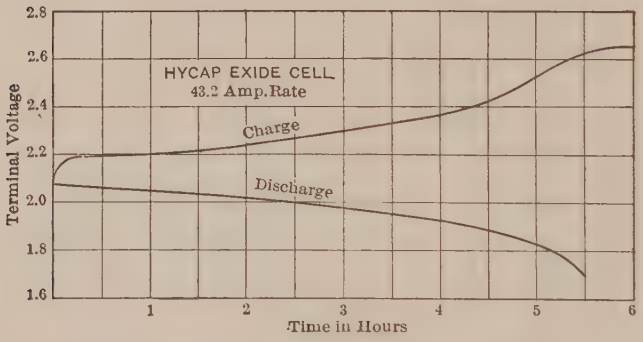

FIG. 242.—Charge and discharge curves of Exide storage cell.

brought out by curves plotted from a series of readings. Fig. 242 shows a set of curves plotted from data taken on an "Exide" cell.

The charge and discharge took place at normal rate. Note the rapid rise in voltage during the first and the last hour of charge. During the intervening five or six hours the voltage was fairly constant.

On the discharge, the voltage falls rapidly during the last hour, but remains nearly constant during the remaining time. This makes the storage cell a first rate source of constant potential.

Fig. 243 shows the variation, on discharge, in specific gravity, E.M.F. and internal resistance.

NOTE:

(1) That the specific gravity of the cell increases on charge, and decreases on discharge.

(2) That the internal resistance increases on discharge slowly at first but rapidly as cell is nearly discharged.

(3) That the E.M.F. of the cell rises slightly on charge and falls slightly on discharge.

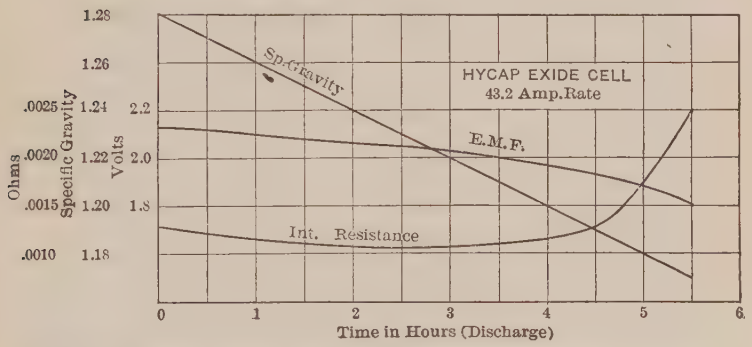

FIG. 243.—Change in specific gravity, E.M.F., and internal resistance during discharge.

A method by which the true ohmic resistance R can be found is as follows: Take terminal voltage V_1 and current I on discharge. Suddenly open the switch while the cell is discharging and record the value which the voltmeter registers on the swing of the needle. (The voltmeter will read instantly E, the E.M.F. of the cell, and then gradually rise as depolarization takes place.) Then,

$$R = \frac{E - V_1}{I};$$

where R = true ohmic resistance of cell;
E = E.M.F. of cell;
V_1 = terminal voltage on discharge;
I = rate of discharge.

Problem 12–12. What is the internal resistance of a cell which has a terminal voltage of 2.00 volts when discharging at a 12-ampere rate, and E.M.F. of 2.32 volts?

Problem 13–12. What voltage would have to be impressed on cell of Problem 12 in order to charge it at same rate?

Problem 14–12. A cell has an internal resistance of .004 ohm. What would be the difference in the terminal voltage on charge and discharge at 20 amperes?

Problem 15–12. A cell of 3.6 sq.ft. positive plate area has an E.M.F. of 2.2 volts, and an internal resistance of .005 ohm. What are terminal voltages at normal rate of charge and discharge?

202. Advantages in Use of Storage Cells. In general, the uses to which a storage battery may be put are three:

(1) To help the generators carry the "peak" of the load. That is, the battery is charged when the load is light. Then, when the load is too heavy for the generators to carry alone, the battery is placed in parallel with the generators to take part of the load. This use allows the installation of much smaller generators, engines and boilers.

(2) To take care of a long-continued light load and allow the engine and generator to be shut down.

The battery is here charged while the generator is taking care of its minimum load, and only thrown in when the generator is shut down. The engine is run steadily at its most efficient load, but no fuel is wasted in keeping it running to supply a light load. Often the battery takes care of its load with no attendant present.

This use saves both in the cost of fuel and labor.

(3) As an auxiliary source of supply in case of accident. Many companies now supply power under heavy forfeits to keep the line "alive" at all times. This necessitates some reserve source of supply which can be drawn upon at a moment's notice. Storage batteries meet this demand wherever a source of Direct Current is required.

203. Use on Constant Potential Line. The voltage of a storage cell is seen to fall off as the cell discharges. If a battery of cells is to be used to supply power to lamps,

some method must be devised to keep the voltage across the line constant.

204. Rheostat Control. One method shown in Fig. 244 is to use a resistance R in a series with the cells. This resistance can be cut out as the voltage falls.

Suppose 115 volts were required across the lamps L. When the cells were nearly discharged, each would have a voltage of 1.8 volts on normal rate of discharge. There would be required then to furnish this pressure $\frac{115}{1.8}=64$ cells.

When the cells were fully charged each would have a terminal voltage of about 2 volts. If 64 cells were used

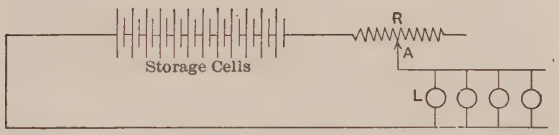

FIG. 244.—Rheostat control of cells.

the line voltage would then be $64\times2=128$ volts, which is $128-115=13$ volts in excess of pressure required. A resistance R is therefore generally inserted between the line and the fully charged cells which will take up this 13 volts in the IR drop across it and leave only 115 volts across the line.

As the voltage gradually falls on discharge, an operator from time to time cuts out some of this resistance R and keeps the voltage across the line constant.

205. End Cell Control. Another method is to cut in and out a certain number of the cells at the end of the series, as more or less voltage is required by the load.

In Fig. 245 cells A, B, C, D, E, F, G are called the END CELLS. As the cells become partly discharged and the voltage of each falls, the arm M, of the end cell switch is moved out toward G, throwing more cells across the line

and thus raising the voltage. (See any Electrical Handbook for description of this switch.)

Problem 16–12. Assume each lamp in Fig. 244, takes 10 amperes at 115 volts. Assume terminal voltage of each of the 64 cells used in series to be 1.8 volts when low and 2.0 volts when fully charged. What value would resistance R have to be in order to keep the voltage across the lamps constant at 115 volts?

Problem 17–12. How many end cells might be used in Problem 16 instead of resistance R?

Problem 18–12. Assume each cell to have an internal resistance of .002 ohm, and E.M.F. of each cell to range from 2.2 to 2.0 volts. How many cells would have to be used to maintain 110

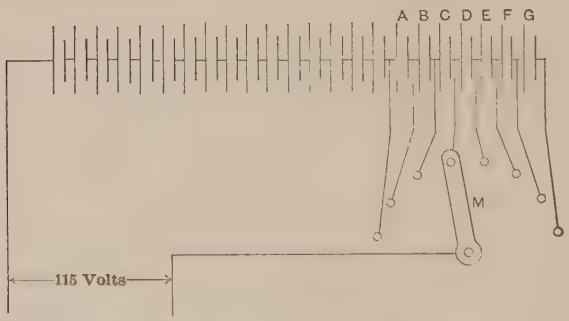

Fig. 245.—End cell control.

volts across a set of lamps as in Fig. 244, if each lamp takes 3 amperes?

Problem 19–12. If a resistance R were used to control voltage in Problem 18, what value would it have to be?

Problem 20–12. How many end cells could be used in Problem 18 to maintain constant voltage at lamps?

206. Methods of Charging Storage Cells. In charging a battery where the End Cell method of control is used, all the cells are thrown in at first. Since the end cells are used a very much shorter time than the rest, they are not discharged to any great extent, and require very little charging. As soon as one of the end cells becomes fully

charged (as determined by methods previously described) it is cut out by moving the switch arm back toward A, Fig. 245.

It has been shown that the voltage required to charge a cell is always greater than the terminal voltage of the cell on discharge, mainly on account of the internal resistance of the cell. The equations previously given, showing the voltage which a cell will give on discharge and the voltage which is required to charge it, bring out this fact plainly.

(1) $$V_c = E + IR_1;$$

(2) $$V_d = E - IR_1,$$

where V_c = terminal voltage required to charge;
V_d = terminal voltage on discharge;
E = E.M.F. of cell;
I = current;
R_1 = internal resistance (virtual).

If we subtract (2) from (1),

$$V_c - V_d = 2IR_1.$$

That is, the voltage required to charge is greater than the terminal voltage on discharge by twice the IR_1 drop of the cell.

Now since the charging voltage is always higher than the discharge voltage, it has been shown that some pressure higher than the line pressure must be used to charge. If the line can be disconnected from the generator during charge, then the voltage of the generator can be raised sufficiently to get the required charging pressure. But if the generator must supply the line and charge the battery at the same time, a small low-voltage generator, designed to carry heavy current, is often connected in series with the main generator and batteries. This small generator is called a BOOSTER. Its purpose is to raise the voltage across the cells. Fig. 246 shows the method of using a booster.

Assume the generator G must maintain 115 volts across the line and at the same time charge battery S, of 64 cells.

The battery would require from 128 to 160 volts to charge (depending largely upon the degree of discharge). The small generator B is put in series with the generator across the battery and raises the voltage enough to charge the cells.

There are many types of automatic and hand control boosters, for which see any book on Electrical Engineering.

For small installations, the battery is usually divided into two or three parallel parts during the charging. The excess voltage of generator can then be taken up by a rheostat. On discharge, of course, the parts are again connected in series.

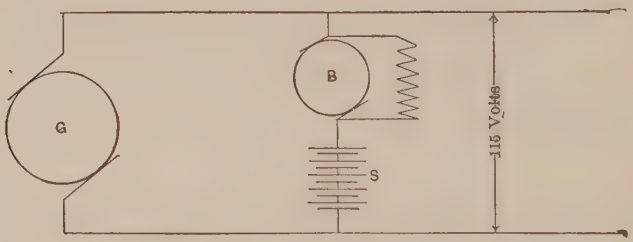

FIG. 246.—Booster control.

207. Floating Batteries. There are certain conditions under which storage batteries may be used continuously on the line and be charged and discharged automatically without any special apparatus such as a booster, etc.

Suppose the batteries are placed directly across the line *at some distance* from the generator, or in such a way that there is considerable resistance in the line wires between the generator and batteries. This resistance would cause a large line drop when a heavy current was sent through the line. If the line drop were enough to cause the impressed voltage across the cells to fall below the E.M.F. of the cells, they would begin to discharge current into the line and so aid the generator. If only a small current is sent through the line, the line drop would be small and the impressed

voltage across the cells would be higher than their E.M.F. The cells would then charge and take current from the line.

In Fig. 247 there is .80 ohm in the line (trolley and track) from generator to car No. 1. When the car is taking 100 amperes, there is a line drop of $.80 \times 100 = 80$ volts. The voltage across the line at this point is then $550 - 80 = 470$ volts.

Assuming the E.M.F. of one cell to be about 2.2 volts, if we place a battery of 220 cells across the line at this point, we should have an E.M.F. of $220 \times 2.2 = 484$ volts; that is, 14 volts $(484 - 470)$ more than the impressed voltage. The battery would then discharge into the line and help the generator to furnish current to the three cars.

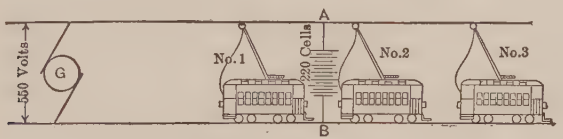

FIG. 247.—Floating battery on railway line.

If, however, car No. 1 were taking only 10 amperes, and cars Nos. 2 and 3 were taking no current, there would be a line drop of only $10 \times .80 = 8$ volts. The voltage across the points AB would then be $550 - 8 = 542$ volts, which is 58 volts $(542 - 484)$ higher than the E.M.F. of the storage battery across the line at that place. The cells would then charge and draw current from the line. (Of course in taking more current from the line the cells would cause a still greater drop in the line voltage from generator to cells, so that there would exist considerably less than the 58 volts difference between the battery E.M.F. and the line voltage.)

Batteries can only be "floated" at a point where there is a large fluctation in the voltage impressed across them, such as railway lines, etc.

Problem 21–12. In a system arranged as in Fig. 247, the generator brush potential is 565 volts. Trolley is No. 00 copper wire. Track has resistance of .03 ohm per mile. How far from generator should a set of 224 lead storage cells be placed in order to "float" satisfactorily? Maximum current in section out to cells is 150 amperes. Average E.M.F. of each cell 2.1 volts.

Problem 22-12. In a system arranged as in Fig. 248: generator has a resistance of .2 ohm, E.M.F. of 120 volts. $R = .4$ ohm. B consists of 56 cells of 2 volts E.M.F. How many amperes must load use in order that the battery B may deliver current to the line? Neglect line resistance.

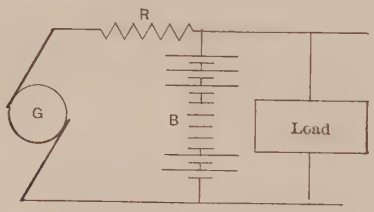

FIG. 248.

208. Edison Storage Battery. A storage battery invented by Thomas Edison and manufactured by the Edison Storage Battery Company has recently appeared on the market. It is designed especially for traction work, being light and compact.

The positive plate consists at first of nickel hydrate, $Ni(OH)_2$, held in steel tubes firmly attached to a steel grid.

The negative plate consists of iron oxide, FeO, held in pockets on a metal plate.

The electrolyte is a 21 per cent solution of caustic potash, KOH, in pure distilled water. Specific gravity 1.20.

The retaining jar, instead of being either glass or vulcanite as in the case of other storage cells, is made of nickel-plated sheet steel. The chemical actions which take place within the cell are not yet clearly understood. The following are supposed to approximate the reactions:

When the cells are *first charged*, the nickel hydrate, $NiOH_2$, is changed to nickel oxide NiO_2, and the iron oxide, FeO, is reduced to metallic iron, Fe. The cell may thus be considered to consist of a positive plate of nickel oxide, NiO_2, and a negative plate of pure iron, Fe.

209. On Discharge. The nickel oxide, NiO_2, probably goes to a lower oxide of nickel, Ni_2O_3; that is, there are only three parts oxygen to two of nickel, instead of two oxygen to one of nickel. The oxygen thus liberated is negatively charged and goes over to the iron plate, gives up its negative charge, and unites with the plate to form iron

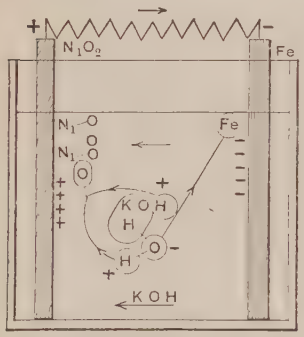

Fig. 250.—Action within Edison cell on discharge.

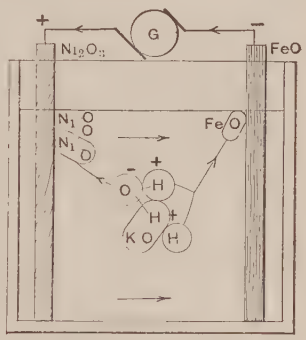

Fig. 251.—Action within Edison cell on charge.

oxide, either FeO or Fe_2O_3. The electrolyte neither loses nor gains in the process and therefore remains at the same density throughout discharge and charge. The potassium hydroxide seems to act merely as a catalyzer or carrier. Fig. 250 shows the action on discharge.

210. On Charge. The iron oxide, FeO, is broken up and the negatively charged oxygen leaves the iron plate, travels back against the current through the cell and unites with the positive nickel oxide, Ni_2O_3, and forms a higher oxide, NiO_2. Fig. 251 shows this action during charge.

The following equations show at a glance the chemical reaction:

DISCHARGE

$$Fe + 2NiO_2 + H_2O + KOH = FeO + Ni_2O_3 + H_2O + KOH.$$

CHARGE.

DISCHARGE.

Positive Plate		Negative Plate
NiO_2 NiO_2		Fe
+		+
H_2	←——	O
+		
KOH		

Produces

Ni_2O_3		FeO
+		
H_2O		
+		
KOH		

CHARGE.

Positive Plate		Negative Plate
Ni_2O_3		FeO
+		+
O	——→	H_2
+		
KOH		

Produces

NiO_2 NiO_2		Fe
+		
H_2O		
+		
KOH		

211. Physical Changes. As stated above, the specific gravity of the electrolyte does not depend upon the state of charge or discharge. Some other method, then, must be used to determine the condition of this form of storage cell as to charge and discharge.

Fig. 252 shows some curves plotted from data taken on an Edison storage cell, type "A6."

Note that on discharge the voltage falls very rapidly at first as in a lead cell. Unlike a lead cell, however, there is no voltage at which it remains nearly constant, but has a general downward trend. Near the point of complete discharge the voltage falls off rapidly again, as in a lead cell. The condition may thus be determined closely by the voltage on discharge.

The average terminal voltage on discharge at normal current is 1.2 volts. The lowest voltage is 1.00 volt.

The voltage curve on charge is slightly irregular, but has the peculiar characteristic of reaching a maximum of 1.84 volts on normal current when fully charged. Average voltage on charge, 1.7 volts.

212. Comparison of Lead and Edison Cells. Average voltage per cell on discharge: Lead cell, 2.00 volts; Edison cell, 1.2 volts. Watt hours per lb. of cell complete: Lead, 8.5 watt-hours; Edison, 16.8 watt-hours.

Internal resistance: The Edison cell has a slightly higher internal resistance; the efficiency is therefore lower.

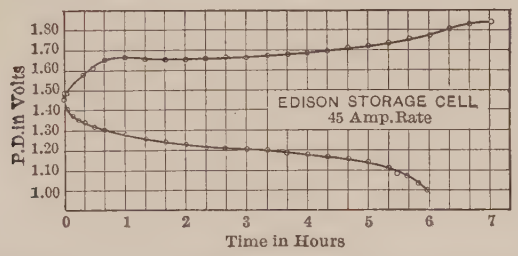

FIG. 252.—Change in terminal voltage on charge and discharge.

The "life" of the Edison cell under commercial conditions is as yet undetermined.

It is claimed for the Edison cell that it can stand for an indefinitely long time, charged or discharged, and not be injured. A lead cell cannot remain any time at all *uncharged*, and only a few months *charged*, without being ruined.

The temperature range at which the Edison cell will work best is limited to the region around 70° F. Any condition of use which tends to greatly lower the temperature decreases the efficiency to a marked extent. The lead cells are not so limited in this respect.[1]

[1] For further information concerning lead cells, the student is referred to Lamar Lyndon's "Storage Battery Engineering."

SUMMARY OF CHAPTER XII

ELECTRIC BATTERIES. Devices for transforming chemical energy into electrical energy. Require two unlike conductors, called positive and negative plates, and an electrolyte which acts chemically upon one of the plates.

PRIMARY CELL. An electric battery in which worn-out element is replaced by another. Negative element generally of zinc.

STORAGE CELLS. An electric battery in which worn-out element is replaced by the electrolytic action of an electric current forced through cell in reverse direction. Elements generally of lead and lead compounds.

ACTION IN CELL. By the chemical action taking place at the negative plate, the electrolyte is broken up into two oppositely charged parts. The positively charged part gives up its charge to positive plate, and negatively charged part gives up its charge to negative plate. A difference in potential between the plates is thus produced.

POLARIZATION. Hydrogen bubbles collect on the positive plate and increase the resistance of cell, and lower the E.M.F. by setting up a counter E.M.F.

DEPOLARIZATION. Some oxydizing agent is used as a depolarizer to remove the hydrogen by uniting with it to form water. Cells which are depolarized rapidly are used for closed circuit work, those which are depolarized more slowly, for open circuit or intermittent work.

LOCAL ACTION. Any impurities in the negative plate cause a difference of potential to exist between plate and impurity, forming a short circuit current, which consumes the plate, but produces no available energy.

ELECTROLYSIS. When an electric current is sent through a solution containing a metal salt, it will deposit the metal on the negative plate. If chemical action takes place between the electrolyte and positive plate, the positive plate is consumed.

ELECTROCHEMICAL EQUIVALENT. The amount of metal deposited on the negative plate, and the amount taken from the positive plate, by an ampere-hour of electricity is a constant, depending upon metals and electrolyte.

ELECTROPLATING. Use is made of electrolysis to plate conducting materials; to refine metals; to make elec-

trotypes, etc., and to restore the active elements in a storage battery.

DESTRUCTION OF WATER MAINS, ETC., BY ELECTROLYSIS. The leakage currents from the return circuits in electric railways travel along iron pipes, and are likely to eat away portions of the iron at the point where they leave the pipe to return to the generator.

STORAGE CELLS. Do not store electricity, but chemical energy. Common types composed of lead peroxide, positive plate; and spongy lead, negative plate.

Action on charge and discharge:

Discharge.

$$PbO_2 + Pb + 2H_2SO_4 = 2PbSO_4 + 2H_2O$$

Charge.

CARE OF STORAGE CELLS. Storage cells are injured by:

(1) Too rapid charging or discharging.
(2) Use of impure electrolyte.
(3) Use of too dense or too light electrolyte.
(4) Over-charging and over-discharging.

CONDITION OF CELLS. Can be ascertained:

(1) By color of plates.
(2) By specific gravity of electrolyte.
(3) By terminal voltage at normal charging or discharging rate (not by the E.M.F.).

USE OF STORAGE CELLS.

(1) To help carry " peak " of load.
(2) To carry all the " light load."
(3) As a reserve supply of electrical energy.

PRACTICE IN USE. Voltage across cells may be controlled by:

(1) Resistance in series with cells.
(2) End cells.
(3) Floating the cells on the line at a point of considerable voltage fluctuation.

EDISON STORAGE CELLS. Positive element, nickel oxide, changes to lower oxide on discharge.

Negative element, spongy iron, changes to iron oxide on discharge.

Electrolyte, 21 per cent solution of potassium hydrate, which remains at same specific gravity throughout charge and discharge.

Is lighter per watt-hour than most lead cells, but has lower E.M.F. per cell. Can remain charged or discharged indefinitely without deteriorating. Efficient temperature range is limited.

PROBLEMS ON CHAPTER XII

23–12. How long must a current of 800 amperes run in a copper refining vat to deposit enough copper to make 1000 ft. of No. 0 trolley wire?

24–12. It is desired to copper plate an iron casting having 2.4 sq.ft. of surface. How long must a current of 40 amperes run in order to make the plate $\frac{1}{16}$ in. thick?

25–12. Zinc-coated iron is called "galvanized iron." An iron sheet 4 ft. square is to be "galvanized." A current of 30 amperes running for 2 hours will deposit how thick a plate of zinc on the iron?

26–12. A primary cell using zinc as negative plate maintains a terminal voltage of 1.0 volt at normal rate of discharge. A steam plant has an efficiency of 6 per cent. Zinc costs 5 c. per lb. Coal $5.00 per ton. What is relative cost per K. W.-hour of zinc and coal as fuels? 1 lb. coal furnishes 14,000 B.T.U.

27–12. A battery of 20 lead storage cells in series, each having 2 sq.ft. positive plate area, is to be charged from a 110-volt generator. How much resistance must be placed in series with the cells, assuming, 2.2 E.M.F. and .005 ohm resistance for each cell?

28–12. If the cells in Problem 27 are to be charged from a 110-volt circuit in two parallel sets of 10 cells in series, what series resistance is needed?

29–12. A set of 80 lead storage cells each having 2.2 volts E.M.F. and .003 ohm internal resistance, is to be charged in two parallel sets of 40 cells in series. Each cell has positive plate area of 4.2 sq.ft. What must be terminal voltage of generator?

30–12. If cells in Problem 29 are discharged in series at normal rate, what will be terminal voltage of set?

31–12. What must be the E.M.F. of generator in Problem 29, if its internal resistance is .02 ohm?

32–12. If generator in Problem 27 has an E.M.F. of 112 volts and an internal resistance of .32 ohm, what series resistance must be used in charging the cells?

33–12. If a generator of 135 volts and .03 ohm internal resistance be used in Problem 29, what resistance must be placed in series with the cells?

34–12. If generator in Problem 48–8 has its field separately excited from 110 volts circuit, at what speed must it be run in order to charge a set of 64 storage cells arranged in two parallel sets of 32 cells in series? Each cell has an internal resistance of .004 ohm and an E.M.F. of 2.3 volts. Normal current is 25 amperes per cell.

35–12. (*a*) What is the ampere-hour capacity of set of cells in Problem 27? (*b*) What is the K.W.-hour capacity?

36–12. (*a*) What is the ampere-hour capacity of each cell in Problem 29? (*b*) What is the K.W.-hour capacity of set?

37–12. A set of 90 lead storage cells, each having 2.2 volts E.M.F. and .004 ohm internal resistance, is to be charged from a 110-volt line. If each cell is to take its normal current of 20 amperes, what would be best arrangement of cells in order to have least power lost in a series resistance in line?

38–12. How could cells in Problem 37 be arranged in order to deliver 120 amperes and not exceed normal current of each cell? At what voltage would they deliver this current?

39–12. A lead storage cell delivers 20 amperes at 2.1 volts. It requires 2.4 volts to charge this cell at 20 ampere rate. What is the virtual resistance of cell?

40–12. A storage battery of 240 cells in series is " floated " at the end of a 4-mile trólley line, the resistance of which (line and return) is .08 ohm per mile. Generator voltage is 550 volts; each cell has an E.M.F. of 2.1 volts and an internal resistance of .001 ohm. What current will cells supply to the line, when there are 5 cars at the battery end of the line, each taking 65 amperes?

41–12. What will be terminal voltage of the set of battery cells in Problem 40?

42–12. What current will cells in Problem 40 take when there is no load on the line?

43–12. What current will generator be delivering when the 5 cars of Problem 40 are half way between station and battery?

44–12. What current will battery be delivering to or receiving from line in Problem 43?

45–12. If there are but 2 cars on line in Problem 43 each taking 75 amperes. (*a*) What power is generator delivering? (*b*) What power is battery receiving or delivering?

CHAPTER XIII

PHOTOMETRY AND ELECTRIC ILLUMINATION

Illumination; Distribution, Color, Intensity—Law of Inverse Squares —Photometry, Candle Power; Horizontal and Spherical—Foot-Candle—Use of Sharp-Millar Illuminometer—Arc Lamp; Ballast and Regulating Resistances—Flaming or Luminous Arcs—Incandescent Lamps, Carbon, Tantalum, Tungsten, Nernst—Life; Effect of Overburning; of Underburning—Mercury Vapor Lamp —Moore Tube.

In taking up the subject of illumination, it is perhaps well to start with some general ideas underlying the whole problem. In the first place, it should be stated that we illuminate objects for the sole purpose of making them visible to the eye. The eye, then, is the natural starting point.

When passing upon the merits of any scheme of ordinary illumination, that which should mark it as a success or failure, should be the general effect of the scheme on the eye. Success should be measured largely by the degree of clearness with which the objects are perceived by the eye, as to shape and color. If certain parts of a room or street are too brilliantly lighted, objects in the dimmer portions are not perceived by the eye. If a certain side of one object be too highly illuminated, the general shape of the object is lost, as the eye does not readily perceive its more dimly lighted parts. This is because the eye automatically adjusts itself to the most brilliantly lighted area within its view, and accordingly, is out of adjustment for perceiving the rest. We should get rid of the idea,

therefore, that a light of intense brilliancy is the thing to be sought. It is, in general, highly undesirable.

The problem, then, resolves itself into two parts: The first object should be to secure a kind of lamp which will cause objects to appear in their accustomed colors; that is, the colors in which they appear by sunlight. The second object is to so distribute the lamps that the several illuminated surfaces receive their share of the light, and yet no bright light is thrown directly into the eyes.

213. Nature of Light. All space is supposed to be filled with a medium infinitely lighter than air, called ether.

The sensation of light is experienced when certain wave motions in this ether are transmitted to the eye. These wave motions are called light waves. Light waves differ from one another in length and violence. The *difference in length* causes a difference in color. Thus short waves may be blue or violet, while longer waves may be red or orange. If we have a source of light which sends out long ether waves, we may expect a predominance of red and orange light in it. The sunlight contains waves of practically all lengths and thus is composed of all colors. The *difference in violence* of the waves gives rise to a difference in intensity of the light.

When these light waves strike any object, they are partly reflected and partly absorbed. Substances differ widely as to the percentage of light they absorb and the percentage they reflect. If two objects are illuminated by the same amount of light, the one which absorbs the less light and reflects the more will appear the brighter.

Some objects reflect light waves of a certain length only, and absorb all the rest. It is this property that gives color to objects. Suppose, for instance, that a piece of cloth were receiving light from the sun, all of which it absorbed except the waves of proper length to cause a sensation of green to the eye. The green waves only would then come from the cloth to the eye, all the rest being absorbed, and

the cloth would appear green. If it absorbed waves of all lengths, it would appear black, because no light would be reflected from it to the eye.

If now the piece of cloth, which absorbs all wave lengths except that of green, were exposed to a source of light which was emitting all colors *except green*, there being no green waves to be reflected from it, the cloth in this light would appear black.

Suppose a piece of cloth absorbed all colors but two, say violet and red. When light having all wave lengths fell upon it, it would absorb all the waves except violet and red. These two, the cloth would reflect as a mixture and would appear purple. If, however, the source of light contained no violet waves, it could only reflect the red waves and appear red. This light, then, would not cause the cloth to show its normal color.

So in choosing an artificial source of light, it is necessary to select one which shall send out all wave lengths, if we wish to have the different objects appear in their normal colors.

214. Light Intensity. Candle-power. Foot-candle. A source of light sends out LIGHT FLUX, just as a magnetic pole sends out MAGNETIC FLUX. The laws concerning INTENSITY OF ILLUMINATION are similar to those concerning INTENSITY OF A MAGNETIC FIELD.

Just as a magnetic pole is of unit strength when it produces a unit force at a unit's distance, so a source of light is of unit intensity when it produces unit intensity of illumination at a unit's distance.

The intensity of illumination which a standard candle will throw upon a surface placed one foot's distance from it (at right angles to rays of light) is the UNIT INTENSITY OF ILLUMINATION. This unit is called a FOOT-CANDLE. The candle itself is said to have an intensity of one CANDLE-POWER. A candle is standard when its intensity is 1.136 times the intensity of a standard Hefner lamp burning

under standard conditions. Thus a light of 16-c.p. intensity would illuminate a *surface* placed one foot from it with an intensity of 16 foot-candles.[1]

Careful distinction should be made between candle-power and foot-candle. CANDLE-POWER is the measure of the intensity of a source of light. The FOOT-CANDLE is the measure of the intensity of illumination of some surface upon which the light falls.

Again, just as the field intensity about a magnet varies inversely as the square of the distance from the pole, so the intensity of illumination of a surface varies inversely as the square of its distance from the source of light. This is called the LAW OF INVERSE SQUARES. Accordingly, a 32-c.p. lamp will illuminate a surface 1 ft. away from it with an intensity of 32 foot-candles, but a surface 4 feet away, with only $\frac{32}{4^2}$ or 2 foot-candles.

Rule. To find the intensity of illumination on any surface (at right angles to light rays) divide the CANDLE-POWER of the lamp by the SQUARE of the distance in FEET. The result will be FOOT-CANDLE illumination.[2]

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[1] For a description of Standard Hefner Lamp, see Reference Books on Photometry.

[2] It is often desired to find the illumination of a surface which is not at right angles to the rays of light. Referring to Fig. 253: let A be a source of light of I candle-power, and hung l_v feet above the floor. It is desired to find the intensity of light on the horizontal surface S. Let θ equal the angle which (l), the ray of light from A to S, makes with the vertical (l_v). Then E, intensity of illumination of S in foot-candles, can be found from the equation $E = \frac{I}{l_v^2}\cos^3\theta$. This is a very convenient equation for finding illumination of different floor areas, etc.

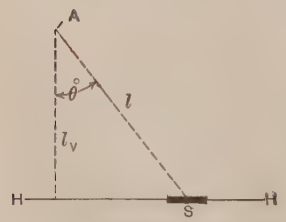


FIG. 253.

Problem 1–13. (*a*) What is the illumination on a surface 5 ft. from a 32-c.p. lamp? (*b*) How far from a 16-c.p. lamp will the illumination be the same as in (*a*)?

Problem 2–13. The illumination required on a printed page for easy reading is about 2 foot-candles. (*a*) How high above a reading table should a 16-c.p. lamp be hung? (*b*) a 32-c.p. lamp?

Problem 3–13. It is desired to illuminate a space on the floor to an intensity of 1.4 foot-candles. How high should a cluster of four 20-c.p. lamps be hung above the space?

Problem 4–13. The average illumination on the ground on a moonlight night is .025 foot-candle. The moon is 238,000 miles from the earth. What is the candle-power of the moon?

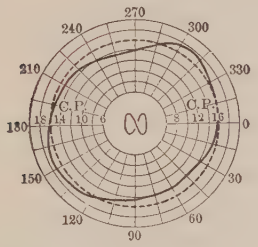


FIG. 254.—Horizontal distribution of candle-power about a carbon incandescent lamp.

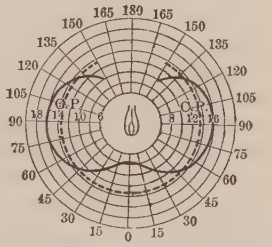


FIG. 255.—Vertical distribution.

215. Distribution of Intensity about a Lamp. Lamps do not show the same brightness on all sides; that is, they have a greater candle-power in one direction than in another. It is customary to obtain the candle-power of a lamp as viewed from all positions and average the results. This average is called the MEAN SPHERICAL CANDLE-POWER of the lamp.

When the average candle-power is taken for points in a horizontal plane only, this value is called the MEAN HORIZONTAL CANDLE-POWER. The latter is the usual method of rating incandescent lamps.

Figure 254 shows the results of some tests taken at Pratt for the distribution of candle-power around a single-

'oop carbon-filament lamp, in a horizontal plane. Notice that the light is very evenly distributed in this plane.

Figure 255 shows the vertical distribution around the same lamp. Note that the lamp has low candle-power directly beneath the tip.

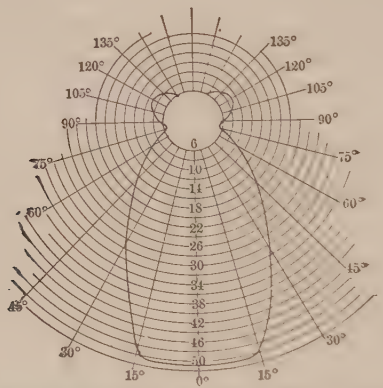


Fig. 256.—Distribution when fitted with special shade.

Figure 256 shows the effect on the distribution when a special shade is used on the same lamp in order to throw the light down.

216. Measurement of Candle-power. Photometers. The candle-power of a lamp is measured by comparing it with another lamp of known candle-power. The comparison is generally made by means of an instrument called a Photometer.

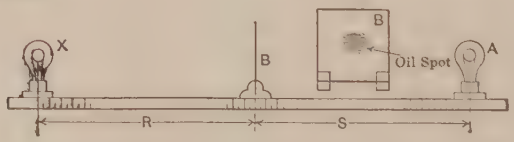


Fig. 257.—Diagram of Bunsen photometer.

We will take up the simplest form of this instrument, the Bunsen Photometer, shown diagrammatically in Fig. 257.

The standard lamp A and the lamp to be tested, X, are placed at the ends of a track. Between the lamps is placed a paper screen B, on which is a paraffin oil spot. There should be no source of light in the room except X and A. If the screen is placed at such a position that it is illuminated equally on both sides, the oil spot disappears.

When this position has been found, let

R = distance of screen from lamp X.
S = " " " " A.
A = intensity of A in candle-power.
X = " X "

Then the intensity of illumination on each side of the screen is as follows:

$\frac{A}{S^2}$ = intensity on side facing A;
$\frac{X}{R^2}$ = " " " X.

But intensity is equal on both sides, hence

$$\frac{A}{S^2} = \frac{X}{R^2}$$

or

$$X = \frac{R^2}{S^2} A.$$

Thus the intensities of the lamps are to each other as the squares of their respective distances from the screen.

Problem 6–13. In Fig. 257 standard lamp A is 16 c-p., $S = 230$ cms., $R = 250$ cms. What is the candle-power of lamp X?

Problem 7–13. If the distance from A to the screen, Fig. 257, is twice as great as the distance from X to the screen, how does the candle-power of A compare with that of X?

In practice the standard lamp is usually an incandescent lamp with a carbon filament, carefully standardized at a

rated voltage. These standard lamps should not be burned for any length of time, as they are likely to change in candle-power. A standard should always be used which is as nearly as possible of the same candle-power as the lamp to be tested, and should always be burned at its rated voltage. Figure 258 represents a common method of obtaining proper voltage regulation. The variable resistance R_1 is adjusted until the desired voltage is obtained across the lamp X, and then variable resistance R_2 adjusts the voltage across the standard lamp A. For this arrangement, lamp A must be of lower rated voltage than lamp X. Since they are in parallel across the same mains, any fluctuation

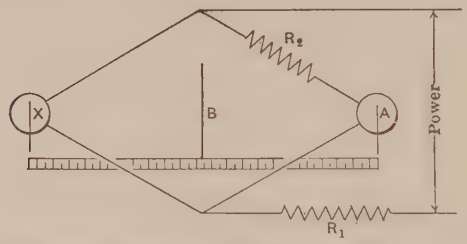


FIG. 258.—Diagram of photometer wiring for testing incandescent lamps.

in the voltage during readings will cause approximately proportional changes in candle-power of both lamps and not affect the results.

The lamp X is often rotated at about 120 revolutions per minute. Its mean horizontal candle power can thus be found from one setting of the screen.

For obtaining the candle power of arc lamps, special photometric apparatus has to be used, for a description of which, see "High Efficiency Electrical Illuminants," by R. V. Hutchinson, Jr.

217. Measurement of Intensity of Illumination. The Illuminometer. The problem of computing or measuring the intensity of illumination on any given surface is very complex. If only one source of light is illuminating the

surface which is perpendicular to the rays of light, a direct application of the law of inverse squares would be all that is necessary. But such is never the case in actual practice. Light generally comes from several sources, either directly or after being reflected from other surfaces.

Several instruments have been devised to measure the intensity of illumination directly.

The portable photometer shown in Figs. 259 and 260 is a most convenient and satisfactory device for the meas-

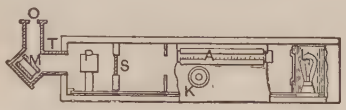


FIG. 259.—Side view of Sharp-Millar photometer.

urement of both the intensity of light sources directly in candle-power, and of intensity of illumination of surfaces directly in foot-candles.

It is called the Sharp-Millar Universal Photometer and consists of a closed box containing a standard lamp *L*, which illuminates an opaque screen *S*, and lights up one side of the photometer screen *P*.

FIG. 260.—Top view of Sharp-Millar photometer.

The light whose candle-power is to be measured is placed at *a stated distance* from the opening in the tube *T*, and illuminates a white matt surface placed in the tube at *M*. The light reflected from this matt surface illuminates the other side of the Photometer Screen *P*.

We thus have a photometer screen illuminated on one side by light from a lamp of known candle-power, and on the other by light from a lamp of unknown candle-power. We merely have to get the illumination the same

on both sides of the screen in order to determine the candle-power of the unknown source.

This is done by drawing the standard lamp along in the box by means of the knurled head K, until the spot on the photometer screen disappears as in the Bunsen photometer. Instead of having now to measure the distance from the lamps to the screen, a spot of light on the scale A indicates directly the candle-power of the unknown source.

If the lamp of unknown candle-power is not placed at the correct distance from the photometer box, of course a correction has to be applied to the indicated candle-power as follows:

$$\frac{\text{The indicated candle-power}}{\text{True candle-power}} = \frac{\text{Square of standard distance of lamp from surface } M}{\text{Square of actual distance of lamp from surface } M}.$$

When it is desired to measure the intensity of illumination on any surface, the photometer is brought to that place, an opaque screen like that at S is placed at the opening O, and a mirror replaces the white matt surface at M. The illumination on the side of the photometer screen P toward lamp L is now regulated as above by moving the standard lamp. The scale now indicates the illumination of the screen at O in foot-candles.

The standard lamp consists of a one candle-power tungsten incandescent lamp run on a small storage battery. It takes about .9 ampere at 4 volts.

In order to measure the candle-power of lamps of high intensity, a smoked glass absorbing plate may be placed between the photometer screen and the white matt surface M. This cuts down the light reaching the photometer screen P by a known amount, so that the indicated candle-power has merely to be multiplied by some constant of about 10, or 100, depending on the absorbing plate used.

When measuring very weak illumination, the absorbing plate is placed between the screen *S* and the photometer screen. The indicated scale reading must then be divided by 10 or 100, as the case may be.

By means of these absorbing screens which can quickly be turned into place, intensities can be measured varying from .004 foot-candle to 2000 foot-candles.

The instrument surely has a right to the name of Universal Photometer.

218. Arc Lamps. When an electric current is made to flow across a small gap from the tip of one carbon rod to another, an electric "arc" is formed. The carbon points are heated to a brilliant incandescence, gradually vaporized and burned up by the oxygen in the air. The electric current passes from one tip to the other in this stream of incandescent carbon vapor. If the rods are composed of pure carbon, by far the greater part of the light is emitted by the glowing tips of the rods, and but little by the carbon vapor. Fig. 261 shows the results of a test on the distribution about a direct current arc lamp. The light is seen to have the greatest intensity at an angle of 40 to 45 degrees below the horizontal. Thus most of the light comes from the "crater" in the upper or positive carbon. The light being thrown downward makes this form of arc especially adapted to street lighting. In an alternating current arc lamp the light is sent out in about equal intensity from both carbon tips as shown in Fig. 262. The light emitted by carbon arcs is an intense white, slightly bluish. They have the great advantage of causing objects to appear in their normal colors. But they also have the disadvantage of any source of light of high candle-power, in that the eye must be shaded from the direct rays. When shades are employed for this purpose, only about half of the total light can be used.

If the rods are formed of a mixture of carbon and calcium fluoride, the vapor given off becomes highly luminous,

a golden orange in color, and often producing light of 1000 mean spherical candle-power. These so-called LUMINOUS or FLAMING ARCS are extensively used for display and advertising purposes. The fumes given off are obnoxious, so that they are rarely used for indoor lighting. Furthermore, the cost of the special carbons and their rapid consumption has hindered their coming into general use, in spite of their high efficiency of .15 watt per candle-power.

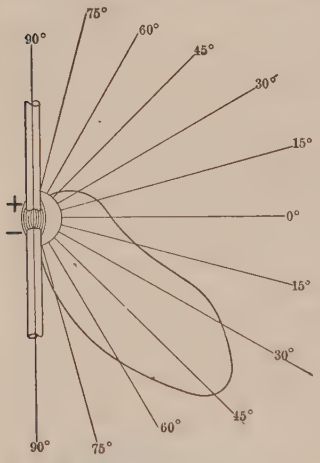


FIG. 261.—Distribution of candle-power about a D.C. arc lamp.

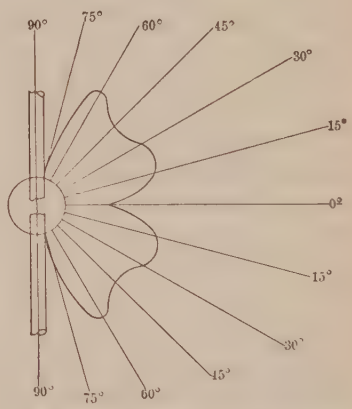


FIG. 262.—Distribution of candle-power about an A.C. arc lamp.

To decrease the rate of consumption of ordinary carbon rods, the arc is sometimes inclosed in a nearly air-tight glass globe. This excludes the oxygen to such an extent that the same pair of carbons lasts 100 hours instead of only 7 or 8.

From 70 to 80 volts can be used to maintain a long arc in the inclosed lamp, while only 35 volts can be used across a short arc in the open air. The inclosed arc can thus be used on a 110-volt circuit. These facts are tending to cause the inclosed arcs to be used in almost all new installations.

218. Operation of D.C. Arc Lamps. Ballasting Resistance. The arc between the tips of carbon rods is naturally unstable and cannot be maintained alone, directly across the supply mains of constant potential circuit. This is evident if we consider the characteristics of the arc.

Suppose the arc to be burning steadily across a constant potential source of supply. If the voltage decreased ever so little, the current would decrease a little and would not vaporize the carbon so rapidly. The path of carbon vapor between the tips would then be less dense or of smaller cross-section; so the resistance of it would increase. This would still further decrease the current, and continue to do so till the arc was extinguished.

Or suppose the voltage increased ever so little. The current would increase and cause a greater flow of carbon vapor between the tips. This would decrease the resistance of the arc and allow still more current to flow. This would continue until practically a short circuit was made.

All of these changes would take place too rapidly for any regulating device to counteract them.

The instability of such an arc is corrected as follows:

A Ballasting Resistance R, Fig. 263, is inserted in series with the arc A, across the line of constant potential M.

The voltage across the mains is then taken up both in the IR drop of the ballasting resistance and in the voltage used in maintaining the arc. If the voltage across the mains becomes a little greater and causes a greater current to flow through the lamp, the IR drop across the ballast becomes enough greater to cause a slight decrease in the voltage across the arc. Any decrease in current through the lamp causes a less IR drop across the ballast and allows a greater part of the line voltage to come across the arc. The ballasting resistance then opposes an increase or decrease of current between the carbon tips. It must always be inserted when lamps are used on constant potential lines. In

A.C. lamps a choke coil may take the place of the ballasting resistance. For action of choke coil, see Chapter XV.

Regulating Coil. The device for regulating the length of the arc is very simple when the lamps are used in parallel on a constant potential line. See Fig. 264. In series with the ballast R and the arc A is placed a Regulating Coil C. The plunger P is attached to a friction clutch F. When no current is running through the lamp the plunger drops to its lowest position and causes the friction clutch to

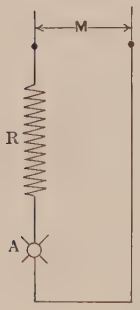


Fig. 263.—Arc and ballasting resistance.

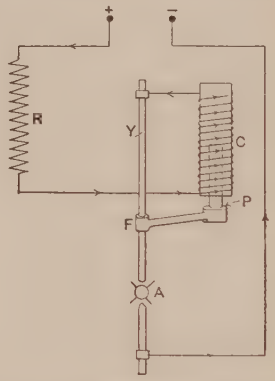


Fig. 264.—Arc with ballasting and regulating resistances.

release the upper carbon and allow it to drop until it touches the lower carbon.

The two carbons are thus in contact when the power is off. So, when the current is turned on, there is a complete circuit through the lamp. But as the current goes through the coil, the plunger P is sucked up into the coil. The friction clutch grips the upper carbon and raises it, on account of this upward motion of the plunger. An arc is thus struck between the positive and negative carbon tips. These parts are shown in Fig. 265.

As the carbons burn away and increase the length of the gap between them, and accordingly increase the resist-

ance of the circuit, less current flows. This makes the regulating coil weaker and allows the plunger to drop. The dropping of the plunger releases the friction clutch on the positive carbon, so that it falls and touches the lower carbon again. The process of striking the arc then takes place as at first. Thus an almost constant length of arc is maintained. The voltage across the arc can be changed by adjusting the amount of resistance in the ballast. The amount of current taken by the lamp can be regulated by adjusting the distance the plunger is allowed to move.

When arc lamps are operated in series, a much more complicated regulating scheme is used. A ballast resistance, however, need not be inserted in the lamps, because an automatic adjustment is made at the generator for keeping the current output constant.

FIG. 265. — Photograph of arc lamp showing ballasting and regulating resistances.

219. Incandescent Lamps. Carbon Filament. When an electric current of sufficient strength is sent through a fine filament of carbon it heats it to incandescence and, if exposed to the oxygen of the air, the filament immediately burns up. For this reason the filament has to be inclosed in a bulb from which the air has been exhausted when this source of light is to be used. Such a lamp can be made of any medium candle-power and emits a soft yellow light, very well suited to indoor illumination.

The resistance of the cold carbon filament is about twice as high as when incandescent, though any further increase in temperature does not greatly tend to further decrease the resistance. Such a lamp is rated as to its voltage, mean horizontal candle-power, and life. If a lamp is burned at any voltage differing but slightly from that

specified in the rating, a great change in the candle-power, efficiency and life will take place.

The following table shows the effect on the candle-power, efficiency and life, when a lamp rated at 110 volts, 16 c.p. is burned at either lower or higher voltages.

Actual Volts.	Per Cent of Rated Voltage.	Mean Horizontal Candle-power.	Percentage of Rated C.P.	Life in Hours.	Per Cent of Rated Life.	Watts per C.P.
	90		55		760	
	91		59		650	
	92		63		545	
	93		67		450	
	94		71		355	
	95		75		270	
105.	96	12.8	80	1,000	200	3.86
106.7	97	13.6	85	875	175	3.72
107.8	98	14.4	90	735	147	3.60
108.9	99	15.2	95	600	120	3.50
110	100	16.0	100	500	100	3.40
111.1	101	17.0	106	400	80	3.28
112.2	102	17.8	111	300	60	3.16
113.3	103	18.9	118	240	48	3.06
114.4	104	19.9	124	200	40	2.96
	105		131		34	
	106		138		29	
	107		145		25	
	108		153		21	
	109		161		18	
	110		169		15	

Note that a rise of 1 per cent above the rated voltage causes:

A rise of 6 per cent in candle-power.
A loss of 20 per cent in life.
A gain of 3½ per cent in efficiency.

A fall of 1 per cent below rated voltage causes:

A loss of 5 per cent in candle-power.
A gain of 20 per cent in life.
A loss of 3 per cent in efficiency.

This illustrates the necessity of avoiding burning a lamp much above or below its rated voltage.

Problem 8–13. A 3.5-watt carbon filament lamp which has 18 candle-power at 110 volts, will have what candle-power at 107 volts?

Problem 9–13. What would be the efficiency of lamp in Problem 8 at 107 volts?

Problem 10–13. How much would the life of the lamp in Problem 8 be affected by being used on 107 volts?

Problem 11–13. What would be the effect upon the life, efficiency and candle-power on a 3.30 watt, 16 c.p., 112-volt carbon filament, if it were burned on 115 volts?

The color of the light emitted by a carbon incandescent lamp is very pleasing to the eyes, but contains too much yellow to give the normal color to objects. In this respect it is inferior to the arc lamp. Its efficiency is rarely better than 3.5 watts per candle-power except in the case of the G.E.M. lamp, which has an efficiency of about 2.5 watts per candle-power. The filament in the latter is composed of carbon which has been "metallized" by a special firing process.

220. Tantalum and Tungsten Lamps. Incandescent lamps have lately come into use, in which the filaments are of metal, either tantalum or tungsten. These filaments have to be made very fine and much longer than a carbon filament, in order to get the resistance necessary on a 110-volt circuit. The arrangement of filament is illustrated in Fig. 266. The efficiency of a TANTALUM lamp is about 2 watts per candle-power; and the horizontal distribution is remarkably uniform. The life on a D.C. circuit is from 700 to 900 hours, but falls to about half that on an A.C. circuit, due to a peculiar granulating effect which the alternating current has on the filament.

The TUNGSTEN lamp has an efficiency of from 1 to 1.5 watts per candle-power, a life of 1000 hours, and is unaf-

fected by an alternating current. The main difficulty in the use of both tantalum and tungsten lamps is the fact that the long slender filaments are so liable to breakage. This is especially true of the tungsten.

The filament of each becomes more or less plastic when burning, and sags to such an extent when placed in a horizontal position, that parts of it easily form short circuits. The extreme high efficiency and long life (barring accidents) of the tungsten has made it a favorite for indoor lighting and it is rapidly displacing the carbon filament lamps. The color also is nearer white and gives more nearly normal color values to illuminated surfaces. The General Electric Company have recently announced a tungsten lamp, the filament of which is drawn and exceedingly tough.[1]

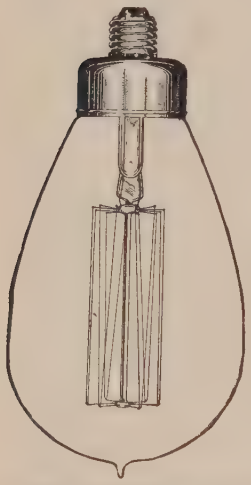


FIG. 266. – Arrangement of long filament in tantalum lamp.

221. The Nernst Lamp. In the Nernst lamp, use is made of the fact that many non-conducting substances become conductors when heated to incandescence. A small pencil-like glower composed of about the same materials as the Welsbach gas mantle is first heated by an auxiliary heating device to make it a conductor. A current can then be sent through it, which causes it to glow with an intense white light. The glower, being in the bottom of the lamp, throws most of the light down. It is generally surrounded by a frosted globe on account of its high candle-

[1] The reason for the higher efficiency of metallized carbon, tantalum and tungsten lamps, lies in the fact that they operate at higher temperatures and thus give out a greater percentage of the energy in light waves and a less percentage in heat waves.

power. The air does not have to be excluded from the glower as from an incandescent filament. Fig. 267 represents the structure of the lamp diagrammatically.

The glower G is connected in series with the coil M and the ballast R. The heating coils C are at first connected in parallel with the glower.

When the power is turned on, the current will not go through the glower, as it is a non-conductor when cold. The coil M is therefore not energized, the current taking the parallel path through the heating coil C by way of the spring steel arms A.

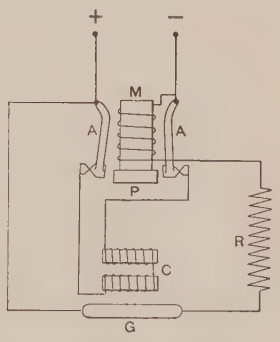


Fig. 267.—Diagram of electric circuits within a Nernst lamp.

In a half minute, the heating coil C has raised the temperature of the glower high enough to cause it to become a conductor. The current now flows through the path containing the glower energizing the coil M, the pole piece of which P, attracts the arms A, and breaks the heating coil circuit. The whole current now going through the glower, heats it to incandescence. When very high candle-power is desired, several glowers are connected in multiple. The lamp has excellent color qualities; and an efficiency of about 2 watts per candle-power. The life of a glower, which can be replaced, is about 700 hours.

The complicated mechanism and high cost of glowers do not allow the Nernst lamp to compete in this country with the highly efficient filament lamps described above.

222. Mercury Arc Lamp. The mercury arc lamp, commonly known as the Cooper Hewitt lamp, makes use of the property of luminescence of metallic vapor. The mercury is held in the lower end, M, Fig. 268, of a vacuum tube 20 to 50 inches long.

Since current will not flow through the tube from B to A unless there is a stream of vapor to carry it, a starting device must be used.

Sometimes the tube is merely tilted back into the horizontal position so that a thread of mercury flows out of M and completes the circuit. It is then allowed to return to a position at a slight angle to the horizontal, so that the mercury will flow back into the end as the vapor condenses.

Generally an automatic starting device is used. It consists of a coil, L, of high inductance, through which the current goes at first. But this circuit is suddenly broken by an ingenious automatic mercury contact breaker D. The high self-inductance of the circuit causes a spark to jump across from G to A. This spark charges the mercury electrically and sets up enough vapor in the tube for the current to pass from B to A. The tube now glows with a light composed entirely of green, blue and yellow waves. This gives a peculiar color effect to most objects, especially those which usually reflect red. The light has, however, a very penetrating effect and brings out the shape and contour of objects very distinctly. Its intrinsic brilliancy is low, being only about 17 c.p. per sq.in. of tube. For this reason, it is used in drafting rooms and wherever a light is needed to make small details stand out clearly.

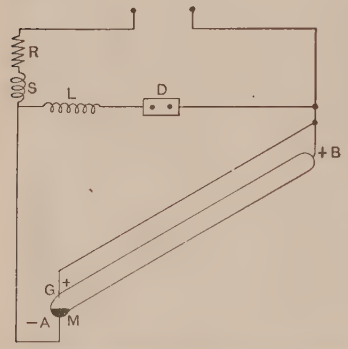


FIG. 268.—Diagram of electric circuits in a mercury vapor lamp.

A 50-inch tube has a hemispherical candle-power of about 800 c.p. and takes 3.5 amperes at 110 volts, or .48 watt per c.p. The life is about 1000 hours, being limited by the loss of the vacuum.

Since the arc is unstable below a certain current strength, and the lamp has the best efficiency at the critical value of current, a ballasting resistance R and inductance S are used in series with the tube to counteract the effects of any variation in the voltage and keep the current above critical value.

223. The Moore Tube. A high voltage will force a current of electricity through a tube which contains gas at a very high vacuum, and cause the gas to glow with a soft luminescence. The color depends upon the gas used, carbon dioxide producing a white diffused daylight, nitrogen an orange light, etc.

The Moore Tube, making use of these characteristics, consists of a long high-vacuum tube, to the terminals of which is supplied an alternating current at high pressure. A special transformer is employed to attain the necessary high voltage.

As in operation, the gas is gradually precipitated on the glass wall to the tube, the vacuum continues to grow greater. Thus it is necessary from time to time to admit gas into the tube.

Mr. Moore has devised a very ingenious valve for this purpose which operates automatically and admits the required amount of gas at the proper time.

The intensity is only about .65 c.p. per sq.in. This requires that long tubes be installed, to give sufficient illumination for large rooms. But the effect is very pleasing, approaching, as it does, so closely to diffused daylight. The efficiency compares favorably with the best carbon filament incandescent lamps. Fig. 355*a* shows this light installed in a silk mill, where it finds special application in bringing out the true color values of the manufactured product.

SUMMARY OF CHAPTER XIII

The merits of any scheme of illumination should be judged by its effect upon the eye. Colors should appear as they do in daylight and objects should be evenly illuminated in such a way as to show their outlines.

NATURE OF LIGHT. Light is caused by waves in the ether; a definite color effect being produced by a wave of a definite length. The violence of the wave determines the intensity of the light. An object has the color effect of the waves it reflects.

PHOTOMETRIC UNITS. The unit of intensity of a source of light is the candle-power, which is the intensity of a standard candle.

The unit of intensity of illumination of a surface is the foot-candle, which is the intensity of illumination on a surface at right angles to rays of light and at one foot distance from a light of one candle-power.

Intensity of illumination on any surface at right angles to rays of light can be found from the equation:

$$E = \frac{I}{d^2},$$

where E = illumination of surface in foot-candles;
I = intensity of source of light in candle-power;
d = distance of surface from source in feet.

LAW OF INVERSE SQUARES. The intensity of illumination on any surface at right angles to light rays, is inversely proportional to the square of its distance from the source of light.

BUNSEN PHOTOMETER. A light of unknown candle-power is placed on one side of a screen of paper with an oil spot on it. A light of known candle-power is placed on the other side, and screen adjusted so that the oil spot disappears. The candle-powers of the two lights are to each other as the squares of their respective distances from the screen.

The Sharp-Millar Universal Photometer is a very satisfactory direct reading Photometer and Illuminometer.

ARC LAMPS, D.C. Light comes mostly from the crater in upper carbon and is thrown down, unless carbons are impregnated with some metallic salt, in which case the arc itself is luminous.

Will not operate satisfactorily on constant potential lines without ballasting resistance.

Arc is struck and maintained by regulating coil generally in series with arc.

High in efficiency but of too great intensity for indoor lighting unless screened. Colors appear nearly the same as in daylight.

INCANDESCENT LAMPS. Carbon filament; pleasant yellow light, low efficiency but of convenient candle-power. Small variation in voltage across terminals makes great variations in candle-power and life. Filament must be in vacuum.

Tungsten Lamp; whiter light than carbon filament, high efficiency, but the filament is fragile.

Nernst Lamp; the glower is heated to incandescence in the air. As glower is a non-conductor when cold, a heating device is added to the lamp. About the same efficiency as carbon filament lamps.

MERCURY ARC LAMP. Luminous mercury vapor in vacuum tube. Lacks red rays so that colors do not appear in true values, but has a very penetrating effect, and brings out details. Highly efficient.

MOORE TUBE. Luminescent gases in high vacuum tube. Requires high voltage. Can use any gas; carbon-dioxide producing a soft, white light which gives colors their true value. Of lower intrinsic candle-power and very pleasing to the eye. About the efficiency of carbon filament lamps.

PROBLEMS ON CHAPTER XIII

12–13. In a photometer arranged as in Fig. 257, the standard lamp is 134 cms. from the screen and the test lamp is 158 cms. from the screen when balanced. If the standard lamp is 20 c.p., what candle-power is the test lamp?

13–13. What is the foot-candle illumination on each side of the photometer screen in Problem 12?

14–13. In using a Sharp-Millar photometer to measure the candle-power of an arc lamp, the opening was held 140 ft. from the light. The photometer had been standardized for 12 ft., and registered 7.2 c.p. What was the true candle-power of the arc lamp?

15–13. What foot-candle illumination would be registered at the place where the photometer was read in Problem 14, assuming all the light came from the arc lamp?

16–13. If a standard lamp of 25 c.p. were used in photometer of Problem 12, how far from screen would it have to be placed in order to balance test lamp?

16(*a*)–13. Each lamp in Fig. 187 is carbon filament, 32 c.p. at 110 volts. What is the candle-power of each as located in Fig. 187?

17–13. Five 16-c.p., 110-volt carbon filament lamps are in series in a street car which is the only one on the line, and is taking 75 amperes. Generator voltage is 550 volts; trolley is No. 00 hard drawn copper; track resistance is .03 ohm per mile; car is 1 mile from station. What is the candle power of each lamp in car?

18–13. In a room lighted by one 32-c.p. lamp hung 8 ft. from the floor, what would be the illumination on the floor directly under the lamp?

19–13. What would be the illumination on the floor in Problem 18, 4 ft. from a point directly under lamp? See footnote to page 366.

20–13. If the room in Problem 18 had two 16-c.p. lamps hung 8 ft. from the floor and 10 ft. apart, what would be the illumination on the floor directly under each lamp?

21–13. What would be the illumination on the floor in Problem 20, at a spot directly under a point half way between the two lamps?

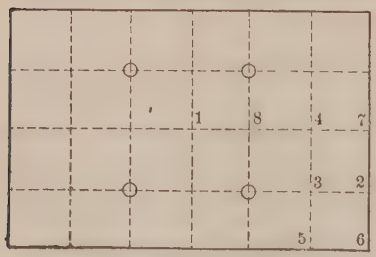


FIG. 270.

22–13. In a 20×30 ft. room there are four 20-c.p. incandescent lamps symmetrically arranged as indicated in Fig. 270, 5 ft. from long side and 10 ft. from short side. Suppose floor to be divided into 5-ft. squares. Find the illumination at corners of squares marked 1, 2, 3, 4. Lamps are each 8 ft. above floor. Assume no reflection from side walls or ceiling.

23–13. Find illumination in room of Problem 22 at points marked 5, 6, 7 and 8.

CHAPTER XIV

ELECTRICAL MEASURING INSTRUMENTS

Galvanometer; D'Arsonval and Thomson Types—Deflecting Force; Control; Damping—Sensibility—Shunts; Ayrton Universal Shunt—Series Resistance; Megohm—Ballistic Galvanometer—Thermal Effects—Ammeters; Resistance of—Types: Solenoidal, Hot-Wire, Permanent Magnets, Two Coil or Electro-Dynamometer—Voltmeters; Resistance of—Types: Solenoidal, Hot-Wire, Permanent Magnet, Two Coil, Electro-Static—Wattmeter—Construction of Weston Type—Thomson Integrating Wattmeter or Watt-hour Meter—Potentiometer; Theory, Construction and Use—Voltameter.

In the following detailed description of electrical measuring instruments, no attempt has been made to include all electrical instruments. It is intended to study those types only which are in general use in this country, in operating stations, laboratories, and testing departments of manufacturing plants.

224. Galvanometer. The oldest instrument for measuring current and voltage is the galvanometer. Most of our modern ammeters and voltmeters are merely adaptations of one form or other of this instrument.

In its earliest form, the galvanometer consisted of a compass needle suspended in the center of a coil. Fig. 271 represents such an instrument, called a tangent galvanometer. This instrument is so called because the current flowing in the coil is proportional to the tangent of the angle through which the needle is deflected.

If instead of reading the angle through which the needle swings, the coil is turned through an angle sufficient to

bring the needle back to zero, the instrument becomes a SINE GALVANOMETER. The current flowing in the coil is then proportional to the sine of the angle through which the coil has to be turned. Such instruments even when fitted with a mirror and telescope were not very sensitive, and have only a historical interest.

225. Thomson Astatic Galvanometer. Sir William Thomson (Lord Kelvin) made many remarkable improvements on the above galvanometers and succeeded in pro-

FIG. 271.—Tangent galvanometer.

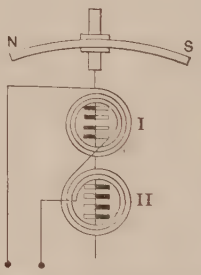


FIG. 272. Thomson astatic galvanometer.

ducing an instrument of high sensibility. Fig. 272 represents a typical galvanometer of the Thomson type.

The two coils I and II are wound in opposite directions. When a current is sent through them, they are magnetized in opposite directions. This allows an arrangement of the needles called ASTATIC.

The needles within coils I and II are joined together by a stiff wire but in reversed positions, as in Fig. 273. If they are of nearly equal strength they nearly neutralize one another. They will then have but little tendency to take a definite position in the earth's field, since they are acted upon by it to nearly equal degrees in opposite directions. In order that such an arrangement of needles

may return to a definite zero point, a controlling magnet, *NS*, Fig. 272, is placed above or below it. As this magnet is nearer to one needle than to the other, it causes the combination to take a definite position. The controlling force of this magnet can be made as small as desired. Thus we have strong magnetic needles in a very weak controlling field. An exceedingly small current in the coils then exerts a torque sufficient to cause the magnets to turn against this controlling field. Galvanometers of this type have been made so sensitive that one fifty-billionth part of an ampere would cause a deflection of one scale division. This was the type used for receiving messages sent through the Atlantic cable.

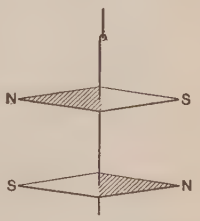


FIG. 273. — Astatic arrangement of two magnetic needles.

But the Thomson galvanometer cannot be made to return quickly to zero, nor can it be kept from swinging back and forth for a long time after each deflection. The means by which the moving part of an instrument is brought back to zero is called its CONTROL.

When the moving part of an instrument is kept from swinging back and forth, it is said to be DAMPED, and the instrument is said to be DEAD-BEAT.

Thomson galvanometers can be damped by mechanical means only, such as a vane attached to the suspension fiber which increases the air resistance to turning. For this reason another form of galvanometer is in much more general use, called after its inventor the D'Arsonval.

226. D'Arsonval Galvanometer. Fig. 274 represents the moving part of an ammeter which is constructed on the D'Arsonval principle. It contains the working principles of most of our modern ammeters and voltmeters, and is built on the reverse principle of the Thomson type. Instead of a stationary coil and moving magnets, this type

consists of a permanent stationary magnet, and a moving coil. The soft iron core is supported rigidly between poles, in order to concentrate the magnetic lines of force. The coil swings freely around this core. Owing to its strong magnetic field, this type is peculiarly free from ordinary magnetic disturbances which greatly affect the Thomson type.

The *Deflecting force* is the motor action of a current flowing in the coil as explained in Chapter VIII.

The *Control* is effected by means of the torsional elasticity of the suspension wire.

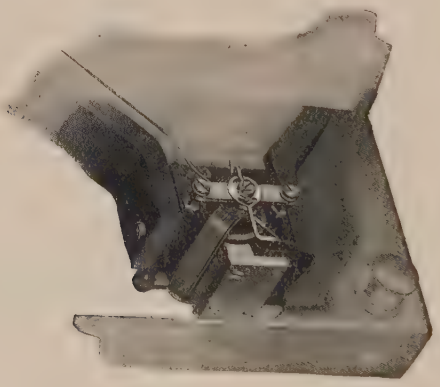


FIG. 274.—Ammeter of D'Arsonval type.

The *Damping* is very easily effected by means of the induced currents set up in a short-circuited coil wound on the moving coil, as explained in Chapter VIII. Galvanometers of this type are made of such a sensitiveness that one-hundred-millionth part of an ampere will cause a deflection of one scale division. Through small ranges, the deflections of this galvanometer are proportional to the current flowing in the coil. And since the resistance of the coil is constant, the voltage must vary as the current. Therefore the deflections are also proportional to the voltages across the coil.

227. Shunts. In order that a galvanometer may be used throughout a wide range of current measurements, shunts are sometimes placed around the instrument so that only a small fraction of the current in the line passes through the moving coil.

Suppose that it is desired to measure the current I_m flowing in the main line in Fig. 275, but that the galvanometer G cannot safely carry all this current. A shunt, S, might be placed across the terminals of the galvanometer, which would conduct a certain proportion of the current around the galvanometer. But in order to know how much current is flowing in the main line, it is necessary to know what fraction of the total current the galvanometer carries and what fraction the shunt carries.

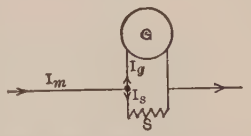


FIG. 275.—Use of shunt (S) with galvanometer (G).

Let S = resistance of shunt in ohms;
G = " galvanometer in ohms;
I_m = current in main line;
I_s = " shunt S;
I_g = " galvanometer G.

The circuit through S is in parallel with the circuit through G. Then

(1) $$I_m = I_s + I_g.$$

(Current in parallel combination equals the sum of currents in each branch.)

The voltage across the Shunt $= I_s S$. (IR)

" " Galvanometer $= I_g G$. (IR)

But voltages across parallel circuits are equal.

Therefore $$I_g G = I_s S,$$

or

(2) $$I_s = \frac{I_g G}{S}.$$

The relation of the current in Galvanometer (I_g) to current in main line (I_m), may be expressed by the fraction $\frac{I_g}{I_m}$.

But from (1)

(3) $$\frac{I_g}{I_m}=\frac{I_g}{I_g+I_s}.$$

Substituting (2) in (3),

$$\frac{I_g}{I_m}=\frac{I_g}{I_g+\frac{I_gG}{S}}=\frac{I_g}{\frac{I_gS+I_gG}{S}}=\frac{I_gS}{I_g(S+G)}=\frac{S}{S+G}.$$

Thus

(4) $$\frac{I_g}{I_m}=\frac{S}{S+G}.$$

The current through the galvanometer holds the same relation to the current in the main line, that the resistance of the shunt does to the resistance of Shunt plus resistance of Galvanometer.

Example. Assume the resistance of the galvanometer, Fig. 275, to be 2500 ohms and the shunt to be 500 ohms. What fraction of the current in the main line goes through the galvanometer?

$$G=2500,$$

$$S=\ 500,$$

$$\frac{I_g}{I_m}=\frac{S}{S+G}=\frac{500}{2500+500}=\frac{1}{6}.$$

Thus $\frac{1}{6}$ of the current in the main line passes through the galvanometer.

The same equation can be used to compute the amount of resistance with which a galvanometer must be shunted in order that a given fraction of the current in the main line may pass through the galvanometer.

Example. Suppose it is desired to shunt a galvanometer in above example so that one-tenth of the current in the main line shall pass through it. What resistance must the shunt be?

$$\frac{I_g}{I_m}=\frac{S}{S+G}=\frac{1}{10};$$

$$\frac{S}{S+2500}=\frac{1}{10};$$

$$10S=S+2500;$$

$$S=\frac{2500}{9}=277.8 \text{ ohms.}$$

Problem 1–14. A galvanometer of 1000 ohms resistance has a shunt of 100 ohms resistance. What part of main current flows through galvanometer?

Problem 2–14. It is desired to shunt galvanometer in Problem 1–14 so that but $\frac{1}{10}$ the main current shall pass through it. Of what resistance must the shunt be?

Problem 3–14. A galvanometer of 2000 ohms resistance is shunted with a 5.00-ohm shunt. On a certain circuit it gives a deflection of 1.4 scale divisions when thus shunted. What deflection would it give on same circuit if unshunted?

Problem 4–14. What resistance shunt would be required to cause a deflection of 14 scale divisions of galvanometer in Problem 3–14 on same circuit?

228. Ayrton-and-Mather Universal Shunt. Fig. 276 represents a shunt arrangement by means of which several shunt values can be obtained throughout a wide range, depending on the position of the arm K. As this arm is swung from right to left, it lowers, by some decimal fraction, the amount of current going through the galvanometer. Since this fraction is always the same regardless of the resistance of the galvanometer, this arrangement is called a UNIVERSAL SHUNT.

These fractions *do not represent the ratio of the current in galvanometer to the current in the main line.* They merely denote ratio of the current in galvanometer with

arm in given position, to current in galvanometer with arm at A.

Thus when the arm is at B, $\frac{1}{10}$ as much current flows through the galvanometer as when it is at A, not $\frac{1}{10}$ of the current in the main line. The fraction of the current that flows through the galvanometer when the arm is at A depends upon the resistance of the galvanometer, and is, of course, different in connection with different galvanometers.

But, regardless of the resistance of the galvanometer, when the arm is at B, C, or D, $\frac{1}{10}$, $\frac{1}{100}$ or $\frac{1}{1000}$ as much

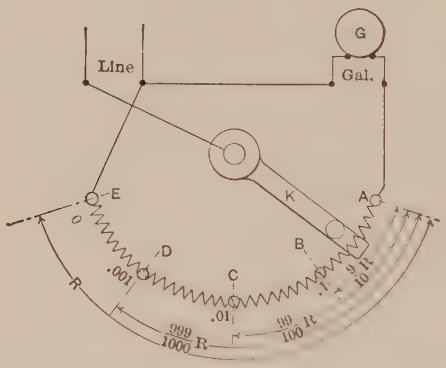


FIG. 276.—Diagram of Ayrton universal shunt.

current flows through the galvanometer as when the arm is at A. The term UNIVERSAL is therefore a little broad. The great advantage of this shunt is the ease with which the same galvanometer can be used to measure a wide range of currents when equipped with it.

It is necessary, however, to prove that the ratios are true as stated, regardless of the resistance of the galvanometer. Note first that as the arm moves from right to left, it cuts resistance out of the shunt and adds it to the galvanometer circuit.

Let G = resistance of galvanometer circuit,
R = resistance of EA.

Then BA is made $\frac{9}{10}\ R$,
and CA " $\frac{99}{100}\ R$,
" DA " $\frac{999}{1000}\ R$.

Let I_g = current in galvanometer,
I_m = current in line.
S = resistance of shunt.

Then $\frac{I_g}{I_m} = \frac{S}{S+G}$; as in previous paragraph.

When arm is at A

$$\frac{I_g}{I_m} = \frac{S}{S+G} = \frac{R}{R+G}.$$

When arm is at B

$$\frac{I_g}{I_m} = \frac{S_1}{S_1+G_1}.$$

But $S_1 = \frac{1}{10}R$,
and $G_1 = G + \frac{9}{10}R$,
Thus

$$\frac{I_g}{I_m} = \frac{S_1}{S_1+G_1} = \frac{.1R}{.1R+(G+.9R)} = \frac{1}{10}\left(\frac{R}{R+G}\right).$$

This ratio is $\frac{1}{10}$ of that when the arm was at A.

This shunt works especially well on ballistic galvanometers, and is in common use in capacity tests.

229. Sensibility of Galvanometers. Working Constant. As there are several uses to which galvanometers are put, so there are several ways of stating their sensibility.

If the galvanometer is to measure current or quantity of electricity, the sensibility is rated as the fractional part of an ampere or coulomb that will produce a deflection of one scale division. Thus it may be stated of a certain galvanometer that its sensibility is .000004 ampere. By

this would be meant that .000004 of an ampere would cause a deflection of one scale division. A galvanometer of a sensibility of .0000004 ampere would be 10 times as sensitive as the first instrument because it would require but $\frac{1}{10}$ as much current to cause the same deflection. The *smaller* the current to produce a deflection of one scale division, the *greater* the sensibility.

Again, if the galvanometer is to be used to measure insulation or high resistance, as described in Chapter V, the sensibility may be expressed as the number of megohms which must be connected in series with it across one volt pressure to produce a deflection of one scale division.

Thus a galvanometer of 8000 megohms sensibility, means a galvanometer which will give a deflection of one scale division when 8000 megohms resistance are in series with it across one volt. If another instrument has a sensibility of 16,000 megohms, it would be twice as sensitive as the first. The *greater* the number of megohms through which one volt pressure can produce a deflection of one scale division, the greater the sensibility.

In *purchasing* a galvanometer, much is learned about its characteristics from a statement of its *sensibility.*

In *using* a galvanometer, however, it is of greater importance to know the WORKING CONSTANT of the instrument. The term "working constant" refers merely to a local "set-up" of the instrument and not to standard conditions as in the case of the term "sensibility." For instance, the sensibility of a galvanometer may be 1000 megohms, while its working constant for a certain set-up may be 100,000 megohms. That is, the galvanometer may be on a 100-volt line and give a deflection of one scale division when 100,000 megohms are in series with it. Knowing the "working constant" it is possible to compute the resistance that is in series with a galvanometer at any given time, by noting the deflection. Thus if the above galvanometer gave a deflection of 4, it would indicate

that only $\frac{100{,}000}{4}$ or 25,000 megohms must be in series with it at that time. For a complete discussion of the method for finding the working constant of a galvanometer as applied to insulation measurement, see Chapter V.

230. Ballistic Galvanometer. A current is often sent through a galvanometer which flows for an exceedingly short time only. This is the case with many induced currents and condenser discharges. Since the current in such cases does not flow long enough to be measured, a galvanometer has been devised, the deflections of which are proportional to the *quantity* of electricity, or the *charge*, passing through the coil. These instruments are said to "throw" rather than "deflect," and for this reason are called BALLISTIC GALVANOMETERS. They are made with a heavier moving coil and can be damped magnetically, without destroying the ratio between the "charge" and "throw." Such an instrument is used in capacity measurements as follows.

A condenser of known capacity is discharged through the ballistic galvanometer and the "throw" noted. Then a source of unknown capacity, charged to same voltage, is discharged through the galvanometer and the throw noted. The capacity of the two sources are to each other as their respective galvanometer throws. See Chapter XI.

231. Thermo-Electric Effects. Before going further into the structural details of electrical measuring instruments, it is necessary to consider briefly some phenomena which may affect the choice of materials employed.

It has been discovered that if two metals are employed in constructing an electric circuit and one juncture of the metals is heated to a higher temperature than the other juncture, that an electric pressure is set up tending to cause a current to flow. The metals may be welded, soldered, or merely held together by mechanical pressure; the E.M.F. depends entirely upon the materials selected

and the difference in temperature of the two junctures, varying greatly with the different combinations of materials and being almost directly proportional to the difference in temperature of the joints. Consider the electric circuit in Fig. 277; BA is a bismuth bar, joined at A to AC, a bar of antimony; two metals often chosen because of the high value of the thermal E.M.F. per degree difference of temperature of junctures. When heated at point A, a current will flow in the direction BAC, because that juncture is at a higher temperature than the other ends B and C. If, however, the points B and C (really the other juncture) were heated, the current would flow in the opposite direction. Such an arrangement is called a THERMO-COUPLE. The

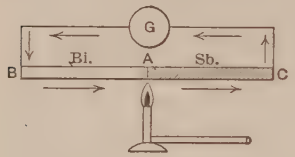


FIG. 277.—A thermo-couple.

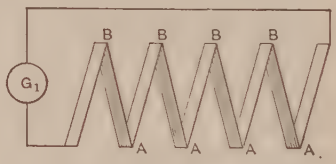


FIG. 278.—A thermo-pile.

current will continue to flow as long as one juncture is maintained at a higher temperature than the other.

232. Thermo-Bolometer. Pyrometer. No successful commercial use has yet been made of this thermal E.M.F. for supplying electric power, though enough current can be generated by means of several joints in series, alternate ones of which are heated by a gas flame, to supply electro-plating vats. The materials used in such a device in a short time seem to undergo a molecular change in their structure which greatly diminishes their thermo-electric power.

The thermo-electric E.M.F. of a pair of bismuth and antimony bars is only about .1 millivolt for every degree centigrade that one juncture is above the other. Thermo-piles therefore have to be built up of many such pairs in series, as in Fig. 278, and one set of junctures must be

maintained at a temperature considerably above the other set, in order to procure a usable amount of pressure.

Use is made of this thermal E.M.F. in making instruments to measure very minute differences in temperature, and also very high or very low temperatures.

A thermo-pile such as that in Fig. 278 may be used as a sensitive instrument for detecting small differences of temperature. If the temperature of the joints A is but a small fraction of a degree above or below that of the

FIG. 279.—The Bristol pyrometer for measuring high temperatures.

joints B, an E.M.F. will be set up which can be detected by the deflections of a sensitive galvanometer, G_1. Since the E.M.F. set up is nearly proportional to the difference in temperature of the joints, the deflections of the galvanometer can be made to measure this difference in temperature.

By inserting in the moving coil of a sensitive D'Arsonval galvanometer, a bismuth-antimony thermo-couple, as small a rise or fall in temperature as one-hundred-millionth of a degree C. can be measured. Such an instrument is called a BOLOMETER.

For measuring high temperatures a thermo-couple of platinum and rhodium is made in such a form that one

end, A, Fig. 279, may be inserted into furnaces of molten metals, etc., and be raised to the temperature of the furnace or metal, while the other end remains at room temperature. The thermal E.M.F. thus set up causes a deflection in a millivoltmeter calibrated to read temperature of end A of thermo-couple.

Figs. 279 and 280 are representations of the Bristol Pyrometer, which employs special alloys for the thermo-couple. Note also the device for compensating for any

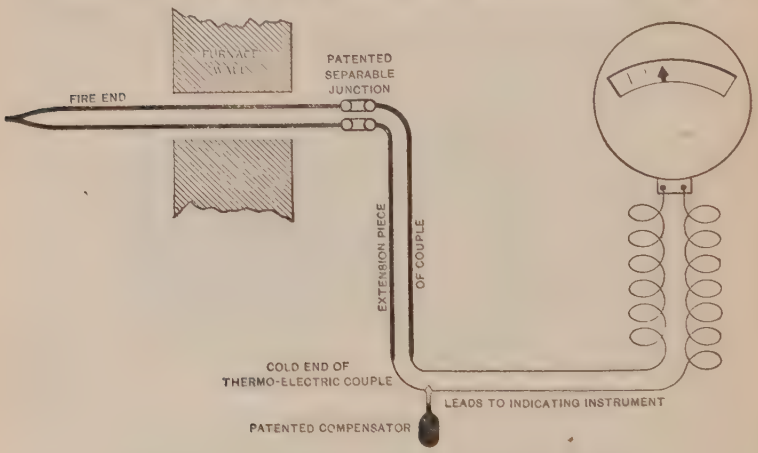


FIG. 280.—The electrical connections of a Bristol pyrometer.

change in temperature which may take place at cold end of couple.

233. Peltier and Thomson Effects. It has also been discovered that the thermo-electric effect is reversible. That is, if we send a current through a thermo-couple, it will either heat or cool the joint, according to the direction of the current. This is called the PELTIER effect and must be carefully distinguished from the I^2R effect, which never *cools* a wire, nor is it reversible.

Sir William Thomson (Lord Kelvin) discovered that in the case of most conductors composed of pure metals, if

one part were raised to a higher temperature than another part, a thermal E.M.F. would be set up between these two points. This effect is also reversible.

In consequence of these discoveries of the thermo-electric effects of different metals and combinations of metals, it is seen that too great care cannot be exercised in the selection of proper materials to go into the construction of accurate electrical measuring instruments. It would not do to have within the instrument itself a source of E.M.F. which would affect its indications, if the temperature of parts of it were changed from any cause.

Ammeters

234. Resistance of Ammeters. Because ammeters are placed in the line and the whole current must be forced through them, the resistance of these instruments must be as low as possible. For instance, the resistance of a Weston D.C. ammeter of 10 amperes capacity is less than .005 ohm.

235. Type of Ammeters. Solenoidal. The simple arrangement shown in Fig. 281 is often used as an ammeter when a cheap instrument for rough measurement is desired. The current flowing through the low resistance coil, *C*, sucks the soft iron plunger *A*, pivoted at *P*, up into the coil. This causes the pointer to move over a scale which is calibrated by sending known currents through coil *C*. The deflections are nearly proportional to the square of the current, since the deflecting force depends upon the product of the strength of the field times the strength of the induced magnetism, both of which are nearly proportional to the current in the coils. The control is effected by means of the weight *W*, called a "gravity control"; damping

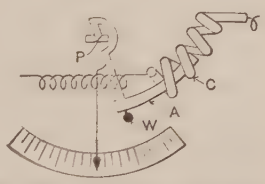


Fig. 281.—Ammeter. Solenoidal.

by means of the eddy currents in the plunger *A*. Neither the damping nor the control is good, due to the comparatively large mass of the moving parts, the large moment of inertia, and the friction on the pivot.

This type can be used equally well on A.C. circuits, if calibrated for that purpose. By making the coil of many turns of fine wire the same design can be used as a voltmeter.

This instrument, with some refinements, is often used on switchboard for the measurement of heavy currents.

Another form of the solenoidal type is shown in Figs. 282 and 283.

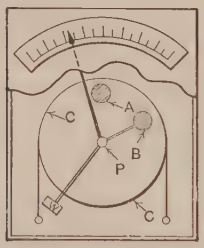


FIG. 282.—Ammeter. Another form of solenoidal type.

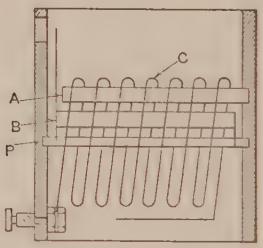


FIG. 283.—Side view of ammeter in Fig. 282.

A and *B* are soft iron rods. *A* is stationary, *B* is pivoted at *P*. When a current is sent through the coil, the rods become magnetized, so that like ends are nearest to one another, and thus *B* is repelled by *A*. This causes a pointer to move over a scale calibrated as in above plunger form of the type. It also has a gravity control and possesses but slight advantage over the model first described. It can be used on A.C. circuit, and changed into a voltmeter by changing coil *C*. The Weston Electrical Instrument Co. are now manufacturing A.C. voltmeters and ammeters in which the above principle is made use of, though in a greatly modified form. Fig. 284 shows one of these instruments. The magnetized iron parts corresponding to the

rods A and B, Figs. 282 and 283, are not in the form of rods, and springs take the place of gravity control.

236. Thomson Inclined Coil Ammeter. The moving element in this meter, shown in Fig. 285, is a light iron vane set at an angle to the shaft to which it is attached. The shaft itself is set at an angle to the axis of the stationary coil. When a current is sent through the coil, the vane tends to swing into such a position that the lines of force passing through it shall be parallel to the lines within the coil. This causes the needle to deflect across a scale calibrated as in previous models. The deflecting force is nearly proportional to the current squared. The control is by means of springs and is very delicate. Damping is effected by air vanes. This instrument is one of the most accurate of those of the solenoidal type and can be used on either D.C. or A.C. circuits.

FIG. 284.—Weston portable A. C. voltmeter.

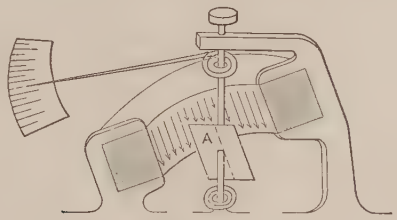


FIG. 285.—Thomson inclined coil meter.

237. Hot-Wire Ammeters. When an electric current passes through a wire, heat is generated. If this heat is allowed to raise the temperature of the wire, expansion will take place. Hot-wire ammeters make use of this expansion of wire by the heat generated, to measure the electric current which generates the heat.

When a current is sent through wire AB, Fig. 286, the heat causes it to sag. This sag is taken up in the arrangement CF. The part CD is a wire, DE a silk thread wound around the pulley P; EF is a spring. As the thread moves around the pulley it causes it to turn, and the pointer attached moves over the scale. The deflections here are almost proportional to the square of the current, since the heating effect is proportional to the current squared, so the scale must be calibrated as in solenoidal instruments. The control is effected by means of the elasticity of the wire AB and spring S. The instrument, however, requires frequent resetting to zero, is very slow in action and unless special attachments are used, is affected by changes in temperature of the room. It measures both A.C. and D.C. currents, and can be used with shunts to increase its range. It can also be used as a voltmeter by placing a high resistance in series with it. Instruments of this type have come into use again in the measurement of alternating currents of the high frequencies used in wireless telegraphy.

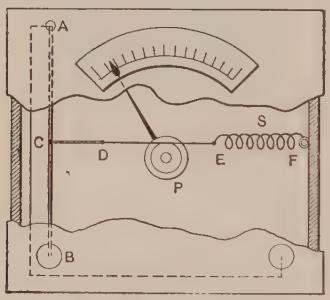


FIG. 286.—Hot-wire meter.

238. Two-Coil Ammeters. Electro Dynamometers. Fig. 287 shows the general appearance of Siemens' Electrodynamometer. Fig. 288 shows the arrangement of the parts of the instrument.

The current, after flowing through the stationary coil C, enters the moving coil D, by means of the mercury cups M. The action of the magnetic field of the stationary coil on the current in the moving coil tends to rotate the moving coil against the spring R. The pointer P thus moves against one of the stops S. If now the indicator I be moved in the

opposite direction until the pointer P floats again, then the deflecting force is measured by the amount the spring has been deflected, in order to restore the moving coil to its zero position. The indicator I shows this amount on the graduated disk T. The deflecting force is proportional

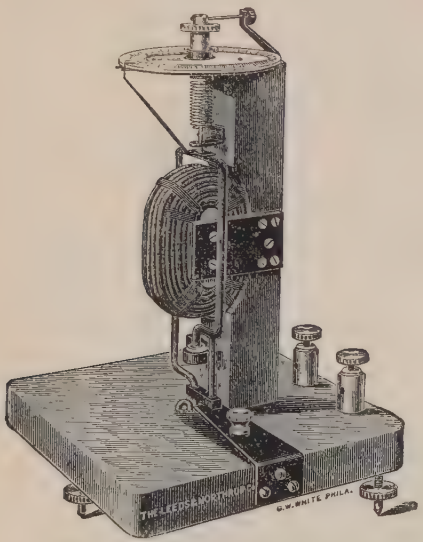


Fig. 287.—Siemens' dynamometer.

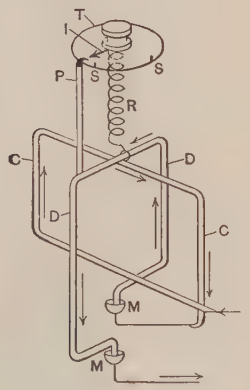


Fig. 288.—Construction of dynamometer.

to the product of the current in the moving coil times the current in the stationary coil. Since these two coils are in series, the same current flows through each, and the deflecting force is therefore proportional to the square of the current. The scale divisions are, therefore, not uniform for a direct reading instrument. This instrument can be used on A.C. circuits; since the current reverses in both coils at the same time, and the relative direction always remains the same. Its indications of A.C. values are correct, even though the instrument has been calibrated on direct current. It can be made extremely sensitive

and accurate. It requires considerable skill, however, to manipulate it; accordingly its use is restricted to laboratories. By making the coils of sufficiently high resistance, this type may also be used as a voltmeter with even better results than as an ammeter.

Fig. 289 shows such an arrangement used as a wattmeter. Note that the scale divisions are uniform.

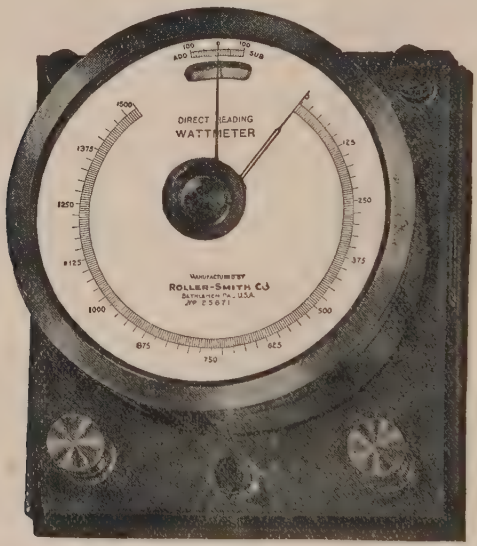


Fig. 289.—Wattmeter constructed on dynamometer principle.

239. Permanent Magnet. Weston Type. The ammeters most used in this country for measuring direct currents are of the permanent magnet and moving coil type. The Weston ammeters are excellent examples of this form. They are merely a portable form of a D'Arsonval galvanometer. The moving coil, however, instead of being supported by a delicate suspension wire, is as accurately fitted into jeweled bearings as the flywheel of a watch. The control is effected by means of two springs which act against each other. This is much more certain than the torsion of a

long suspension wire and less liable to injury. For view of moving element see Figs. 29, 290 and 291.

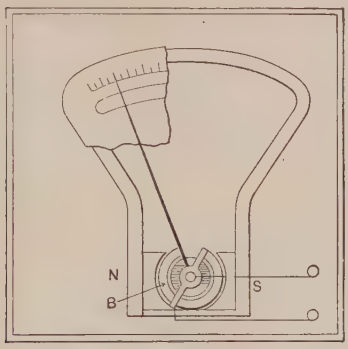


FIG. 290.—Construction of Weston D.C. meters. Permanent magnet type.

As the fundamental principles of this instrument have been explained in Chapter II, only the finer points of construction will be taken up here. Unlike the ammeters

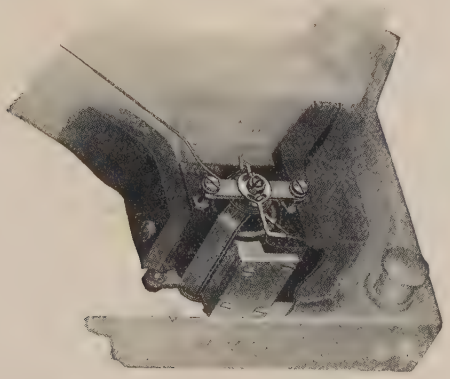


FIG. 291.—Cut-away view of moving parts of Weston meter.

previously described, the scale divisions of the Weston ammeters are uniform throughout. This is brought about by producing an absolutely uniform field, in which the

coil swings. A soft iron core is placed between the poles N and S of the permanent magnet. This concentrates and bends the magnetic lines in such a way as to make them radial in direction (at right angles to the arc in which the coil moves), as in Fig. 292. In whatever position the coil may be, there are always acting upon it the same number of lines at right angles to its motion. The deflections are therefore exactly proportional to the current flowing in the coil. The instrument is made dead beat by the damping effect of eddy currents set up during any motion of the coil in the aluminum bobbin on which the moving coil is wound. See page 202, Chapter VIII.

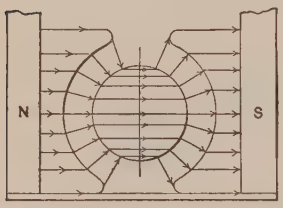


FIG. 292.—Showing uniform radial field of permanent magnet type.

We have seen that these instruments are merely D'Arsonval galvanometers; they could therefore be either ammeters or voltmeters according as the resistance of the moving coil is low or high. As a matter of fact, they are really all MILLIVOLTMETERS. That is, a few thousandths of a volt are sufficient to cause a maximum deflection of the needle. In fact, 45 millivolts or .045 volt across the moving coil gives a full scale reading on the more common type. Furthermore, the wire composing the moving coil is so fine that it can carry but a very small fraction of an ampere, never more than .05 ampere.

In order, therefore, to make an ammeter out of this device, which shall carry and indicate a moderately large current, a shunt must be placed across the coil as in Fig. 293. The instrument now becomes a shunted galvanometer, and a small fraction only of the main current flows through the moving coil. Thus, according to Ohm's Law, if the shunt had .01 ohm resistance and the coil gave its maximum deflection, it would mean that there was .045 volt across

a .01 ohm resistance and that $\frac{.045}{.01} = 4.5$ amperes must be flowing through the shunt. Of course the instrument

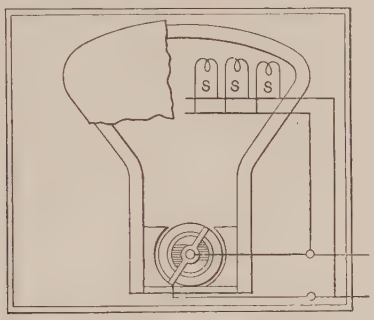


FIG. 293.—Ammeter with shunt, *SSS*, inclosed.

is calibrated so that the scale reads not millivolts, but the amount of current flowing through the instrument, both moving coil and shunt.

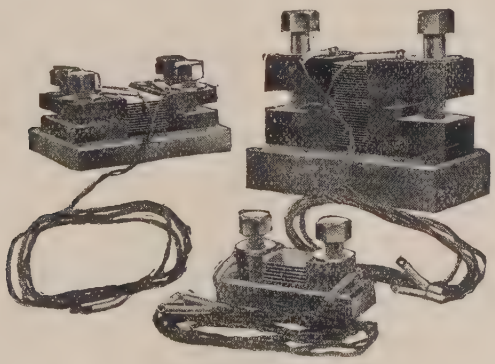


FIG. 294.—Separate shunts, for ammeters.

This shunt must be of such a material or combination of materials that it does not appreciably change in resistance as its temperature changes. In other words, it must have a low Temperature Coefficient of Resistance. It also must

have low Thermoelectric power where soldered to copper. Such a substance is the alloy MANGANIN, invented by Dr. Edward Weston. It consists of copper, nickel and ferro-manganese. Several alloys have nearly as low a temperature coefficient, but do not combine with this a low thermoelectric power. The result on the deflections of a millivoltmeter of using a metal having a high thermoelectric power can readily be imagined. The invention of this alloy for use in shunts, etc., is one of the causes of the success of the Weston instruments.

When currents larger than 25 amperes are to be measured, the shunt S is not placed inside the ammeter case on account of the large amount of heat energy which must be dissipated. Separate shunts are made as shown in Fig. 294. An instrument and connecting leads are calibrated to be used with the shunt as shown in Fig. 295, and the deflections indicate directly the amperes flowing through the line in which the shunt is placed. The shunt and millivoltmeter together constitute an ammeter.

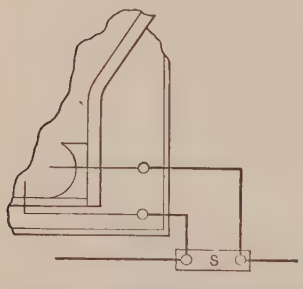


FIG. 295.—Method of using separate shunt.

Example. What resistance must a shunt S be in order that it may be used with a .0450 volt millivoltmeter to measure 100 amperes? Resistance of moving coil and connecting leads is 1.000 ohm.

$$\text{Maximum current through coil} = \frac{.04500}{1.000} = .045 \text{ ampere.}$$

$$\text{“ “ shunt} = 100 - .045 = 99.955 \text{ amperes.}$$

Resistance of shunt must be, by Ohm's Law,

$$\frac{.045}{99.955} = .0004502 \text{ ohm.}$$

Problem 5–14. What resistance must a shunt be, in order that it may be used with a millivoltmeter of 50 millivolts range, to measure 5 amperes? Resistance of moving coil and leads = 1.025 ohms.

Problem 6–14. If leads were used which increased the "coil and lead" resistance to 1.125 ohms, what error would be introduced at full scale reading in Problem 5–14, using same shunt?

Problem 7–14. Assume a copper wire is used for the shunt in Problem 5–14, and the temperature of it rises 10° C. during a reading. What error is introduced at full scale reading?

Problem 8–14. What would the true reading be when instrument in Problem 7–14 read 2 amperes?

240. Multiplier. The millivoltmeter described above can be used for the measurement of voltage, by placing a high resistance in series with the moving coil. This series resistance is called a MULTIPLER. Whenever the instrument is to be used entirely as a voltmeter of given range, the multiplier is placed inside the case, and the instrument is called a voltmeter.

Suppose the millivoltmeter of example, page 410, having a resistance of 1 ohm, were to be used as a voltmeter of 150 volt range. What resistance must be placed in series with the coil? Assume .0075 ampere to pass through meter at maximum reading.

By Ohm's law. Resistance of coil and series resistance = $\frac{150}{.0075}$ = 20000 ohms.

Resistance of series resistance = 20,000 − 1 = 19,999 ohms. Weston voltmeters of the common type have approximately 100 ohms to the volt (maximum scale reading).

Problem 9–14. The coil of a Weston millivoltmeter of 50 millivolts capacity has 5 ohms resistance. What resistance multiplier must be placed in series with it in order that it may have a maximum capacity of 150 volts?

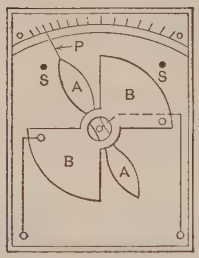
Problem 10–14. What resistance shunt must be used with millivoltmeter in Problem 9–14, in order that it may read 150 amperes? Maximum current through coil should be .01 ampere.

Problem 11–14. A voltmeter has a range of 100 volts. Resistance is 10,000 ohms. What resistance must a multiplier be, in order that the voltmeter may have a range of 300 volts?

Problem 12–14. Assume multiplier in Problem 11–14 to be made of pure copper. What error is introduced by a change of temperature of 8° C. if instrument is reading 250 volts?

Problem 13–14. If multiplier in Problem 11–14 were made of German silver, what error is introduced by a change of 8° C., when instrument reads 250 volts? Temperature coefficient of resistance for German silver is .00025.

241. Electrostatic Voltmeter. One of the disadvantages common to all types of voltmeters described above, is that they all take power to operate them. Fig. 296 shows an example of the electrostatic type of voltmeter which takes no power from line. It is based on the principle that unlike electrostatic charges of electricity attract each other. The stationary vanes *BB* are connected to one side of the line and the movable vanes *AA* to the other side. The vanes *AA* are now attracted to the vanes *BB* in direct proportion to the charge on them. Since the charge upon each depends upon the voltage between them, the deflecting force is proportional to the square of the voltage. Of course as the vanes *AA* approach nearer the vanes *BB* the attraction becomes greater because of the decreased distance between them. The scale divisions therefore are not uniform. A special static voltmeter has been constructed in which the moving vanes float in oil. In this instrument the oil acts as a damper and a dielectric. It can be used on higher voltages because of the greater Dielectric Strength, i.e., insulating strength, of the oil, and is more sensitive because of the greater Dielectric Power of the oil. Electrostatic voltmeters of course can be used equally well on D.C. and A.C. circuits. They are more often used in laboratories than on commercial switchboards.



SS, Stops for Pointer.P.

Fig. 296.—Construction of electrostatic voltmeter.

242. Best Arrangements of Voltmeters and Ammeters to Measure Power, etc. Since all ammeters and all voltmeters (except the electrostatic) consume power when in use, and thus introduce some error, it is well to consider the ways in which they may be arranged to make this error as small as possible.

Suppose it is desired to measure the voltage and amperage of an ordinary incandescent lamp. Assume the connections are made as in Fig. 297. *A* is a Weston milliammeter, resistance .045 ohm. *V* is a Weston voltmeter, resistance 15,000 ohms. Assume both instruments to be correctly calibrated. Suppose the voltmeter reads 110 volts and the ammeter .500 ampere. The voltmeter would read the voltage across the lamp correctly because it is directly across the lamp *L*. But the ammeter measures the current flowing through both the lamp and the voltmeter. It must, therefore, read too high. Let us see how much too high. The current through the voltmeter, by Ohm's law, equals,

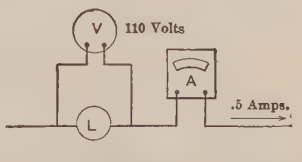


FIG. 297.—Ammeter measures current in lamp plus current in voltmeter.

$$\frac{110}{15{,}000} = 0.0073 \text{ ampere.}$$

The current through the lamp *L* equals,

$$.500 - .0073 = .493 \text{ ampere.}$$

Per cent error,

$$\frac{.0073}{.493} = 1\tfrac{1}{2} \text{ per cent.}$$

The error of $1\frac{1}{2}$ per cent is altogether too large for any such simple measurement.

Suppose we connect up the same instruments as in Fig. 298, with the voltmeter around both the lamp and ammeter. Assume the same readings as before.

The ammeter now reads the current of the lamp L only, and is therefore correct. But the voltmeter reads the voltage across both the lamp and ammeter and therefore reads too high. Let us see how much too high.

Voltage across ammeter,

$$.500 \times .045 = .0225 \text{ volt.}$$

Voltage across lamp,

$$110 - .0225 = 109.98 \text{ volts.}$$

Per cent error,

$$\frac{.0225}{109.98} = .0002 \text{ or } \tfrac{1}{50} \text{ of 1 per cent.}$$

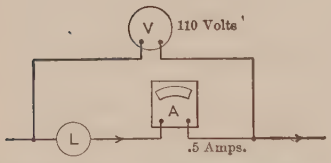


FIG. 298.—Voltmeter measures drop across lamp plus drop across ammeter.

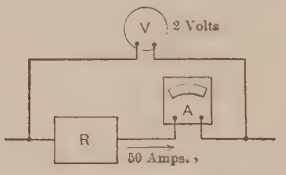


FIG. 299.—Voltmeter measures drop across ammeter and resistance R.

This error is allowable in any grade of commercial work and is too small to be considered. It is evident that when measuring a low current and high voltage, the voltmeter should be placed around both the ammeter and the apparatus under test. This is because the voltage across the ammeter is too small to appreciably affect the reading of the high reading voltmeter.

Suppose, however, that we wish to measure the voltage and amperage in a low resistance armature circuit R.

Assume that they are connected as in Fig. 299, which arrangement, we found, had the smaller error in the previous case.

V is a 3-volt Weston voltmeter of 300 ohms resistance; A a Weston ammeter of .0009 ohm resistance. Assume the voltmeter reads 2.00 volts and the ammeter 50.0 amperes.

The ammeter will read correctly the current through R, but the voltmeter will read the voltage across both R and the ammeter. It will thus read too high. This error can be computed as before.

Voltage across ammeter,

$$.0009 \times 50.0 = .0450 \text{ volt.}$$

Voltage across R,

$$2.00 - .0450 = 1.955 \text{ volts.}$$

Per cent error,

$$\frac{.0450}{1.955} = 2.3 \text{ per cent.}$$

This error is too high for most purposes.

Suppose we arrange the instruments as in Fig. 300, and assume the readings to be as before. The voltmeter now reads correctly, but the ammeter reads both the current through R and the voltmeter. The ammeter reading is then too high. The error is found as before.

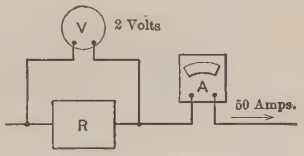


FIG. 300.—Ammeter measures current through resistance R and voltmeter.

Current through voltmeter,

$$\frac{2.00}{300} = .00667 \text{ ampere;}$$

Current through R,

$$50.0 - .00667 = 49.993 \text{ amperes.}$$

Per cent error,

$$\frac{0.00667}{49.93} = .00013 \text{ or about } \tfrac{1}{100} \text{ of 1 per cent.}$$

This error is so small that it is allowable in all commercial work. It is evident that when measuring the power consumed by a piece of apparatus, through which a large cur-

rent at low voltage is flowing, the voltmeter should be placed immediately across the piece of apparatus under test, and not across the ammeter also. The reason for this is that too small amount of current flows through the voltmeter to appreciably affect the reading of the large current ammeter.

243. Summary Concerning Voltmeters and Ammeters.

(1) Solenoidal Instruments (containing soft iron).
- (*a*) Are simple and cheap.
- (*b*) Can be used on both A.C. and D.C. circuits, though must be calibrated for each separately.
- (*c*) Are not particularly accurate.

(2) Hot Wire.
- (*a*) Are simple and cheap.
- (*b*) Can be used interchangeably on both A.C. and D.C. circuits.
- (*c*) Slow acting and "dead beat."
- (*d*) Uncertain control, but fairly accurate.

(3) Electro-Dynamometers.
- (*a*) Are simple.
- (*b*) Not easily manipulated.
- (*c*) Accurate.
- (*d*) Can be used interchangeably on A.C. and D.C.

(4) Permanent Magnet (moving coil).
- (*a*) Extremely accurate.
- (*b*) Dead beat.
- (*c*) Scale uniform.
- (*d*) Can be used on D.C. circuits only.
- (*e*) Positive control.

(5) Electrostatic.
- (*a*) Voltmeter only.
- (*b*) Can be used interchangeably on A.C. and D.C.
- (*c*) Not particularly accurate.
- (*d*) Takes no power.
- (*e*) Best instrument available for high voltage measurement.

244. Indicating Wattmeter. Electro-Dynamometer. In discussing the electro-dynamometer we have seen that the deflecting force is proportional to the product of the current in the stationary coil times the current in the moving coil. When used therefore as an ammeter or a voltmeter the current in each coil would be the same and the deflections proportional to the square of the current in either.

If, however, we wish to measure the power (watts) consumed in a given circuit we may send the main current

FIG. 301.—Weston indicating wattmeter Note heavy terminals for current leads.

through the stationary coil, which we would make of low resistance, and have the current in the moving coil, which would be of high resistance, proportional to the voltage. That is, we would put the stationary coil in series in the line as an ammeter and the moving coil across the line as a voltmeter. The deflections would then be proportional to the product of current times the voltage, or the watts, and the instrument would be a WATTMETER.

The Weston wattmeter, constructed on the above plan, is shown in Fig. 301. The construction of the instrument is like that of their permanent magnet ammeters and voltmeters. The main difference is, that the stationary

coils *cc*, Fig. 302, of this instrument take the place of the permanent magnets in the other.

Suppose the power consumed by the arc lamp L were to be measured. The stationary coils *cc* of low resistance are placed in series with the arc, and a magnetic field is set up proportional to the current flowing through the arc. The moving coil, with a high resistance in series, is placed across the arc like the moving coil of a voltmeter. The current through this coil is proportional to the voltage across the arc. The deflections then are proportional to the product of the current through the arc by the voltage across the arc, or the watts. The scale is graduated to read directly in kilowatts.

245. Compensation in a Wattmeter. By referring to the diagram of Fig. 302, it will be seen that the wattmeter will read too high when connected as shown, because the current flowing in the stationary coils is not only that through the arc, but also the current taken by the moving coil. Thus the current is too large. A correction can easily be made for this by subtracting from the reading the power consumed by the moving coil. This value should be found as follows:

If R_m be the resistance of moving coil and E the voltage across the lamp, and therefore also across the coil, the power consumed by the coil would be $\frac{E^2}{R_m}$.

If, however, we should connect the voltage coil of the wattmeter across both the lamp and the current coils, then the current flowing through the stationary coils would be that of the lamp only. But the voltage across the moving coil would be the voltage across both the lamp and the stationary coils. The wattmeter would still read too high because this voltage is higher than the lamp voltage. The power measured would be the power consumed by the lamp plus the power consumed by the stationary coils. The correction can be made by sub-

tracting from the reading the power consumed by the stationary coils. This correction is found as follows:

Let I be current through stationary coils, and R_s be resistance of stationary coils. Then power consumed by stationary coils $= I^2R_s$.

In using the Weston wattmeter neither of these corrections has to be made, because of a special compensating device shown in Fig. 303. The voltmeter, or moving coil, terminals are connected as in Fig. 302, but a compensating coil, M, is placed in series with the moving coil. The field of this coil opposes the field of the stationary coils cc, and weakens

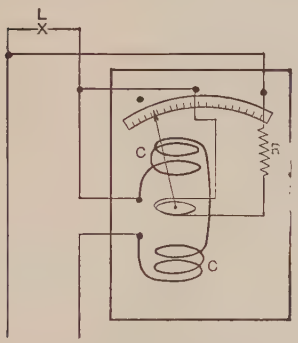


Fig. 302.—Diagram of wattmeter without compensating coil.

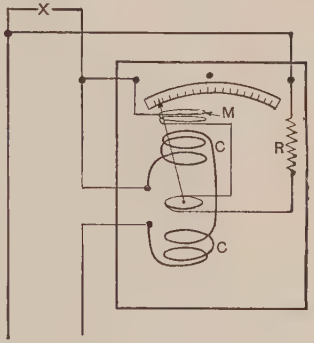


Fig. 303.—Wattmeter equipped with compensating coil.

it by an amount proportional to the current in the moving coil. The field then is exactly proportional to the current flowing through the lamp alone, and the instrument now indicates the watts consumed by the lamp alone.

In some cases it is desirable not to use this compensating coil, but to apply corrections computed as above. For this reason Weston wattmeters have an extra binding post, marked "Ind" (for Independent), which cuts out this coil. In almost all commercial tests, however, the regular connections are used, which include the compensating coil.

Of course this wattmeter can be used on A.C. as well as D.C. circuits, one calibration doing for both.

246. Thomson Integrating Wattmeter or Watthour Meter. It has been stated in Chapter IV that electric energy is paid for by the kilowatt-hour; one kilowatt-hour being the quantity of electric energy consumed when power is consumed for one hour at the rate of one kilowatt. An interesting instrument called an integrating wattmeter

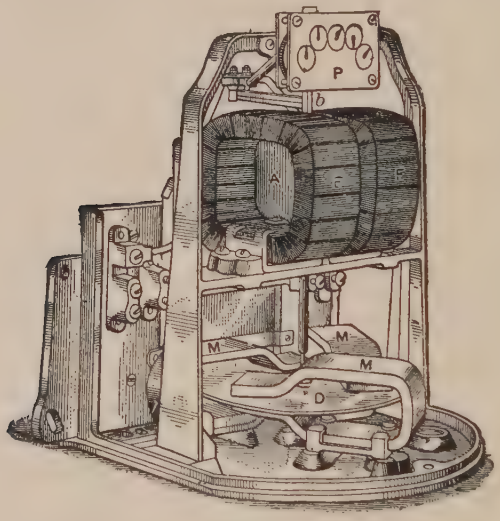


FIG. 304.—Thomson watthour meter.

(since it adds up the work done at all instants) is used to measure electric energy.

An illustration of this instrument is shown in Fig. 304. It is primarily a small shunt motor, the armature of which revolves at a speed proportional to the rate at which electric energy is passing through it. The armature is geared to recording dials, arranged like the dials on a gas meter, which register the number of kilowatt-hours of energy which have passed through the meter.

The field of this instrument is made by current in the stationary coils *FF*, Fig. 304, which are in series with the line. There being no iron in the field, the field strength is proportional to the current flowing in the main line. The armature *A* is connected, by means of a commutator, *b*, across the line, so that the current in the armature is proportional to the voltage across the line.

The torque, then, must be proportional to the product of the voltage times the current, or the watts in the line.

The friction is reduced to a minimum by means of jeweled bearings. Since there is no iron in the magnetic circuit, the field is weak, and therefore no counter E.M.F. of any appreciable value is set up by the rotation of the armature. Under these conditions the armature would race if there were no retarding force applied to the armature.

This retarding force must be proportional to the speed so that it will decrease as the speed decreases. Such a force is secured by attaching an aluminum disk, *D*, to the armature, and causing it to rotate between the poles of permanent magnets *M*, as the armature rotates. This motion sets up eddy currents in the aluminum disk which are proportional to the speed of the disk. The retarding torque of these currents must then be proportional to the speed of the disk, as explained in Chapter VIII.

Thus the speed of the armature will at all times be proportional to the torque. To illustrate this, suppose we assume the driving torque to be increased suddenly; the speed would naturally rise, but as the speed rises the opposing torque due to the increased eddy currents also rises until it is equal to the driving torque and the armature speed becomes constant. The driving torque and the retarding torque are thus always equal.

The above discussion is correct only when we consider the armature to rotate without any friction. Since there is always some friction present, some arrangement must be made to compensate for it. This is done as follows: A

small coil *C*, Fig. 304*a*, is put in series with the armature and placed so as to strengthen the field due to the coils *FF*. The position of this coil is adjustable so that it can

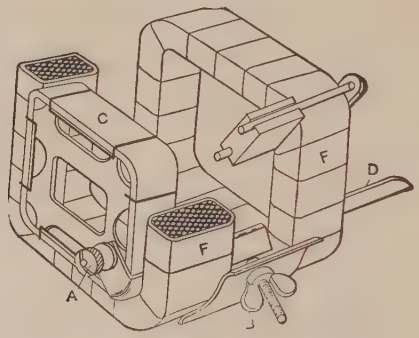


Fig. 304*a*.—Adjustable coil for compensating for friction.

be moved toward, or away from, the armature as the need may be. It is placed at a distance such that the field due to it is not strong enough to cause the armature to rotate when no current is flowing through the main field coils *FF*, but still the lightest current through these coils will be sufficient to start the motor. This compensating coil, then, just overcomes the friction of the armature.

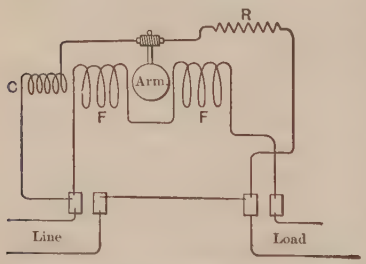


Fig. 305.—Watt-hour meter connected to line to measure power consumed by set of lamps.

If the compensating coil is too near the main coils, and the meter runs when no current is being used, it is said to "creep." This causes the dials to register more energy than was used by the consumer. Fig. 305 shows how a watthour meter is connected to the circuit in order to measure the power consumed by a set of lamps.

The question is often asked: If this meter is a shunt motor, why is it that the armature speed *increases* as the field *increases*, when it is a well-known fact that the armature speed of a shunt motor *decreases* as the field *increases*? The answer to this question is simple, when we consider the reason why a shunt motor decreases in speed as the field strength increases. The increase in the field strength of a shunt motor is *not* the direct cause of the decrease in speed. The large decrease in the *armature current*, due to the increased counter E.M.F., is the reason for this falling off in speed. In the case of the watthour meter, there is no counter E.M.F. of any appreciable value as compared with the impressed E.M.F. Thus any increase in the counter E.M.F. does not appreciably decrease the *armature current*. An increase in the field strength, therefore, accompanied by no decrease in the armature current, causes a larger torque, and, therefore, increased speed.

247. Voltameter. We have seen that a current of one ampere flowing for an hour deposits a given amount of metal from a given solution. Wherever the voltage is constant, use can be made of this fact to measure the total amount of energy consumed in a given time. From the amount of metal deposited, the number of ampere-hours may be computed. The number of watt-hours is then found by multiplying by the known constant voltage.

The apparatus required consists of a jar of electrolyte and two plates. The " gain " or negative plate is weighed before and after the run. The gain in weight divided by the electrochemical equivalent gives the number of ampere-hours. The area of the gain plate in a copper voltameter should be about 4 sq.in. to the ampere.

248. Potentiometer. For very precise measurement of voltage, the potentiometer is the instrument most used. The principle on which it operates may be seen from an inspection of Fig. 306.

Two sources of E.M.F. cell I and cell II are joined in parallel and are connected to the slide wire AB. Cell I must be of higher E.M.F. than cell II, and be able to deliver a steady current. In series with cell I is the variable resistance R. Cell II is a standard cell, generally a Weston, and must be allowed to deliver no appreciable current.

In series with standard cell II is a galvanometer G and a contact maker C. If contact maker C is not touching the wire AB, then cell I delivers a current through AB from A to B. Cell II not having a closed circuit delivers no current.

If we place a weak cell like cell II directly across the terminals of a strong cell like cell I, the stronger cell will force a current through the weak cell in the reverse direction to the E.M.F. of the weak cell.

Suppose contact maker C touches the wire AB at such a point that the IR drop along AB is *greater* than the E.M.F.

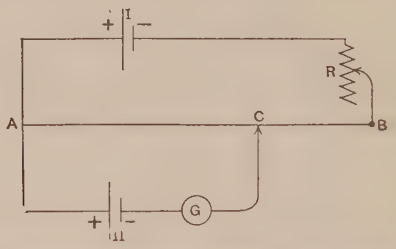


Fig. 306.—Construction of potentiometer for measuring low voltages.

of cell II. The two cells, being in parallel, the stronger cell I will force a current in the reverse direction through cell II. For while cell II is not directly across cell I, still it is across enough of the voltage of cell I to have its weak E.M.F. overpowered. The galvanometer G will then deflect in a certain direction. Cell I is then delivering two parallel currents, one from A to B, through the slide wire, and the other from A to C, through cell II, against the E.M.F. of cell II.

On the other hand, suppose we touch C to AB at such a point that the IR drop from A to C is less than the E.M.F. of cell II. Then cell II aids cell I in sending current from A to C, and raises the IR drop by increasing the current I. Current will now flow out of cell II, and the galvanometer will be deflected in the opposite direction.

Suppose, however, that we touch C to the wire AB at such a place that the IR drop along AB will be exactly *equal* to the E.M.F. of cell II. The force tending to send a current in the reverse direction through cell II will be exactly counterbalanced by the E.M.F. of the cell, and no current will flow in either direction through cell II. The galvanometer will then show no deflection and the potentiometer is said to be "balanced."

As the value of the E.M.F. of the cell II is known accurately, the value of the IR drop from A to C is known accurately. A cell of unknown E.M.F. can now be put in the place of cell II and a balance obtained as before. Since the IR drop along the wire is proportional to its length (I remaining constant), the E.M.F. of the unknown cell is to the E.M.F. of cell II as the distances AC, in the respective settings. By means of the rheostat R, the IR drop in AC can be regulated so that the scale divisions along the slide wire are some decimal multiple of the drop along the wire. For instance, in using a Weston cell, the E.M.F. of which is 1.01985 volts, the resistance R would be regulated until the potentiometer is balanced with C at 1019.9 millimeters from A. Whenever a cell of unknown E.M.F. was put in the place of cell II and balanced, the distance of point C from A in millimeters, divided by 1000, would be the E.M.F. of the unknown cell. This is called "setting the potentiometer."

NOTE. The potentiometer is merely a special case of two generators in parallel feeding into a common line AC. When the voltage across the line is less than the E.M.F. of each generator, they both feed into the line. When the voltage across the line is *less* than the E.M.F. of one generator but greater than that of the other, the generator of the higher E.M.F. feeds into the line, and backs up a current through the other generator.

But when the voltage across the line happens to just *equal* the E.M.F. of one generator, that generator neither delivers nor takes current. The other generator feeds into the line.

The voltage of the line could not be greater than nor equal to the E.M.F. of both generators, since one generator must feed into the line to cause any drop across it.

249. Calibration of Voltmeter by Potentiometer. In order to calibrate a voltmeter, the range of which may be several times greater than the greatest IR drop along the slide wire, an arrangement shown in Fig. 306a is used. The wire AB is calibrated, as described above, to read direct, or have some simple constant. The voltmeter is placed across E_2, a variable source of E.M.F. For readings which do not exceed the IR drop along AB a plug is placed at D and balance obtained as described above. When the voltage desired for calibration becomes greater than the IR drop along AB, the plug D is withdrawn and

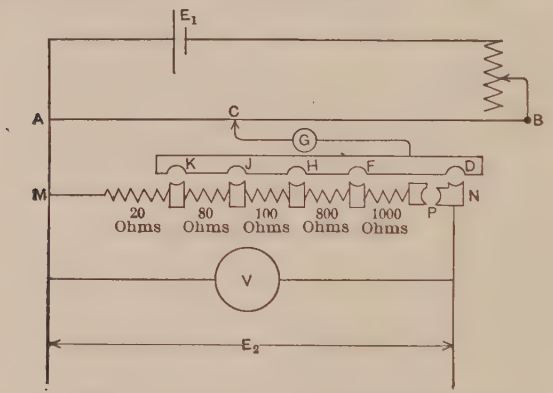


FIG. 306a.—Potentiometer for measuring high voltages.

placed in P. This gives an IR drop of E_2 across the 2000 ohm resistances MN. If now the plug F is put in, but half of this voltage E_2 is put across AC. The reading of AC, multiplied by 2, would then be the correct voltage across E_2. If the plug is placed in K, for instance, only $\frac{1}{100}$ of the voltage of E_2 is thrown across AC, and thus the setting AC, multiplied by 100, gives the true voltage E_2.

This arrangement enables voltmeters to be quickly and accurately calibrated through a wide range of values.

In connection with a standard resistance, the potentiometer affords an accurate method for calibrating ammeters.

SUMMARY OF CHAPTER XIV

GALVANOMETERS.

(1) Tangent,
  (a) Not sensitive.
(2) Astatic (Thomson),
  (a) Sensitive;
  (b) Not dead-beat;
  (c) Slow action, and fair control.
(3) D'Arsonval,
  (a) Moving coil, permanent magnet;
  (b) Sensitive;
  (c) Dead-beat;
  (d) Positive control.

With shunts, galvanometers can be used to measure current; with series coil, to measure voltage.

EQUATION FOR SHUNTS.

$$\frac{\text{Current through Galvanometer}}{\text{Current through main line}} = \frac{S}{S+G}.$$

SENSIBILITY. (a) Number of amperes or coulombs to produce a deflection of one scale division. (b) Number of megohms which may be placed in series with galvanometer across one volt pressure and have the instrument give a deflection of one scale division.

A BALLISTIC GALVANOMETER. "Throws" are proportional to quantity of electricity discharged through it.

THERMO-ELECTRIC PRESSURE. Is set up when one juncture of two unlike metals is at a higher temperature than the other. Possessed by different combinations of metals in a greatly varying degree. Effect used in pyrometers to measure high temperatures. Such combinations of metals must be avoided in ordinary ammeters and voltmeters.

AMMETERS AND VOLTMETERS. See summary on page 416.

ARRANGEMENT OF INSTRUMENTS FOR POWER MEASUREMENT. In using wattmeters, or ammeters and voltmeters, for accurate power measurement, they should be connected to the circuit in such a way as to cause as small error in readings as possible. In very accurate work the errors due to power consumed by instruments must be corrected.

WATTHOUR METER—THOMSON. A small shunt motor, the speed of which is proportional to rate of power consumption. Armature current proportional to voltage; field proportional to current. Opposing torque supplied by eddy-currents in disk which revolves between permanent magnetic poles.

POTENTIOMETER. A device for measuring voltage or E.M.F.; takes no current from source, the pressure of which it measures. E.M.F. to be measured is balanced against the fall of potential along a wire.

By balancing a definite fraction of the E.M.F. against this fall of potential, high voltages may be measured, and high reading voltmeters accurately calibrated.

PROBLEMS ON CHAPTER XIV

14–14. The scale of an electro-dynamometer is divided into 360°. The instrument is to be used as an ammeter, and 2 amperes are found to give a deflection of 15°. How many degrees deflection would the following currents give: (*a*) 1 ampere; (*b*) 4 amperes; (*c*) 5.5 amperes?

15–14. How many amperes would the following deflections of the electro-dynamometer of Problem 14 indicate: (*a*) 10°; (*b*) 40°; (*c*) 75°?

16–14. It is desired to measure the power taken by an electric device. Instruments are arranged as in Fig. 297. The ammeter has a resistance of 0.09 ohm; the voltmeter has a resistance of 16,500 ohms. Ammeter reading = .465 amperes. Voltmeter reads 112 volts. If instruments are correct, what is per cent error in power measurement due to the arrangement of the instruments?

17–14. If above instruments were arranged as in Fig. 298, (*a*) what would each read? (*b*) What would be the per cent error in power measurement for this arrangement?

18–14. In measuring the core loss of a 5 K.W. transformer the wattmeter, ammeter, and voltmeter were connected as shown in Fig. 307. *C* and *D* are the ammeter or current connections to

the wattmeter, and E and F the voltmeter or potential connections, The compensating coil is not used.

Voltmeter has a resistance of 2200 ohms;
Current coil of wattmeter, .0045 ohm;
Potential coil of wattmeter, 3300 ohms.

The readings were as follows:

Ammeter = .634 ampere;
Voltmeter = 110 volts;
Wattmeter = 69.7 watts.

What was the true core loss in watts?

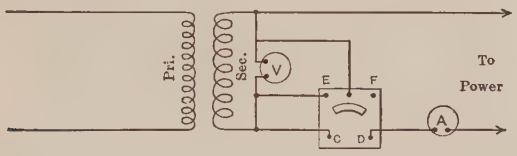


FIG. 307.—Measurement of core loss with ammeter, voltmeter, and wattmeter.

19–14. If instruments used in Problem 18 are connected to measure transformer core loss as in Fig. 307*a*, what will each instrument indicate?

20–14. A potentiometer arranged as in Fig. 306*a* is being used to calibrate a voltmeter V. 1 millimeter of the slide wire = 1 millivolt drop. The slide C is at 1428.3 mm. when the instrument is balanced. If plug D is in place, what is the voltage across E_2?

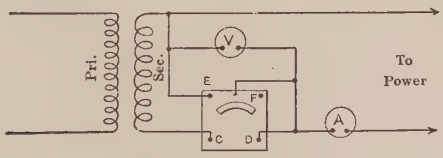


FIG. 307*a*.—Another arrangement of ammeter, voltmeter, and wattmeter for measuring core loss.

21–14. Assume that the same reading were obtained in Problem 20, with plug D withdrawn and plug J in place. What would be voltage across E_2? Plug P is in place.

CHAPTER XV

ALTERNATING CURRENTS

Definition: Cycle; Frequency; Phase; A.C. Generator; Vector Diagrams, Average, Maximum, Effective and Instantaneous Values of E.M.F. and Current—Computation of Current; Current and Voltage in Phase; Leading Current; Lagging Current—Causes and Effects of Lead and Lag—Reactance, Inductance, and Capacity—Computation of Reactance; Impedance; Computation of Ohm's Law for A.C. Circuits—Series and Parallel Circuits—Power in A.C. Circuits—General Law for A.C. Circuits; when Current and Voltage are in Phase; when out of Phase—Power Factor—Effect of Inductance and Capacity on Power Factor—Use of Ammeter and Voltmeter for A.C. Power Measurements—Use of Wattmeter.

250. Cycle: Frequency. An electric current which flows back and forth at regular intervals in a circuit is called an Alternating Current. When the current rises from zero to a maximum, returns to zero, and increases to a maximum in the opposite direction and finally returns to zero again it is said to have completed a Cycle. For convenience a cycle is divided into 360 degrees. Any point in the cycle is spoken of as a certain Phase. Thus when the cycle is half completed, it is said to be at the 180-degree phase; when one-fourth completed at the 90-degree phase. This cycle of changes is repeated over and over again. Fig. 331 is a graphical representation of the cycle of such a current. The maximum value in one direction is reached at the 90-degree phase, and in the other direction at the 270-degree phase. The number of times this cycle takes place in one second, is called the Frequency of the current. Thus a current which rises to a maximum in each direction 60 times

a second is said to make 60 cycles per sec., or to have a frequency of 60. The symbol for frequency is f.

251. Water Analogy. If we liken the flow of a direct current to the flow of water in a river, we may liken the

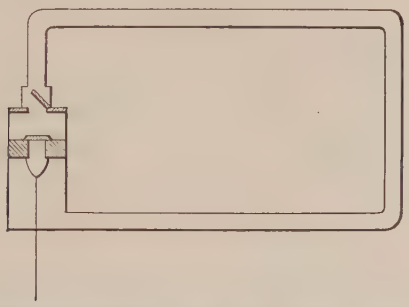


FIG. 308.—Pipe circuit containing a pump with valves. Corresponds to a D.C. electric circuit.

flow of an alternating current to the ebb and flow of the tide in a narrow channel. Of course the frequency of such a tidal flow would be extremely small. Some ideas

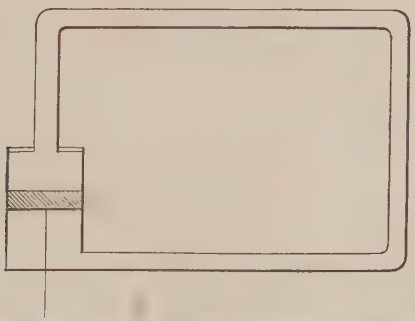


FIG. 309.—Pipe circuit containing a valveless pump. Corresponds to an A.C. electric circuit.

of the nature of direct currents and alternating currents can be derived from the study of Figs. 308 and 309.

Fig. 308 represents a pipe circuit containing a pump with valves so arranged that the water in the pipe flows

in one direction only, independent of the direction of the piston motion. This is the water analogy to a direct current system. The pipe takes the place of the wires, the pump of the generator. The valves of the pump represent the commutator of the generator. Both valves and commutator allow the current to flow in but one direction.

The pump in Fig. 309 has no valves. The direction of the current in the pipe then depends upon the direction of the piston motion. The pump without valves represents a generator without a commutator. Such a generator would deliver an alternating current to the line, as explained in Chapter VII.

FIG. 310.—Two pole generator, with single coil armature.

We will now consider, in greater detail, the E.M.F. and current set up in a single coil armature of such an alternating current generator.

252. Alternating Current Generator. Single Coil. Fig. 310 is a simple diagram in perspective of a single coil armature. An alternating E.M.F. will be set up in the coil if it is revolved as explained in Chapter VII.

The following details in connection with the E.M.F. induced should be studied carefully.

When the coil is in the position shown in Fig. 311, it is cutting no lines of force. There is therefore no voltage induced in it. This position, or phase angle, is zero.

When the coil has moved 30 degrees from the zero as shown in Fig. 312, it is cutting force lines, and there is an E.M.F. induced of OUT at *A*, and IN at *B*.

Suppose we plot the E.M.F. as ordinates and the position of the coil in degrees from zero as abscissæ, Fig. 316.

The line e_1 represents the instantaneous voltage at the 30° phase, or when coil is at 30° as shown in Fig. 312.[1]

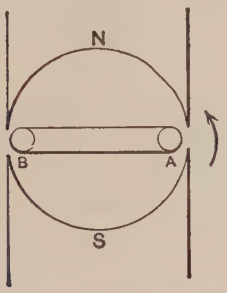


FIG. 311.—Coil passing through zero position and cutting no force lines. Fig. 317 is vector diagram for E.M.F. at this instant.

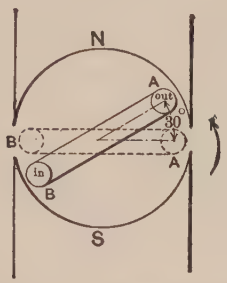


FIG. 312.—Coil passing through 30-degree position. Fig. 318 is vector diagram for E.M.F. at this instant.

When the coil has moved 60° from the zero position as in Fig. 313, the instantaneous E.M.F. has increased to a value represented by the line e_2, Fig. 316.

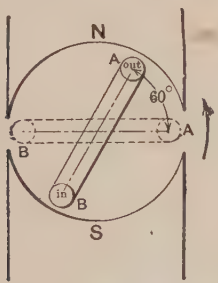


FIG. 313.—Coil passing through 60-degree position. Fig. 319 is vector diagram for E.M.F. at this instant.

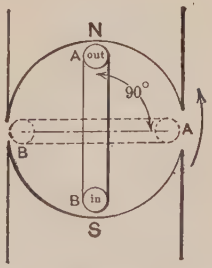


FIG. 314.—Coil passing through 90-degree position. Fig. 320 is vector diagram for E.M.F. at this instant.

[1] The angular position of the coil and the phase angle are the same in a two-pole machine, but not in others. When the phase differs from the angular position, it is customary to speak of the position of the coil as "so many degrees" and of the phase as so many "electrical degrees." This causes no confusion in actual practice.

When the coil has moved 90° from the zero position, it is then cutting lines at the fastest rate as it is moving across them at right angles, as shown in Fig. 314. The line E, Fig. 316, now represents the instantaneous induced voltage.

As the coil moves on from this point, the induced voltage across it begins to decrease, and at length becomes zero again, as the coil reaches a point 180° from zero position. Lines e_3 and e_4, Fig. 316, show the instantaneous value of the E.M.F. at 120° and 150° respectively. At 180° the instantaneous E.M.F. is zero, but as soon as it passes this point the coil begins to cut lines in the opposite direction, and an E.M.F. is now induced which is IN at A, and OUT at B, as shown by Fig. 315. The instantaneous value of this induced E.M.F. is represented by the line $-e_1$ in Fig. 316. The value continues to increase in this reverse direction, until a position 270° from zero is reached. Here the voltage becomes a maximum and is represented by the line $-E$, Fig. 316. From here on, it gradually decreases until it becomes zero again as it reaches 360° (which is really the zero point at which we started to follow the movement of the coil). The lines $-e_3$ and $-e_4$ represent the values of the E.M.F. at 300° and 330° respectively. The curve, Fig. 316, plotted from these values, represents the various instantaneous values of the E.M.F. at all times, during one complete cycle. Note that the E.M.F. is continually changing, and if we wish to indicate its value we must state it, as for a given instant only. At any other instant it will be greater or less than this value.

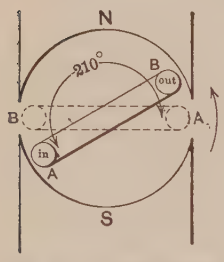


FIG. 315. — Coil passing through 210-degree position. Fig. 312 is vector diagram for E.M.F. at this instant.

As the armature turns around it merely repeats this cycle of values, shown by the vertical lines in Fig. 316.

Such a curve is known as a SINE CURVE and approximates more or less closely the curves of the E.M.F. values of most A.C. generators.

253. Vector Diagram. Figs. 317–326 show another and a simpler way of representing the relation of the E.M.F. induced in an armature coil to the phase angle, when the E.M.F. follows a sine curve.

The greatest value which the E.M.F. attains is represented by the line E, called a VECTOR. This line is supposed to be rotating in a counter-clockwise direction about the

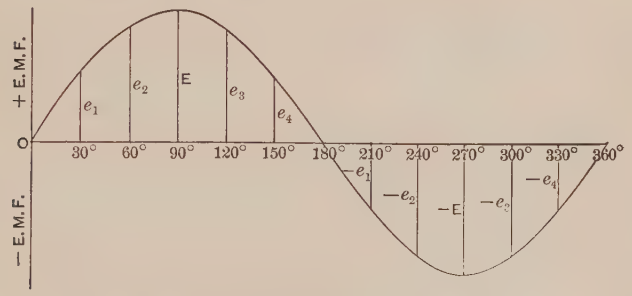


FIG. 316.—Sine curve showing relation between E.M.F. and phase (or, in this case, angular position of coil).

end o. By drawing this vector to some scale, and at angle to an axis, equal to phase angle, we determine the instantaneous value at that position, or phase, by measuring the vertical component to the vector E. The X axis has been used for convenience in Figs. 317–326 to represent the zero position of the vector.

Thus in Fig. 317, which is the vector diagram for coil in position shown in Fig. 311, the vector E has not moved from the zero position and has no vertical component. Thus the value of the E.M.F. in this position is 0. This is the value of the E.M.F., as we have seen, when the coil is in the position of Fig. 311.

In Fig. 318, the vector diagram represents the instantaneous E.M.F., when the position of the coil is as shown

in Fig. 312. The coil has moved 30° from the zero position. The vector E is therefore at an angle of 30° to the horizontal. The vertical component of E is now e_1, which equals the instantaneous value of the induced E.M.F., when the coil

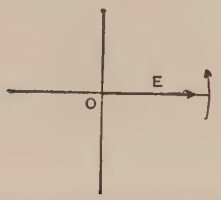


FIG. 317.—Vector diagram for E.M.F. when coil is passing through zero position. See Fig. 311.

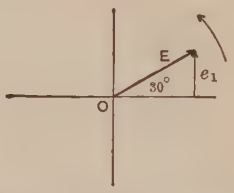


FIG. 318.—Vector diagram when coil is passing through 30-degree position. See Fig. 312.

is 30° from the zero position. The vertical component e_2 at 60° is shown in Fig. 319.

When the vector reaches the 90° position, the vertical component is equal to the vector itself, and reaches its maximum value when the coil is 90° from the zero position.

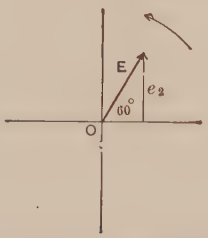


FIG. 319.—Vector diagram for E.M.F. at 60-degree phase. See Fig. 313.

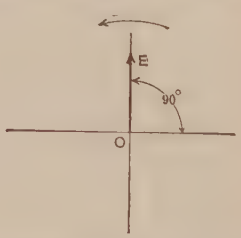


FIG. 320.—Vector diagram for 90-degree phase. See Fig. 314.

Fig. 314 represents the coil in this position and Fig. 320 represents the vector diagram of the E.M.F. at this time.

Figs. 322 and 323 are the vector diagrams for the coil in positions 150° and 180° respectively. Note that e_4 is the instantaneous value for the E.M.F. when the coil is at the 150° point and its value corresponds to E_4 of Fig. 316. At 180° the instantaneous value of the E.M.F. is

zero. Accordingly, in the vector diagram, Fig. 323, there is no vertical component of E, showing that the instantaneous value of the E.M.F. is zero.

When the coil passes the 180° point, the E.M.F. reverses. This is shown in Fig. 324, by the fact that the line e_1, which

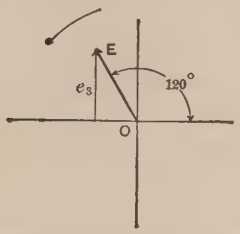


FIG. 321.—Vector diagram for E.M.F. at 120-degree phase.

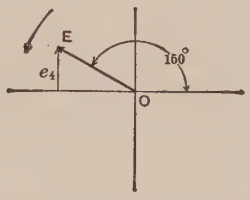


FIG. 322.—Vector diagram for E.M.F. at 150-degree phase.

is the vertical component of E, is below the X axis. At 270°, the instantaneous value of the E.M.F. becomes a maximum in this reversed direction. This appears in Fig. 325, where

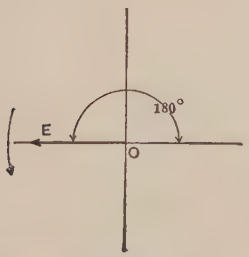


FIG. 323.—Vector diagram for E.M.F. at 180-degree phase.

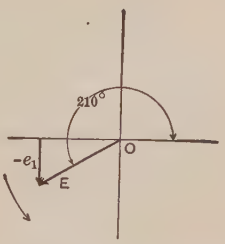


FIG. 324.—Vector diagram for E.M.F. at 210-degree phase. See Fig. 315.

the vertical component of E equals $-E$, and is below line, thus being a minus quantity.

In the same manner, $-e_4$, in Fig. 326, corresponds to $-e_4$ in Fig. 316, and represents the instantaneous value of the E.M.F., when the coil is at the 330° point.

Note carefully:

(*a*) That the lines E, e_1, e_2, e_3, e_4, and $-e_1$, $-e_2$, $-e_3$, $-e_4$, represent merely instantaneous values of the E.M.F. at the various positions of the coil, or at various phases in a cycle.

(*b*) That E represents the maximum value that these instantaneous values ever reach.

(*c*) That the maximum is reached twice during each cycle.

It follows from this that e_1, e_2, and $-e_1$ $-e_2$, etc., can be regarded as merely certain fractions of the maximum value

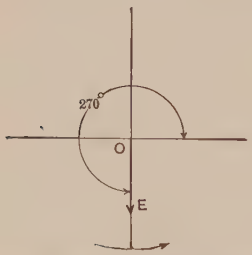


FIG. 325.—Vector diagram for E.M.F. at 270-degree phase.

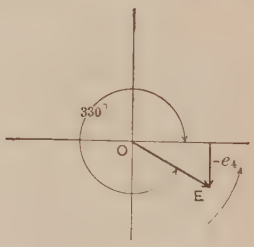


FIG. 326.—Vector diagram for E.M.F. at 330-degree phase.

E. Now we have stated that the curve, Fig. 316, was called a sine curve. It is so called because the ratio between the instantaneous values e_1, e_2, etc., and the maximum value E, equals the *sine of the phase angle at that instant.*

Thus in Fig. 318, e_1, the instantaneous value at the 30° phase, is a certain fraction of E. The value of this fraction is the sine of the angle 30°, since the phase is 30°. By consulting a table of sines, we find that the sin 30° = .5. Thus e_1 is $\frac{1}{2}$ of E. So if $E = 150$ volts, the instantaneous value of the voltage when only 30° of the cycle had been completed would be $\frac{1}{2}$ of 150 or 75 volts.

In Fig. 319, e_2 is that fraction of E, which is equal to the sine of the angle 60°. Sin 60° = .866. Therefore, $e_2 = .866 \times E$.

If $E = 150$ volts,

$e_2 = .866 \times 150 = 130$ volts.

254. Equation for Instantaneous Value of E.M.F. The rule then for finding any instantaneous value of the voltage in an alternating current circuit, when the voltage follows the sine wave, is:[1]

Multiply the maximum value of the voltage by the sine of the angle of phase. This is generally stated in the form of an equation:

$$e = E \sin \phi;$$

where e = instantaneous voltage;
E = maximum voltage;
ϕ = phase angle (in degrees).

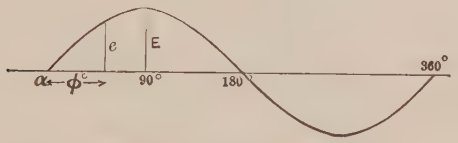


Fig. 327.—Sine curve of E.M.F.

There are, then, *three* general ways of representing an alternating E.M.F. and of determining its value at various instants.

(1) By the graphical representation of the Sine Curve, as in Fig. 327.

(2) By the vector diagram, as in Fig. 328.

(3) By an equation, as

$$e = E \sin \phi.$$

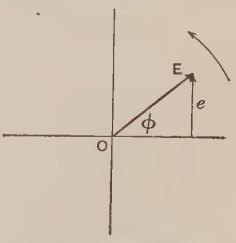


Fig. 328.—Vector diagram of E.M.F.

It is best for one starting the subject of Alternating currents, to do the examples by all three methods, using

[1] Unless otherwise stated, the wave form of an alternator is assumed to be that of a sine curve.

(1) to get a general idea of the different phase relations, etc., (2) and (3) to obtain correct mathematical results.

Example. What is the instantaneous value of an alternating E.M.F. when it has reached the 45° phase of its cycle? The maximum value is 600 volts.

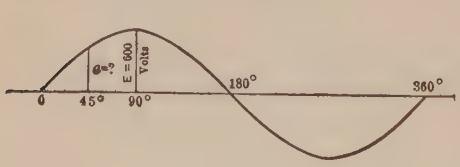


Fig. 329.

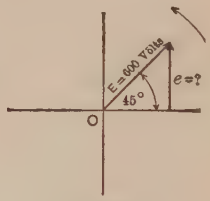


Fig. 330.

Method (1)

Method (2)

Method (3)

$$e = E \sin \phi,$$

$$e = 600 \sin 45°$$

$$= 600 \times .707 = 424 \text{ volts.}$$

Solve as per Example by Three Methods

Problem 1–15. The maximum value of an alternating E.M.F. is 1200 volts. What is the instantaneous value when 65° of the cycle have been completed?

Problem 2–15. What is the instantaneous value of E.M.F. of Problem 1–15 when it has reached the 200° phase? (Note by method (1) and (2), e is seen to have the same numerical value as at the 200° − 180°, or 20° phase. The sign, however, is seen to be negative.)

Problem 3–15. The instantaneous value of an alternating E.M.F. is 400 volts at 75°. What is the maximum value?

Problem 4–15. The value of an alternating E.M.F. is 250 volts at 35°. What is it at 135°?

Problem 5–15. Plot the sine curve, to some convenient scale for one complete cycle for Problem 4–15.

Problem 6–15. The maximum value of an alternating E.M.F. is 600 volts. What are the instantaneous values at the following phases: 20°, 80°, 130°, 210°, 300°, 340°?

255. Average Value of an Alternating E.M.F. Since half of the instantaneous values of an alternating E.M.F. are negative and half are positive, and since the negative values are exactly equal to the positive values, the average of a complete cycle of values must be zero. But the actual average value is not zero. It would be as reasonable to say that the actual average value of the pressure exerted by the piston of the water pump of Fig. 309 is zero, just because the pressure alternates equally in each direction. The actual average value in both cases is the average of all instantaneous values *regardless of signs*, which only indicate direction.

The average value of an alternating E.M.F. can be found very easily by plotting a number of the instantaneous values throughout the cycle and finding their average. Or the average may be computed from the equation by calculus. The results of both of these methods show that the average value of an alternating E.M.F. is .636 times the maximum value. In the form of an equation, this may be written,

$$\text{Average } e = .636\, E.$$

The average of an alternating E.M.F. is little used except to compute the maximum value.

Example. What is the maximum voltage generated in a drum armature consisting of 300 series conductors in each path which has a speed of 1200 R.P.M. Each conductor cuts twice through a field of 1,500,000 lines of force during each revolution.

$$\text{Average } e = \frac{\text{lines cut per sec.}}{10^8}$$

$$= \frac{1{,}500{,}000 \times 2 \times 1200 \times 300}{10^8 \times 60}$$

$$= 180 \text{ volts.}$$

But $\text{Average } e = .636\, E.$

Then $$E = \frac{\text{Average } e}{.636}$$

$$= \frac{180}{.636} = 283 \text{ volts.}$$

Therefore $E = 283$ volts.

Problem 7–15. A 2-pole generator with a drum armature has a speed of 2400 R.P.M. The field has 20,000,000 lines of force. Number of series conductors in each path on armature is 500. What is the maximum value of the E.M.F. generated?

Problem 8–15. What is the instantaneous value of the E.M.F. at the 60° phase in Problem 7–15?

Problem 9–15. A 4-pole generator has a drum wound armature with 2000 series conductors in each path. Speed is 1200 R.P.M. Flux in each pole is 24,000,000 lines. What is average value of E.M.F.?

Problem 10–15. What is maximum value of E.M.F. of Problem 9–15?

Problem 11–15. What is the instantaneous value of the E.M.F. of Problem 9, at 45° phase? Would the 45° phase of the E.M.F. be reached at the instant in which the armature had turned 45° from its zero or neutral position?

256. Computation of Current in A.C. System. The diagrams, Figs. 316–330, which have been drawn to show the relation between instantaneous values of alternating E.M.F. and the maximum value, serve just as well to show the relation between instantaneous values of alternating current and the maximum value. We have merely to put i_1, i_2, I, i_3, i_4, $-i_1$, etc., in the place of e_1, e_2, etc., and all diagrams show current instead of voltage values.

The same three methods can be used to solve problems in current values that we used for E.M.F. values.

(1) Thus in Fig. 331, which is identical with Fig. 316 except for the lettering, i_1 is the instantaneous current at the phase 30° from the zero, i_2 the value of the current at 60°, and I at 90°, etc. The current thus follows a curve of the same shape as that of the E.M.F.

(2) The instantaneous values can also be represented by a vector or clock diagram as in Fig. 332, which is identical with Fig. 317 except for the lettering. I represents the maximum value of the current, and i_1 represents the instantaneous value of the current when it is at the 30° phase; i_1 is equal to I sin 30°, and can be found from the general equation:

$$i = I \sin \phi;$$

when i = instantaneous value of an alternating current;
I = maximum value;
ϕ = phase angle in degrees.

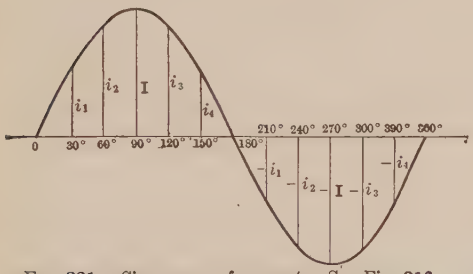


Fig. 331.—Sine curve of current. See Fig. 316.

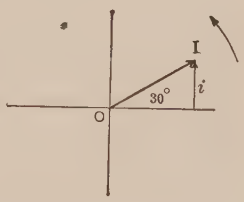


Fig. 332.—Vector diagram of current. See Fig. 318.

As in the case of the alternating E.M.F. it is best to use all three methods for solving each problem, until the ideas of each are thoroughly mastered.

Problem 12–15. What is the instantaneous value of an alternating current at 20°? The maximum value is 45 amperes.

Problem 13–15. What is the maximum value of an alternating current when the value at 65° is 14 amperes?

Problem 14–15. What are the instantaneous values of current in Problem 13–15 at the 180° phase; 200°; 300°?

257. Average Value and Effective Value of Alternating Currents. As the average value of an alternating E.M.F. equals .636 of the maximum value, so likewise, we may

write the equation for the average value of an alternating current:

$$\text{Average } i = .636\ I.$$

Thus if the maximum value of an alternating current is 50 amperes, the average value is $50 \times .636 = 31.8$ amperes.

The average value is of little use, however, as we rate an alternating current on its EFFECTIVE VALUE.

An alternating current has *no unit* of its own, but is measured in terms of the unit of direct current, the AMPERE. Now an ampere is defined as that *steady rate of flow* which will deposit a standard amount of silver from a standard solution in one hour. But an alternating current is *not a steady current*, and neither will it deposit any silver from a solution; since whatever it deposits during one-half a cycle it takes off the next half, when it is flowing in the reverse direction.

Accordingly, in order to compare the alternating with the direct current, we must use some other property which both kinds of current possess. The most natural is the heating effect of each.

Therefore an alternating current is said to be equivalent to a direct current when it produces the same average heating effect, under exactly similar conditions. This value is called the EFFECTIVE VALUE of an alternating current, and is measured in amperes. It is somewhat greater than the *average value*, being equal to .707 of the maximum value. This is because the heating effect of a current depends upon the *average of the squares* of each instantaneous value of the current. The effective value of an alternating current is thus often called the "Square root of the mean squares," because it can be found by squaring a number of the instantaneous values in a cycle, finding the average of these squares, and then extracting the square root.

For all practical purposes the effective value of an alternating current may be found from the following equation:

$$I_f = .707\, I \quad \text{or} \quad \frac{I}{\sqrt{2}};$$

where I_f = effective value of an alternating current;
I = maximum value.

258. Effective Value of Alternating E.M.F. In the same way, the value of alternating E.M.F., by which the voltage is rated, is the EFFECTIVE value, and is found by the equation:

$$E_f = .707\, E,$$

where E_f = effective value of alternating E.M.F.,
E = maximum value.

When no special value is designated, the EFFECTIVE value is always understood, for both current and voltage.

Proof.. That the *effective* value of an alternating current equals the "square root of the average squares."

Suppose two calorimeters are so arranged that one measures the heat generated per hour by an alternating current flowing through a resistance R. The other measures the heat generated per hour by a direct current flowing through an equal resistance R.

Let the two currents be so regulated that the amount of heat generated per hour by each current be equal.

Then by definition:

$$I_d = I_f;$$

where I_d = Direct current in amperes;
I_f = effective value of alternating current.

The heat generated by the direct current equals,

$$H = .24 I_d^2 Rt;$$

The heat generated by an alternating current equals,

$$H = .24\ (\text{av. } i^2) Rt.$$

where i = instantaneous value of the current.

But the heat generated by the direct current equals the heat generated by the alternating current.

Therefore,

$$.24 I_d^2 Rt = .24 \text{ (av. } i^2) Rt;$$

$$I_d^2 = \text{av. } i^2;$$

$$I_d = \sqrt{\text{av. } i^2}.$$

But $I_d = I_f$,

Thus,

$$I_f = \sqrt{\text{av. } i^2}.$$

Effective value $= \sqrt{\text{Average of squares of instantaneous values.}}$

Since the effective value of voltage and current is always a definite fractional part of the maximum values, the EFFECTIVE value may be used in VECTOR DIAGRAMS. To use the effective value this way is merely equivalent to reducing the scale of the diagram.

Example. What is the effective value of an alternating current whose maximum value is 48 amperes?

$$I_f = .707 \times 48 = 33.9 \text{ amperes.}$$

Problem 15–15. The effective value of an alternating current is 25 amperes. What is the greatest instantaneous value of this current?

Problem 16–15. What is the average value of the current in Problem 15–15?

Problem 17–15. The effective value of an alternating current is 200 amperes. What is the instantaneous value of 30° phase?

Problem 18–15. What is the effective value of an alternating E.M.F. when the instantaneous value at 150° is 500 volts?

Problem 19–15. What is the effective E.M.F. in Problem 7–15?

Problem 20–15. What is the effective voltage in Problem 9–15?

259. Phase Relations of Current and Voltage. Whenever an alternating E.M.F. is allowed to send a current through a circuit, of course this current is an alternating current. The curve and equation representing the current

will have the same form as the curve and equation for the E.M.F., as explained above. The vector diagrams will be similar also. Accordingly, the curves for the alternating voltage and current in a circuit can both be drawn on the same pair of axes, as in Fig. 333. Their vector diagrams also can both be drawn on one pair of axes as in Fig. 334.

There are *three* relations possible between the current and E.M.F.:

(1) Current and voltage curves may be "*in phase.*" Figs. 333 and 334.

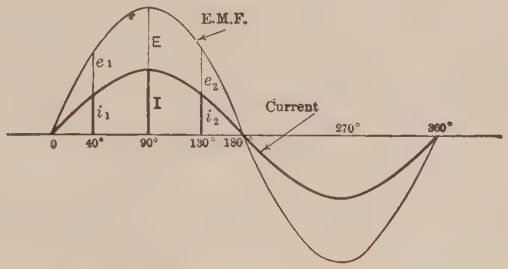


FIG. 333.—Sine curve of E.M.F. and current.

(2) Current curve may "*lag*" behind voltage curve. Figs. 337 and 338.

(3) Current curve may "*lead*" voltage curve. Figs. 340 and 341.

260. (I) E.M.F. and Current in Phase. The current and voltage may be "*in phase*" as in Figs. 333 and 334. That is, they both are zero at the same instant, both pass through their maximum values at the same instant, and, in fact, are in the same phase throughout their entire cycles. This is the case when the *inductance* and *capacity* in the circuit are balanced, or when the circuit contains *resistance* only.

Fig. 333 represents the relations of the current and voltage when they are "*in phase.*" The heavy line represents

the current. Note that both the curves are at zero at the same instant, and that they reach their maximum values E and I at the same instant of phase. The value e_1, at the 40° phase, corresponds to the value i_1 at the 40° phase and is the same fractional part of E that i_1 is of I.

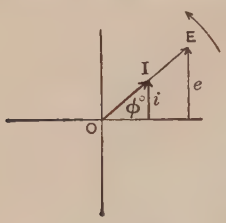


Fig. 334. — Vector diagram; current and voltage in phase.

Fig. 334 also shows the conditions or relations of E.M.F. to current when they are "in phase." The lines I and E coincide in direction at all times, and thus are always at the same angle ϕ, from the zero axis. The instantaneous value e is the same fractional part of E that the instantaneous value i is of I. This fraction is represented by the expression "sin ϕ". That is,

$$i = I \sin \phi$$

and

$$e = E \sin \phi.$$

261. Ohm's Law in A.C. Circuits. There is always likely to be some confusion about the use of Ohm's law, when the student reaches alternating current work. This confusion disappears when one understands that Ohm's law deals with VOLTAGE, CURRENT AND RESISTANCE only, and always holds true as far as the relative values of these three alone are concerned. The law says nothing about *inductance* or *capacity* and is not concerned with the effect of these. Accordingly the student may be sure that as far as the RESISTANCE alone is concerned, the voltage required to force a certain current through a certain *resistance* is always the product of the *current times the resistance*, whether the voltage is direct or alternating.

If some other factor like a counter E.M.F. due to inductance, etc., is in the circuit, allowance must be made for it according to some other law, not Ohm's law. Therefore

do not have any doubts about the validity of Ohm's law. It *always applies* in so far as RESISTANCE affects the results.

Example. A circuit containing resistance only has an alternating E.M.F. impressed upon it, of which the effective value is 110 volts. If the resistance is a lamp of 55 ohms, what is the current through the lamp, when the voltage is at the 30° phase?

Solution:

$$E_f = .707\ E,$$

$$E = \frac{E_f}{.707} = 155 \text{ volts.}$$

Since there is resistance only in the circuit, the current will be in phase with the voltage and will be at a maximum value when the voltage is at a maximum.

Thus by Ohm's law,

$$I = \frac{E}{R}$$

$$= \frac{155}{55} = 2.82 \text{ amperes.}$$

The instantaneous value at the 30° phase, drawn to scale, as in Fig. 335, =1.4 amperes.

Or by the equation:

$$i = I \sin \phi$$
$$= 2.82 \sin 30$$
$$= 2.82 \times .500 = 1.41 \text{ amperes.}$$

Problem 21–15. The average voltage in an alternating current circuit containing 20 ohms resistance (no capacity and no inductance) is 125 volts. What is the average current?

Problem 22–15. What is the effective current in Problem 21–15?

Problem 23–15. When the instantaneous voltage in Problem 21 is 120 volts, what is the current value?

Problem 24–15. At what phase would the voltage of Problem 21 be 120 volts?

Problem 25–15. Solve by vector diagram. The maximum voltage is 200 volts, resistance = 2 ohms. What is current, when the voltage is at the 200° phase?

Problem 26–15. Coil has 200 turns, Fig. 336; resistance, 32 ohms; R.P.M.=2400; ϕ=1,000,000.

(*a*) What is average voltage?
(*b*) " effective voltage?
(*c*) " maximum voltage?

Problem 27–15. (*a*) How many degrees from zero position will coil in Problem 26 be, when instantaneous current equals 6 amperes? (*b*) What will voltage be at this point?

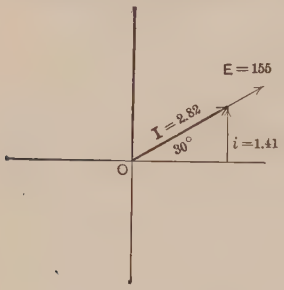


Fig. 335.—Vector diagram.

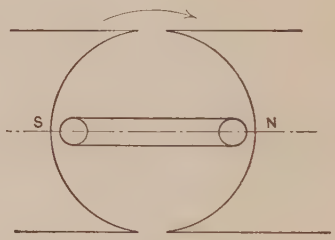


Fig. 336.—Single coil generator.

262. (II.) Lagging Current. The current may "lag" behind the voltage as in Figs. 337 and 338. Note in Fig. 337 that the current curve is 50° behind the voltage curve; that

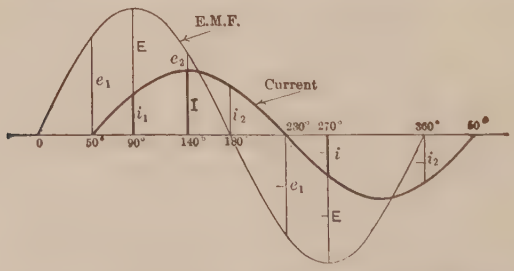


Fig. 337.—Sine curve of current lagging 50° behind the E.M.F.

is, the voltage curve has a value e_1 at 50° while the current has a value of zero at 50°. Also, the voltage curve reaches its maximum at 90° point while the current curve reaches it at the 140° point, 50° later. So when the voltage curve

has become zero again at 180°, the current curve has but just passed through the maximum value and still has a value of i_2.

Fig. 338 shows how this same phase difference is represented by a vector diagram.

Since the current reaches its maximum value 50° later than the voltage, the vector I lags 50° behind the vector E. Thus when the vector E has reached 65°, the vector I has reached only 15°. The instantaneous values at this instant are e and i.

$$e = E \sin 65°,$$

and

$$i = I \sin 15° \text{ or } I \sin (65° - 50°)$$

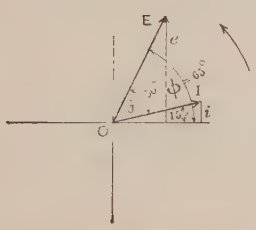


FIG. 338.—Vector diagram of current lagging 50° behind the E.M.F.

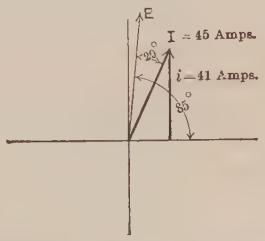


FIG. 339.—Vector diagram for determining the instantaneous value of the current when the E.M.F. is at the 85° phase.

The general equations for a "lagging" current are

$$e = E \sin \phi,$$
$$i = I \sin (\phi - \theta);$$

where ϕ = phase of the voltage in degrees;
θ = difference in phase between E and I.

Example. In an inductive A.C. circuit the current lags 20° behind the voltage. The maximum value of the current is 45 amperes. What is the instantaneous value of the current, when the voltage is at the 85° phase?

Solution. Draw to scale a vector diagram as in Fig. 339. The value of i when voltage is at 85° equals 41 amperes.

Or by the equation:

$$\begin{aligned} i &= I \sin(\phi - \theta); \\ &= 45 \sin(85° - 20°) \\ &= 45 \sin 65° \\ &= 40.8 \text{ amperes.} \end{aligned}$$

Problem 28–15. What instantaneous value will the current and voltage have in above example when the voltage is at the 110° phase? Maximum voltage is 150 volts.

Problem 29–15. The phase difference in an inductive A.C. circuit is 50°. The current has an instantaneous value of 25 amperes when the voltage is at its maximum. (*a*) What is the maximum value of the current? (*b*) What is the effective value of the current?

Problem 30–15. (*a*) What is the maximum value of the voltage (Problem 29–15), if the instantaneous value is 500 volts, when the current is at its maximum? (*b*) What is the average value of the voltage?

Problem 31–15. The maximum value of an alternating voltage is 1500 volts, the maximum value of the current is 80 amperes. If the instantaneous value of the current is 25 amperes, when the instantaneous value of the voltage is 800 volts, what is the "phase difference" between current and voltage?

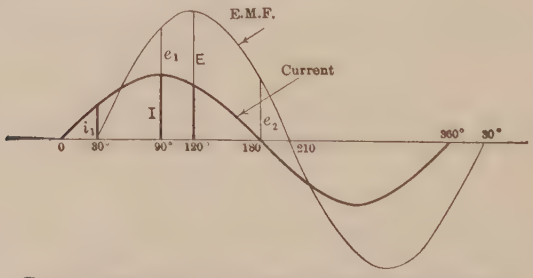


FIG. 340.—Sine curve of current leading E.M.F. by 30°.

263. (III.) Leading Current. The current curve may "*lead*" the voltage curve as in Figs. 340 and 341.

Notice that the current, Fig. 340, has reached a value i_1, at 30° while the voltage is still zero. The current thus "leads" the voltage by a "phase difference" of 30°.

Also, while the current reaches its maximum at 90° the voltage does not reach its maximum until 120°; again a difference of 30°. And so on throughout all phases of the cycle.

In the vector diagram Fig. 341, the current vector I is 30° ahead of the voltage vector E, so that when vector I has moved 55° from the X axis, vector E has gone but 25°. The equations for the instantaneous values at this instant would be:

$$e = E \sin 25°;$$
$$i = I \sin 55° \text{ or } I \sin (25° + 30°).$$

The general equations for a "leading" current are:

$$e = E \sin \phi,$$
$$i = I \sin (\phi + \theta),$$

where θ = phase difference between E and I.

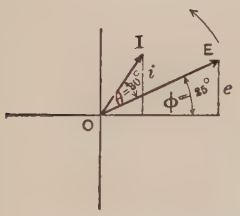


FIG. 341.—Vector diagram of current leading E.M.F. by 30°.

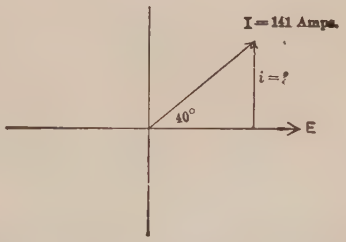


FIG. 342.—Vector diagram for determining value of current when E.M.F. is zero. Current leading by 40°.

Example. In a certain A.C. circuit the current leads the voltage by 40°. The effective current is 100 amperes. What is the instantaneous current when the voltage is 0?

Solution. Find maximum value of current from equation:

$$I_f = .707\, I;$$
$$I = \frac{I_f}{.707};$$
$$I = \frac{100}{.707} = 141 \text{ amperes.}$$

Draw the vector diagram to scale as in Fig. 342. i measures 91 amperes.

Or by the equation:

$$i = I \sin (\phi + \theta);$$
$$i = 141 \sin (0° + 40°);$$
$$= 90.7 \text{ amperes.}$$

Problem 32–15. What will be the instantaneous value of the current in above example when the voltage is at a maximum?

Problem 33–15. The angle of lead of current is 80°. The maximum current is 56 amperes. What is current value when voltage is at 135°?

Problem 34–15. What is value of voltage in Problem 33, when current is maximum, if average voltage is 250 volts?

264. Cause of a Lagging Current. Inductive Reactance. We have seen in Chapter X that, if an electric circuit contains inductance, an E.M.F. is induced in it whenever there is a *change* in the current flowing in the circuit. This *induced* E.M.F. always opposes the *change* in the *current.*

Now in an A.C. circuit we have a current which is *always* changing, so, if there is any inductance in the circuit we must always have an E.M.F. induced opposing this *change.* When the current is rising in value, the induced E.M.F. opposes this rise and the value of the current flowing is less than the "impressed voltage divided by the resistance," just as the current in a motor which is running is less than the impressed voltage divided by the resistance of the motor, on account of the opposing E.M.F. generated in the rotating armature. Accordingly, the rise of current in the A.C. line takes place *later than the rise in the impressed voltage.* This causes the current curve to lag behind the voltage curve on the rising side of the curve.

But when the current is decreasing, the induced E.M.F. opposes the decrease and tends to keep the value of the current greater than the impressed E.M.F. alone would. Thus the current falls off later than the impressed voltage. This causes the current curve to lag behind the voltage curve on the descending side. Thus the current curve "lags" behind the voltage curve throughout the entire

cycle. If a circuit contains nothing but the inductance (that is, if the resistance and capacity are negligible) the current lags 90° behind the voltage.

This is evident, if we consider that the only voltage required would be that necessary to overcome the induced E.M.F. This induced E.M.F. would be greatest, not when the current was greatest, but when the current was *changing at the fastest rate.* By inspecting Fig. 343, it is seen that the current is *changing in value most rapidly as it passes through the zero value.* Thus the voltage is greatest when the current is zero. But the voltage is always greatest

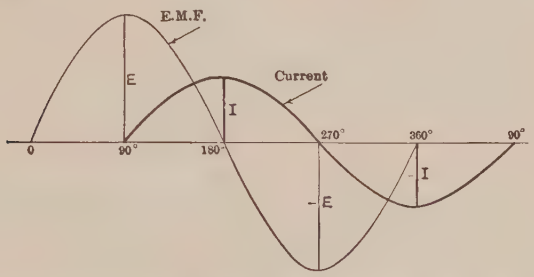


FIG. 343.—Sine curves showing E.M.F. leading current by 90°. Circuit contains inductance only.

when it is at its own 90° phase. The current then must be zero when the voltage reaches its 90° phase. The current therefore lags 90° behind the voltage.

Again, the voltage must be least when the current is changing at its slowest rate. The current changes at the slowest rate when it has a maximum value. The voltage, then, is zero when the current is maximum.

Thus the current and voltage always differ in phase by 90°, which is the difference in phase between all maximum and minimum values. Whenever the voltage is zero, the current is maximum, and whenever the voltage is maximum the current is zero. Fig. 344 shows the vector diagram for voltage and current in a circuit containing inductance only.

Since simple resistance in a circuit does not cause a current to "lag" or "lead," we can say that inductance has an effect which is 90° or at right angles to the effect of resistance. The result of this will be seen later when the combined effect of inductance and resistance is taken up. Of course, if there is also resistance or capacity in the same series circuit, the angle of lag will not be as great as 90°.

Fig. 345 is the conventional way of representing a circuit containing inductance only.

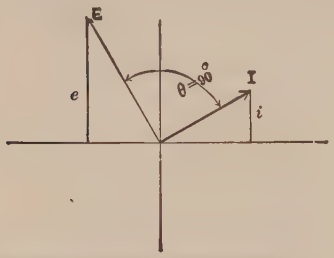


FIG. 344.—Vector diagram when current lags 90°.

FIG. 345.—Conventional diagram of circuit containing A.C. generator, *G*, and inductance, *L*.

The current equation for a circuit having inductance only is

$$i = I \sin (\phi - 90°).$$

This opposition of self inductance to the flow of the current is called INDUCTIVE REACTANCE, and is measured in OHMS just as RESISTANCE is. Also, just as the current flowing through a resistance is found by the equation

$$\text{Current} = \frac{\text{Voltage}}{\text{Resistance}},$$

so the current flowing through an INDUCTIVE REACTANCE is found by the equation

$$\text{Current} = \frac{\text{Voltage}}{\text{Reactance}};$$

or

$$I=\frac{E}{x_L}, \quad I_f=\frac{E_f}{x_L},$$

where I = current in *amperes;*

x_L = inductive reactance in *ohms;*

E = voltage necessary to force current through inductive reactance.

It must be remembered that if we use a maximum value of voltage we get a maximum value of current; if we use the effective value of voltage, we get the effective value of the current; if we use the average value of the voltage, we get the average value of the current; if we use any instantaneous value of the voltage, we get an instantaneous value of the current 90° *behind in phase.*

The current value found in this way is always 90° *behind the voltage producing it.*

Example. In an A.C. circuit containing 40 ohms inductive reactance (no resistance or capacity), the effective voltage is 220 volts. (*a*) What is the effective current? (*b*) What is the maximum current? (*c*) What is the current when the voltage is at 140° phase?

Solution. Effective current $=\frac{\text{Effective voltage}}{\text{Reactance}}$,

or

$$I_f=\frac{E_f}{x_L}.$$

(*a*) $$I_f=\frac{220}{40}=5.5 \text{ amperes.}$$

and

$$I_f=.707I;$$

$$I=\frac{I_f}{.707}$$

(*b*) $$=\frac{5.5}{.707}=7.8 \text{ amperes.}$$

Maximum current = 7.8 amperes.

Draw vector diagram to scale as in Fig. 346, when E.M.F. is at 140° phase, then by measurement

$$I = 7.8 \text{ amperes};$$

$$i = 6 \text{ amperes};$$

or by the equation

$$\begin{aligned} i &= I \sin (\phi - 90) \\ &= 7.8 \sin (140° - 90°) \\ &= 7.8 \sin 50° \\ &= 5.97 \text{ amperes.} \end{aligned}$$

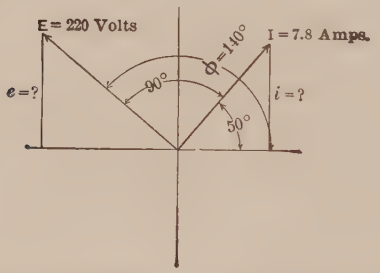


FIG. 346.—Vector diagram of E at 140° and I lagging 90°.

Problem 35–15. The average voltage in an A.C. circuit containing 20 ohms inductive reactance is 500 volts. Find: (*a*) Effective current. (*b*) Instantaneous value of voltage when current is 15 amperes.

Problem 36–15. The maximum value of the current in an A.C. circuit containing inductance only is 28 amperes. The average value of the voltage is 220 volts. What is the inductive reactance?

Problem 37–15. At what phase will the voltage be, in Problem 36, when the instantaneous value of the current is 10 amperes?

Problem 38–15. What effective voltage is required to force a maximum current of 20 amperes through 8 ohms of inductive reactance?

Problem 39–15. What instantaneous value will voltage of Problem 38 have when the current is at its maximum value of 20 amperes?

Problem 40–15. In an A.C. circuit containing inductive reactance only, the voltage is 1200 and current 45 amperes. What

is the instantaneous value of the current when the voltage is 300 volts?

265. Cause of a Leading Current. Capacity Reactance. In Chapter XI, it was stated that capacity in a circuit acts like an air chamber in a pump circuit; it tends to oppose any change in pressure, and keep the pressure constant. Thus condensers in a circuit might be thought of as reservoirs, in which electricity is being stored at the instants when the pressure is rising. These reservoirs or condensers then use the pressure of the stored charge, to maintain the current at the instants when the pressure is dying out. A current will thus be flowing into a condenser, until it

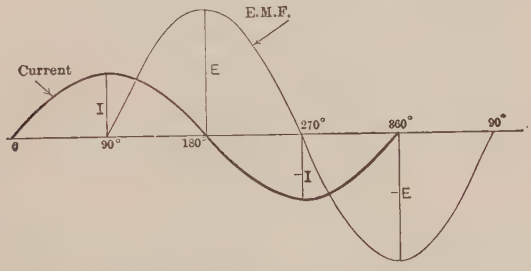


FIG. 347.—Sine curves showing current leading voltage by 90°. Circuit contains capacity only.

is fully charged and then cease. As long as the voltage across the condenser is rising, current will be flowing into the condenser, and the *most current will be flowing, when the voltage is rising most rapidly.* The voltage is rising most rapidly when it is at zero value as shown in Fig. 347. Thus the current must be greatest, when the voltage is zero, that is, the current's phase must be 90° when the voltage phase is zero. The current thus "leads" the voltage by 90°. In the same way the current flows out from the condensers at the greatest rate when the voltage is falling at the fastest rate. The voltage falls fastest at 270°, where it is zero. Thus current becomes a maximum at this point, and is still 90° ahead of the voltage.

Thus if an A.C. circuit contains capacity only, the current "leads" the voltage 90°. Fig. 348 shows the vector diagram of a current leading the voltage by 90°.

Fig. 349 is the conventional way of representing an A.C. circuit containing capacity only, the current equation for which is

$$i = I \sin (\phi + 90°).$$

Of course, if there is resistance or self inductance in series with the capacity, the "angle of lead" will not be 90°.

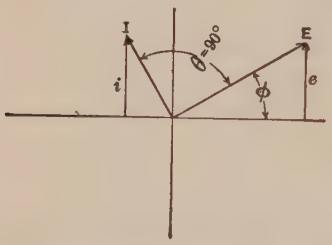


Fig. 348.—Vector diagram when current leads voltage by 90°.

Fig. 349.—Conventional diagram of A.C. circuit containing generator, *G*, and capacity, *C*.

The opposition which capacity offers to the flow of the current is called Capacity Reactance, and is measured in *Ohms* just as resistance is. The symbol for capacity reactance is x_c, and the equation for voltage and current relations in a circuit containing a capacity reactance of x_c, is similar to Ohm's law for the current-voltage relations in a circuit containing a resistance R. Thus we have the equation:

$$\text{Current} = \frac{\text{Voltage}}{\text{Capacity Reactance}},$$

or

$$I = \frac{E}{x_c}, \quad I_f = \frac{E_f}{x_c}, \text{ etc.}$$

where I = current through capacity reactance;
x_c = capacity reactance in ohms;
E = voltage necessary to force current through capacity reactance.

In using the equation, the *maximum* value of the current must be used with the *maximum* value of the voltage; the *effective* current with the *effective* voltage, etc.

It must be remembered that *the current value in this equation is always* 90° *ahead of the voltage value.*

Example. The voltage across a capacity reactance of 4 ohms is 120 volts. What is the current through reactance?

$$I_f = \frac{E_f}{x_c}.$$

$$= \frac{120}{4} = 30 \text{ amperes.}$$

Example. What is the maximum value of the current in above example?

$$I_f = .707I;$$

$$I = \frac{I_f}{.707}$$

$$= \frac{30}{.707} = 42.4 \text{ amperes.}$$

A vector diagram for this example is shown in Fig. 350. Note that the maximum value of the current (I) is 90° ahead of the maximum value of the voltage E.

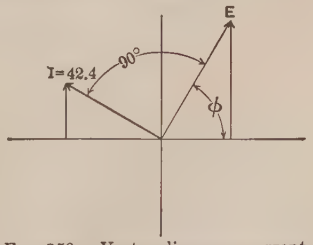


FIG. 350.—Vector diagram; current leading by 90°.

Problem 41–15. Draw vector diagram for above example with voltage at 15° phase and find instantaneous values of current and voltage.

Problem 42–15. A 110 volt A.C. circuit contains a condenser only. If the current is 2 amperes, what is the capacity reactance of the condenser?

Problem 43–15. When the instantaneous voltage in Problem 42 is 120 volts, what instantaneous value will the current have?

Problem 44–15. What voltage is necessary to force a maximum current of 20 amperes through a circuit containing 50 ohms of capacity reactance?

Problem 45–15. (*a*) Draw vector diagram and determine instantaneous value of voltage when instantaneous current in Problem 44 is 5 amperes. (*b*) In what phase will voltage be at that instant?

266. Computation of Inductive Reactance. In taking up the computation of Inductive Reactance, that is, the opposition offered to the flow of an alternating current by an inductance, it is necessary to have well in mind the meaning of the term INDUCTANCE and its measurement.

Inductance has been defined in Chapter X as the *inertia reaction* to any change in the current flowing in an electric circuit.

There is one unit of INDUCTANCE, that is, one HENRY, when a *change* of one AMPERE per sec. induces an E.M.F. of one VOLT.

This is expressed by the equation:

$$\text{Av. } E = L\frac{I}{t};$$

where average E = average *voltage* induced in circuit;
L = inductance of circuit in *henrys;*
I = change in current in *amperes;*
t = time of change in *seconds.*

This equation may be stated in words as follows:

The voltage induced in any circuit by change in the current is equal to the product of the inductance in henrys times the rate of change in amperes per sec.

Now in an alternating current, *during one cycle*, the current makes the following *four* changes, as per Fig. 351.

(1) The current rises from zero to maximum.
(2) " falls from maximum to zero.
(3) " rises from zero to maximum in opposite direction.
(4) " falls from this maximum to zero.

In each of the four changes, the current makes a complete change from zero to the maximum value I, or vice versa, in one direction or the other. The time of each complete change can be found as follows:

The number of cycles per sec. $=f$.

The time for one cycle $=\frac{1}{f}$ sec.

" each change $=\frac{1}{4}$ time for each cycle.

" " $=\frac{1}{4f}$ sec.

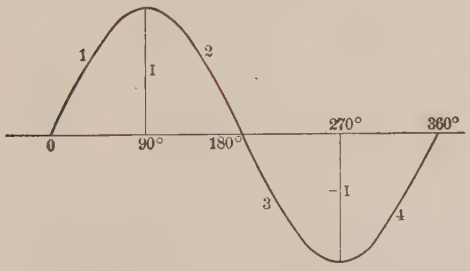


FIG. 351.—Sine curve of current during one complete cycle.

The rate of change, then, of an alternating current, must be I amperes in $\frac{1}{4f}$ sec., or $(4fI)$ amperes per sec.

Thus the expression,

$\frac{I}{t}$ (rate of change of current) must equal $4fI$.

The expression

$$\text{ave. } E = L\frac{I}{t};$$

becomes

$$\text{ave. } E = 4fLI.$$

That is, the average *voltage* induced in an inductive circuit equals *four* times the product of the *frequency* by the *inductance* by the *maximum current.*

If a circuit contains nothing but inductance, the impressed voltage merely has to overcome the induced voltage, and is therefore equal to it; just as in starting an unloaded machine which has no friction, an impressed force has to overcome merely the inertia reaction and must be equal to it.

Thus the average impressed voltage must equal the average induced voltage. The maximum voltage E is generally used in the equation, and this is found as follows:

$$\text{average } E = 4\,fLI,$$

but $$\text{average } E = .636E,$$

then $$.636\,E = 4\,fLI,$$

or $$E = 6.28\,fLI.$$

Since $$6.28 = 2\pi$$

the equation is generally written

$$E = 2\pi\,fLI.$$

This is the general equation for the voltage necessary to send an alternating current through an inductance, that is, through an inductive reactance.

But we have seen that the voltage necessary to send an alternating current through an inductive reactance was found by the following equation:

$$I = \frac{E}{x_L};$$

or $$E = IX_L.$$

But $$E = 2\pi\,fLI,$$

then $$IX_L = 2\pi fLI;$$

and $$X_L = 2\pi fL.$$

Therefore the value of an *Inductive reactance* **in *Ohms*** is equal to 2π times the *Frequency* times the *Inductance.*

Example. What is the inductive reactance of an A.C. circuit of .08 henry inductance, if the frequency is 60 cycles?

$$X_L = 2\pi fL$$
$$= 2 \times 3.14 \times 60 \times .08$$
$$= 30.2 \text{ ohms.}$$

Problem 46–15. What current will flow in circuit of above example if the voltage is 110 volts?

Problem 47–15. Draw vector diagram for Problem 46, when instantaneous value of the current is 2.5 amperes.

Problem 48–15. The inductance of an A.C. circuit is .2 henry, the voltage is 110 volts. At what frequency will the current be 3 amperes?

Problem 49–15. What will be the reactance of circuit in Problem 48–15?

Problem 50–15. A 550 volt A.C. circuit has a frequency of 133 cycles. What inductance must be placed in the circuit to limit the current to 40 amperes?

Problem 51–15. What instantaneous value will current in Problem 50 have when the voltage is at the 40° phase?

Probelm 52–15. A coil 40 cms. long, containing 85 turns of wire, is wound on a wrought-iron core 15 cms. in diameter ($\mu = 1000$). The wire is of such size that the resistance is practically zero. If this coil were put directly across a 110 volt A.C. circuit of a frequency of 60 cycles, what current would flow through coil? See equation in Chapter X for computing inductance of a coil.

267. Computation of Capacity Reactance. In a circuit containing capacity, such as a condenser, the current charges and discharges the condenser *four* times each cycle, as seen from Fig. 351, which represents the current curve in such a line:

(1) Charging condenser (positively).
(2) Discharging condenser.
(3) Charging condenser (negatively).
(4) Discharging condenser.

The time consumed for each complete charge or discharge, then, must be $\frac{1}{4f}$ sec. The quantity of charge in a condenser can be found by the equation (Chapter XI)

$$q = EC;$$

where q = charge in *coulombs;*
E = voltage across terminals of condenser;
C = capacity or (number of *coulombs* per volt).

But the quantity of water or electricity or anything that flows, must equal the average rate of flow times the time of flow; thus

$$q = \text{Av. } I \times t.$$

But

$$t = \frac{1}{4f}.$$

Therefore

$$q = \text{Av. } I \times \frac{1}{4f} = \frac{\text{Av. } I}{4f}.$$

Since also

$$q = EC,$$

then

$$EC = \frac{\text{Av. } I}{4f};$$

or

$$E = \frac{\text{Av. } I}{4fC};$$

but

$$\text{Av. } I = .636I.$$

Therefore

$$E = \frac{.636I}{4fC} = \frac{I}{6.28fC} = \frac{I}{2\pi fC} = \left(\frac{1}{2\pi fC}\right)I.$$

The usual form of the equation is

$$E = \left(\frac{1}{2\pi fC}\right)I.$$

The law may be stated in words as follows: The *voltage* necessary to cause an alternating current to flow through a line containing capacity only, equals the product of the *current* times the *reciprocal* of 2π times the *frequency* by the *capacity*.

But we have seen that the voltage to force a current to flow in a circuit containing capacity only, also equals:

$$E = X_c I$$

where X_c = the capacity reactance.

Therefore

$$X_c I = \left(\frac{1}{2\pi f C}\right) I;$$

and

$$X_c = \frac{1}{2\pi f C}.$$

Thus the value of *Capacity Reactance in Ohms* equals the *Reciprocal* of 2π times the *Frequency* times the *Capacity*.

Example. What is the capacity reactance of an A.C. circuit containing 35 microfarads capacity if the frequency is 60 cycles?

$$X_c = \frac{1}{2\pi f C}$$

$$= \frac{1}{6.28 \times 60 \times 35 \times 10^{-6}}$$

$$= \frac{1}{.0132}$$

$$= 75.8 \text{ ohms.}$$

Problem 53–15. What current will 550 volts force through circuit of above example?

Problem 54–15. What instantaneous value will voltage in Problem 53–15 have when the instantaneous current is 8 amperes?

Problem 55–15. Find the average value of the voltage across a condenser of 20 mfs. capacity, if a current of 3 amperes flows, when the frequency is 133 cycles.

Problem 56–15. What will the instantaneous current value be in Problem 55, when the voltage is at 80° phase?

Problem 57–15. The voltage in a 25~A.C. circuit is 550 volts. If a condenser of the following dimensions be placed directly across the line, what current will flow in circuit? 2000 plates of lead foil 80 cms. by 60 cms. Dielectric of waxed paper .008 cm. thick. See Chapter XI for equation for capacity of condenser.

Problem 58–15. Three condensers of 20, 80, and 50 mfs. capacity respectively, are placed in parallel across a 60 cycle A.C. circuit of 220 volts. What is maximum current in circuit?

268. Reactance, a Combination of Inductive and Capacity Reactance. Inductive reactance and capacity reactance have exactly opposite effects upon the phase relation of the current curve to the voltage curve.

Inductive reactance causes the current to "lag."

Capacity reactance causes the current to "lead."

Thus when the inductive and capacity reactance are combined in series, they tend to neutralize each other. The combined effect then is the difference between them, and is called the REACTANCE. The REACTANCE of such a series circuit is found by subtracting the capacity reactance from the inductive reactance. This is expressed by the equation:

$$X = X_L - X_c;$$

where X = reactance in *ohms;*
X_L = inductive reactance in *ohms;*
X_c = capacity reactance in *ohms.*

When the result is positive, the reactance is due to a larger inductive; when negative, to a larger capacity reactance.

Example. What is the reactance of a circuit containing 14 ohms inductive reactance in series with 5 ohms capacity reactance?

$14 - 5 = 9$ ohms reactance (inductive).

Example. What is the reactance of a 60 cycle A.C. circuit containing an inductance of 1.5 henrys in series with a capacity of 40 microfarads?

$$X = 2\pi fL$$
$$= 6.28 \times 60 \times 1.5$$
$$= 565 \text{ ohms.}$$
$$X_c = \frac{1}{2\pi fC}$$
$$= \frac{1}{6.28 \times 60 \times 40 \times 10^{-6}}$$
$$= \frac{1}{.0151}$$
$$= 66.3.$$
$$X = X_L - X_c$$
$$= 565 - 66$$
$$= 499 \text{ ohms reactance.}$$

Problem 59–15. What is the reactance of a 133 cycle A.C. circuit, which contains 20 mfs. capacity in series with 4 henrys inductance?

Problem 60–15. What current flows in circuit in Problem 59, if the voltage is 550 volts?

Problem 61–15. What current will flow in a 60 cycle, 220 volt circuit containing 2 henrys inductance and 25 mfs. capacity in series?

Problem 62–15. What current will flow in circuit of Problem 61, if frequency falls to 30 cycles, but everything else remains unchanged?

Problem 63–15. Draw vector diagram for Problem 61, and determine value of current when voltage is at 30° phase.

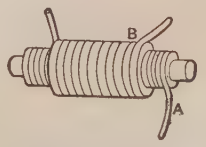


FIG. 352.—Induction coils with iron core.

Problem 64–15. If coil *B* of Fig. 352, containing 2 henrys inductance, and condenser of Problem 57–15 were placed in series in a 500 volt 60 cycle circuit, what would be the maximum current flowing in circuit?

269. Impedance, a Combination of Resistance and Reactance. In a circuit containing RESISTANCE only,

the current is in phase with the voltage. In a circuit containing REACTANCE only, the current either "leads" or "lags" 90°. The effect of reactance on the phase of the current is thus at an angle of 90° to the effect of resistance. The result, therefore, of a series combination of the two *cannot be found by simple addition or subtraction.* It can easily be found by the graphical method as follows:

Suppose it were required to find the result of a series combination of 4 ohms RESISTANCE and 3 ohms inductive REACTANCE. Draw horizontal line R to scale to represent 4 ohms resistance (Fig. 353). Then, since the effect of reactance is at right angles, 90°, to resistance, draw the line X_L to scale at right angles from end of R, to represent 3 ohms reactance. The line Z then represents the IMPEDANCE, or resultant of R and X_L. In this case, when scaled off, $Z=5$ ohms, which is neither the sum of 4 and 3 nor the difference.

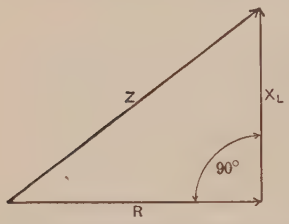


FIG. 353.—Graphical method of finding impedance from resistance and inductive reactance.

Since the lines R, X_L and Z always form a right triangle, of which Z is the hypothenuse, an equation can be formed, from which the value of Z can be found directly, without drawing the lines to scale. In every right triangle "the square of the hypothenuse equals the sum of the squares of the other two sides."

Thus

$$Z^2=R^2+X^2{}_L\,;$$

and

$$Z=\sqrt{R^2+X_L{}^2}.$$

In above problem,

$$\begin{aligned} Z&=\sqrt{4^2+3^2}\\ &=\sqrt{16+9}\\ &=\sqrt{25}\\ &=5 \text{ ohms} \end{aligned}$$

If the reactance is a capacity reactance, it is customary to draw the diagram as in Fig. 354, with the line representing the capacity reactance, X_c, drawn downward to show that it acts in the opposite direction to inductive reactance, which is drawn upward. The mathematical result and the equation are the same as in the case of inductive reactance.

$$Z = \sqrt{R^2 + X_c^2}.$$

If the circuit contains both inductance and capacity in series with the resistance, then the impedance can be

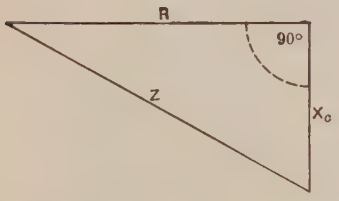


FIG. 354.—Graphical method of finding impedance from resistance and capacity reactance.

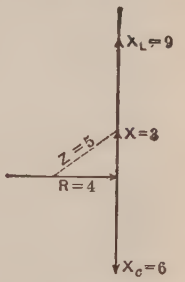


FIG. 355.—Graphical determination of impedance.

found by first subtracting the capacity reactance X_c from the inductive reactance X_L, and combining the result, X, with the resistance, R as per Figs. 353 and 354. The equation would then become:

$$Z = \sqrt{R^2 + (X_L - X_c)^2}$$
$$= \sqrt{R^2 + X^2}.$$

Fig. 355 shows the graphical method of obtaining the impedance in a series circuit of resistance, inductance and capacity.

Suppose 9 ohms inductive reactance and 6 ohms capacity reactance, are in series with 4 ohms resistance.

The resistance R is represented by the horizontal line R. The inductive reactance X_L by the line X_L drawn *up*

from end of R, at right angles to R. The capacity reactance is represented by the line X_c drawn *down* from end of R, at right angles to R. The resultant of X_L and X_c is found by subtracting X_c from X_L and is equal to X. That is:

$$\begin{aligned} X &= X_L - X_c \\ &= 9 - 6 \\ &= 3 \text{ ohms.} \end{aligned}$$

FIG. 355*a*.—Moore light installed in silk mill for matching colors.

The line Z is then drawn as the resultant of R and X. Z by scale $= 5$ ohms.

By the equation

$$\begin{aligned} Z &= \sqrt{R^2 + (X_L - X_c)^2} \\ &= \sqrt{4^2 + (9-6)^2} \\ &= \sqrt{16 + 9} \\ &= \sqrt{25} \\ &= 5. \end{aligned}$$

When inductance and resistance exist in the same piece of machinery as in an induction coil, the same current

must be forced through both reactance and resistance. The resistance and reactance are therefore considered to be in series.

The same applies to the capacity and resistance of a condenser, etc.

270. General Law for A.C. Circuits. The impedance of a circuit represents the total opposition to flow of the current. Thus the relation between voltage and current can be stated simply as follows:

The CURRENT flowing in an alternating current circuit equals the *quotient* of the VOLTAGE divided by the IMPEDANCE. This is sometimes called Ohm's law of the alternating current. It is represented by the equation

$$I = \frac{E}{Z},$$

where I = maximum, average or effective current in amperes;
E = respectively maximum, average, or effective voltage in volts.
Z = impedance in ohms.

Example. What current flows in an A.C. circuit of 110 volts and 4 ohms impedance?

$$I = \frac{E}{Z},$$
$$= \frac{110}{4} = 27.5 \text{ amperes.}$$

Example. A series circuit contains an induction coil of 10 ohms resistance, 6 ohms reactance, and a condenser of 25 ohms capacity reactance, negligible resistance. If voltage is 220 volts, what current flows?

The reactance of the induction coil must be inductive, thus:

$$X = X_L - X_c$$
$$= 6 - 25$$
$$= -19 \text{ ohms reactance.}$$

$$Z = \sqrt{R^2 + X^2}$$
$$= \sqrt{10^2 + (-19)^2}$$
$$= \sqrt{100 + 361}$$
$$= \sqrt{461}$$
$$= 21.5 \text{ ohms impedance.}$$
$$I_f = \frac{E_f}{Z}$$
$$= \frac{220}{21.5}$$
$$= 10.24 \text{ amperes.}$$

Problem 65–15. What is the impedance in a circuit containing 25 ohms inductive reactance, in series with 70 ohms capacity reactance and 40 ohms resistance?

Problem 66–15. What current will flow in circuit of Problem 65 if the voltage is 550 volts?

Problem 67–15. A 60 cycle 110 volt circuit has 12 ohms resistance in series with .2 henry inductance and 24 mfs. capacity. What is impedance of the circuit?

Problem 68–15. What is the maximum current in Problem 67?

Problem 69–15. How much would impedance of circuit be, if the frequency in Prob. 68 changed to 120 ~?

Problem 70–15. What would be the current in circuit of Problem 69?

271. Voltage and Current Relations in Series and Parallel Circuits. The relations existing between voltage and current in series and parallel DIRECT CURRENT circuits have been explained in Chapter III.

The same method of computing these relations will apply to alternating current circuits. Care must be taken, however, to take into account the effect of any difference in phase between the voltage and current.

272. Series Circuits. Voltage and Current Relations. The current flowing in a series circuit is of course the same throughout, whether it be D.C or A.C.

The VOLTAGE across a series combination equals the VECTOR SUM of the voltages across the separate parts.

This is exactly analogous to the rule for direct currents; the sum of the voltages in direct current being the *algebraic sum*, since the voltage and current are always in phase.

The meaning and application of the expression VECTOR SUM will be made clear by a study of the following illustrations.

Assume a series circuit (Fig. 356) to be made up of a reactance x, and a resistance R.

Let $E = IZ$ = voltage across the combination.
$E_1 = IX$ = voltage across the reactance.
$E_2 = IR$ = voltage across the resistance.

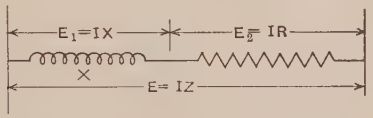


FIG. 356.—Series circuit containing resistance and capacity reactance.

Electromotive force E does *not equal the algebraic sum of* E_1 and E_2, because of the effect of the reactance on the phase relations between current and voltage.

Yet the electromotive force E equals the *vector sum* of the two electromotive forces E_2 and E_1, in exactly the same manner that a single mechanical force equals the *vector sum* of its two components, which are at an angle to each other. Thus the force F (Fig. 357), is equal to the VECTOR SUM of its two components F_1 and F_2. If, therefore, we represent the two electromotive forces E_1 and E_2 by lines drawn at the proper phase angles to each other, the electromotive force E will be represented by their *vector sum.*

In Fig. 358 OA is drawn to scale equal to $IR = E_2$. OB is drawn at right angles to OA equal to $IX = E_1$. The resultant or vector sum of OA and OB is OC, equal to $IZ = E$.

OB is drawn at an angle of 90° to OA, because the effect of reactance is always at 90° angle to the effect of resistance.

In Fig. 354 the line X_c is drawn at right angles to the line R for the same reason.

By scaling off the line OC, or E, we can find the value of the voltage across the series combination X and R.

Or the value of E may be computed from the equation for the right triangle

$$E^2 = E_1^2 + E_2^2 \text{ (since line } OB = \text{line } AC).$$

$$E = \sqrt{E_1^2 + E_2^2}.$$

Furthermore, the angle θ is equal to angle of phase difference between voltage and current in the circuit.

FIG. 357. — Graphical solution for resultant of two forces at right angles to each other.

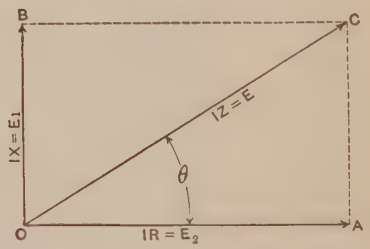


FIG. 358.—Graphical solution for voltage across series combination of resistance and reactance.

This will be apparent on consideration of the following points:

Resistance alone does not throw the current out of phase with the voltage. Therefore the current must have the same phase as the component IR or E_2, which is merely that component of the voltage across the resistance R.

Since the direction of E represents the direction of the voltage across the combination, and the direction of E_2 the direction of the current, the angle between E and E_2 must equal the angle of phase difference between voltage across the combination and the current through the combination.

If X is inductive reactance, the angle θ represents an angle of "lag," or the number of degrees the current is behind the voltage.

If X is capacity reactance, the angle θ represents an angle of "lead," or the number of degrees the current is ahead of the voltage.

This angle may either be measured or be ascertained by means of the following equation:

$$\frac{AC}{OA} = \tan \theta;$$

but $$AC = E_1 \text{ and } OA = E_2.$$

Thus $$\frac{E_1}{E_2} = \tan \theta;$$

or since

$$E_1 = IX \text{ and } E_2 = IR,$$

then $$\frac{E_1}{E_2} = \frac{IX}{IR} = \frac{X}{R} = \tan \theta.$$

Knowing the value of X and R, the tangent of the angle θ can be computed, and the angle θ found by reference to a table of tangents.

Example. In a series A.C. circuit consisting of 8 ohms inductive reactance and 10 ohms resistance a current of 6 amperes is flowing. (*a*) What is the voltage across each part of the circuit? (*b*) What is the voltage across the combination? (*c*) What is the difference in phase between the current and voltage in the combination?

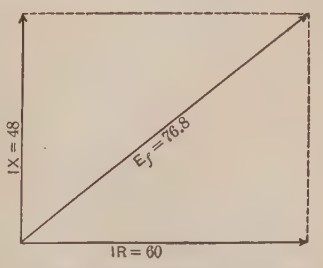


FIG. 359.—E equals effective voltage across X and R.

Solution. (*a*) $IX = 6 \times 8 = 48$ volts across inductive reactance X. $IR = 6 \times 10 = 60$ volts across resistance R. Draw IR to scale horizontally (Fig. 359). Draw IX to scale vertically.

Then E_f (the resultant voltage of IR and IX) to scale = 76.8

volts. Angle θ, the angle of difference in phase between voltage and current in circuit, measures 39°. By computation

$$Z=\sqrt{R^2+X^2}$$
$$=\sqrt{64+100}$$
$$=\sqrt{164.}$$
$$=12.8 \text{ ohms.}$$
$$E_f=I_fZ$$
$$=6\times12.8$$
$$=76.8 \text{ volts.}$$
$$\tan\theta=\frac{8}{10}=.8;$$
$$.8=\tan 38°\ 40';$$
$$\theta=38°\ 40'.$$

The current lags 38° 40′ behind the voltage, since X is INDUCTIVE reactance.

The vector diagram for current and voltage would therefore be like that of Fig. 360.

The diagram of Fig. 361 shows the phase relation of current and voltage by means of sine curves for current and voltage.

When voltage has reached the instantaneous value e_1, at the 39° phase, the current has a value of zero. The current reaches its maximum value I, 39° later than the voltage reaches its maximum value E. The current curve, then, is everywhere approximately 39° behind the voltage curve.

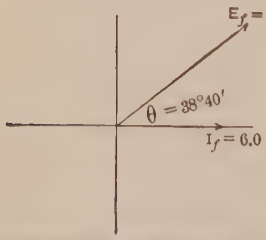


FIG. 360.—Vector diagram of current lagging 38° 40′ behind the voltage.

Note the fact, startling to students who have worked on D.C. circuits only, that the algebraic sum of the voltages across the separate parts (48 + 60 = 108) is greater than the actual voltage across the combination (76.8 volts).

THE COMBINED RESISTANCE of a number of separate resistances in series equals the algebraic sum of the separate resistances.

THE COMBINED REACTANCE of a number of reactances in series equals the algebraic sum of the separate reactances. (Inductive reactances being positive and capacity reactances negative.)

THE IMPEDANCE, or combined effect of series resistance and reactance, has been dealt with earlier in the chapter.

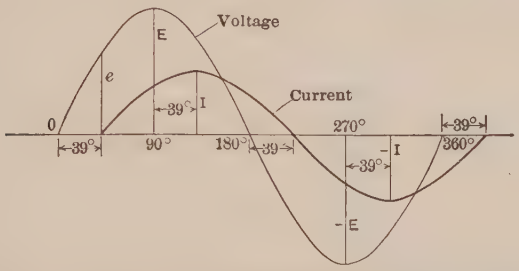


FIG. 361.—Sine curves of current lagging 39° behind the voltage.

THE COMBINED IMPEDANCE of a number of impedances in series equals the VECTOR SUM of the separate impedances.

Problem 71–15. Find the current flowing in an A.C. series circuit containing 16 ohms resistance and 20 ohms capacity reactance. Voltage is 220 volts.

Problem 72–15. Find the voltage across each part of circuit in Problem 71.

Problem 73–15. Find the phase relation between the current and voltage in Problem 71.

Problem 74–15. A 550 volt A.C. circuit has the following pieces in series across the line: (1) A resistance of 14 ohms: (2) a capacity reactance of 20 ohms: (3) 12 ohms inductive reactance. (4) 10 ohms resistance. (5) 4 ohms capacity reactance. Find: (*a*) What current flows through combination. (*b*) Voltage across each part. (*c*) Phase relation of current and voltage.

Problem 75–15. The voltage across R, Fig. 362, is 20 volts A.C. (*a*) Find voltage (E_f) across each part and across the

combination. (*b*) Find phase difference between current and voltage.

Problem 76–15. A coil containing 4 ohms resistance and .5 henry inductance is placed in series with a 15 mf. condenser of

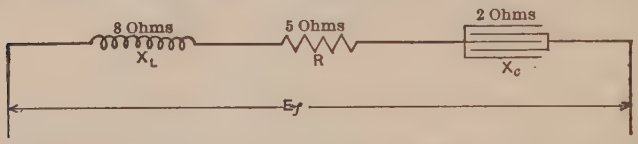


FIG. 362.—Series combination of resistance, inductive reactance, and capacity reactance.

2 ohms resistance, across a 60 cycle A.C. circuit of 550 volts. What current flows in circuit?

Problem 77–15. What is phase relation between current and voltage in Problem 76–15?

273. Parallel Circuits. Voltage and Current Relations. The VOLTAGE across a parallel combination is the same as the voltage across each branch, as in a D.C. circuit.

The CURRENT through a parallel combination is the *vector sum* or resultant of the currents through each branch.

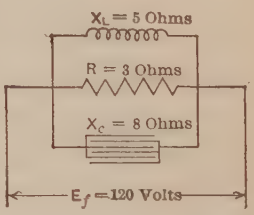


FIG. 363.—Parallel combination of resistance, inductive reactance and capacity reactance.

Example. In the parallel combination, Fig. 363, it is desired to find the current flowing through each branch, and the total current through the combination.

$$I_c = \frac{120}{8} = 15 \text{ amperes (current through } X_c),$$

$$I_R = \frac{120}{3} = 40 \quad \text{“} \quad (\quad \text{“} \quad R),$$

$$I_L = \frac{120}{5} = 24 \quad \text{“} \quad (\quad \text{“} \quad X_L).$$

$I_f \oplus I_c \oplus I_R \oplus I_L =$ current through combination.

Note. The sign $\oplus$ is to stand for the VECTOR SUM of the two quantities between which it is placed.

Draw I_R to scale, Fig. 364, to represent the current through the resistance R. At right angles to I_R (ahead of I_R), draw I_c to represent the current through X_c, leading the current in I_R by 90°. At right angles to I_R (behind I_R), draw I_L to represent the current through I_L, lagging 90° behind current in I_R. Construct the parallelogram $OACG$ and draw the diagonal OC. This represents the resultant of I_L and I_R.

That is, $OC = I_R \oplus I_L$.

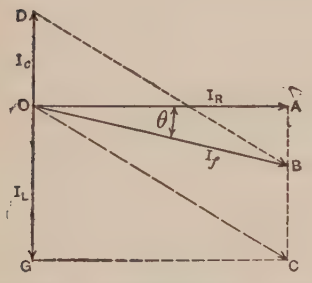


FIG. 364.—Graphical solution for the current in parallel combination of Fig. 363.

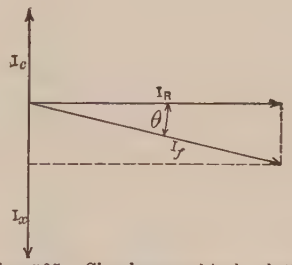


FIG. 365.—Simpler graphical solution for current in parallel combination of Fig. 363.

Construct a parallelogram $ODBC$, and draw the diagonal OB or I_f. This diagonal I_f represents the resultant of I_L and OC. That is, $I_f = OC \oplus I_c = I_R \oplus I_L \oplus I_c$.

Therefore I_f represents (to scale) the amount of the current in the parallel combination of R, X_c and X_L. The voltage must be in phase with I_R, the current through R. Thus the angle θ between I_f and I_R is a measure of the phase difference between voltage and the current of the combination. In this case θ represents an angle of lag, since the inductive reactance X_L is not entirely counteracted by the capacity reactance X_c. By measurement,

$$I_f = 41 \text{ amperes};$$
$$\theta = 13°.$$

Instead of Fig. 364, the simpler Fig. 365 may be drawn. The line I_x represents the current through the combined reactances, X_L, and X_c. The resultant current, I_f, through the whole combination, can then be found by the construction of but one parallelogram. Of course the results obtained are the same. The values may be obtained by computation as follows:

$$I_f = I_L \oplus I_c \oplus I_R;$$
$$= \sqrt{(I_L - I_c)^2 + (I_R)^2}$$
$$= \sqrt{(24-15)^2 + 40^2}$$
$$= \sqrt{9^2 + 40^2}$$
$$= \sqrt{1681}$$
$$= 41 \text{ amps. (current through combination).}$$

$$\tan \theta = \frac{I_L - I_c}{I_R} = \frac{9}{40} = .225;$$

From tables of tangents

$$.225 = \tan 12^\circ 41'.$$

274. Adding Alternating Current Curves. Currents in the different branches of a parallel circuit may be either

(*a*) in phase.

or

(*b*) out of phase.

(*a*) **In Phase.**

We will consider first the case where the currents are in phase. Assume that the currents I_1 and I_2 in the parallel circuits (Fig. 366), are in phase.
Then the equation

becomes,
$$I = I_1 \oplus I_2;$$
$$I = I_1 + I_2.$$

The curves for this condition are represented by Fig. 367. The current flowing in the part BC of the circuit is represented by the curve I. The current flowing in ADB is represented by the curve

I_1; the current through AKB, by the curve I_2. Note that the curve I is a combination of curve I_1 and I_2. Both currents I_1 and I_2 do not flow at once in BC, but merely one current, their resultant I.

Any instantaneous value of I is the sum of the corresponding values of I_1 and I_2. For instance, $i=i_1+i_2$.

or,

$$I \sin \phi = I_1 \sin \phi + I_2 \sin \phi.$$

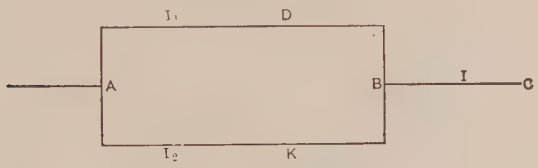


FIG. 366.—Parallel circuit.

(*b*) **Out of Phase.**

We will consider the following cases:

(1) When the current phases differ by 180°
(2) " " " 90°
(3) " " " θ° (general case).

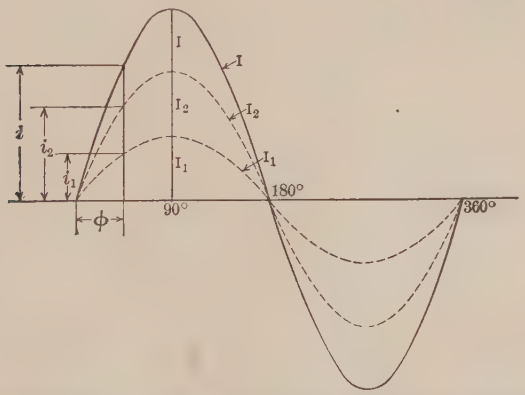


FIG. 367.—Current curve I equals sum of sine curves I_2 and I_1. I_2 and I_1 are in phase.

(1) **Currents 180° Different in Phase.**

If the current I_1 lags 180° behind I_2 Fig. 368 will represent the curves.

The equation

$$I = I_1 \oplus I_2;$$

becomes,

$$I = I_2 - I_1.$$

Any instantaneous value, such as i, equals $i_2 - i_1$, but i_2 and i_1 are not in the same phase. The equation for instantaneous value is,

$$i = I \sin \phi = I_2 \sin \phi + I_1 \sin (\phi + 180°).$$

or

$$i = I_2 \sin \phi - I_1 \sin \phi.$$

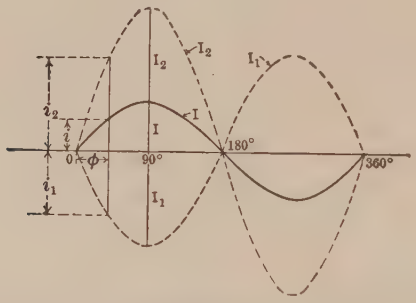


FIG. 368.—Current curve I equals the algebraic sum of the sine curves I_2 and I_1. I_1 is 180° behind I_2.

(2) **Currents 90° Different in Phase.** If the current I_2 lags 90° behind I_1, Fig. 369 represents the curves of the current in the different parts of the circuit. The equation for the resultant current I in part BC is,

$$I = I_2 \oplus I_1.$$

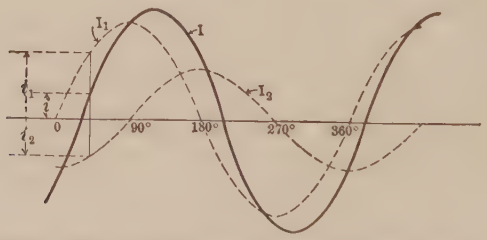


FIG. 369.—Current curve I equals the sum of the sine curves and I_1. I_2 lags 90° behind I_1.

Any instantaneous value can be found by adding algebraically the values of I_1 and I_2 at the same instant.
Thus in Fig. 369

$$i = i_1 - i_2$$

Thus, if two currents which differ in phase by 90° are added together, the resulting current will be the sum of the instantaneous

values of the two currents, and will not be in phase with either current. It may be said to differ in phase from either current by an angle of $\theta°$.

The instantaneous value can be found from the equation:

$$i = I \sin(\phi - \theta) = I_1 \sin \phi + I_2 \sin(\phi_2 - 90°).$$

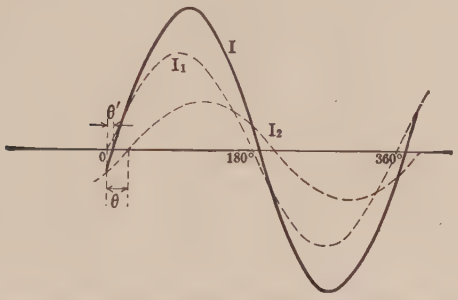


FIG. 370.—Current curve I equals the sum of sine curves I_2 and I_1. I_2 lags $\theta°$ behind I_1.

(3) **Currents $\theta°$ Different in Phase.** Let currents I_1 and I_2 differ in phase by $\theta°$.

Then I, Fig. 370, represents the sum of these two currents according to the equation,

$$I = I_1 \oplus I_2.$$

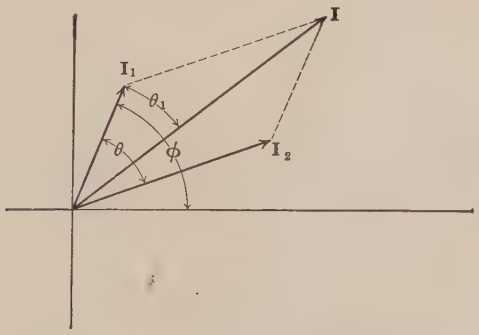


FIG. 371.—Vector diagram of currents in Fig. 370.

Since I_2 lags $\theta°$ behind I_1, the resultant of the two will be in phase with neither, but may be considered to lag $\theta_1°$ behind I_1. The equation for any instantaneous value then becomes,

$$i = I \sin(\phi - \theta_1) = I_1 \sin \phi + I_2 \sin(\phi - \theta).$$

Fig. 371 represents vector diagram of this case corresponding to curves in Fig. 370.

Thus the current flowing in the circuit of the above example on page 480 would be represented by a single curve I, Fig. 372, made up of curves I_R, I_L and I_c.

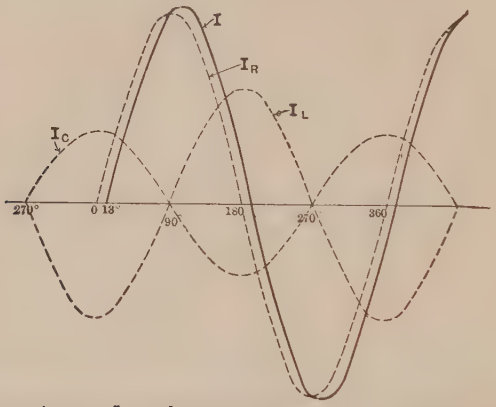


FIG. 372.—Current curve I equals sum of currents I_c, I_L, and I_R. I_L lags 90° behind I_R and 180° behind I_c.

Any instantaneous value on this curve would be made up of corresponding instantaneous values on the other two curves.

The curve I also lags 13° behind the voltage, which of course must be in phase with curve I_R. Any instantaneous value may be found from the equation

$$i = I \sin(\phi - 13°) = I_R \sin\phi + I_L \sin(\phi - 90°) + I_c \sin(\phi + 90°).$$

275. THE IMPEDANCE of a parallel combination can always be found without any special rule. Merely divide the CURRENT through the combination (found as above) by the VOLTAGE across the combination. If the voltage is not given, *assume* a voltage and proceed as above.

Example 1. What is the impedance of the parallel circuit of above example, page 480?

$$Z \text{ (of combination)} = \frac{E_f \text{ (across combination)}}{I_f \text{ (through combination)}},$$

$$Z = \frac{120}{41} = 2.93 \text{ ohms.}$$

Example 2. A circuit arranged as in Fig. 373 presents all fundamental combinations. (*a*) Find the value of the impedance when the following values are known. (*b*) What is the difference in phase between current and voltage?

$X_L = 6$ ohms. $R_2 = 8$ ohms. $X_{L2} = 3$ ohms.
$R_1 = 4$ ohms. $X_c = 10$ ohms. $R_3 = 5$ ohms.

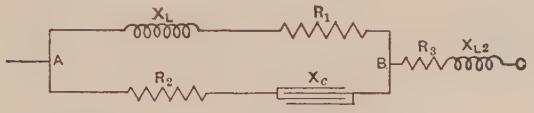


FIG. 373.

To find impedance (Z_1) and phase angle of branch composed of X_L and R_1 in series, draw Fig. 374.

$$Z_1 = 7.2 \text{ ohms},$$
$$\theta_1 = 56°.$$

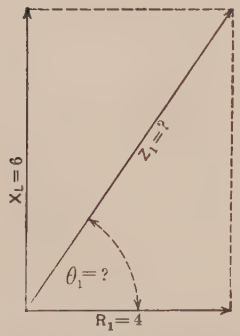


FIG. 374.—Graphical solution for impedance of series combination X_L and R_1 in Fig. 373.

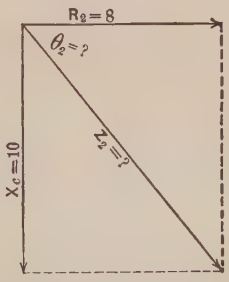


FIG. 375.—Graphical method of finding impedance of series combination R_2 and X_c; Fig. 373.

By computation:

$$Z_1 = \sqrt{R_1^2 + X_L^2}$$
$$= \sqrt{16 + 36}$$
$$= \sqrt{52}$$
$$= 7.22 \text{ ohms}.$$
$$\tan \theta_1 = \tfrac{6}{4} = 1.5,$$
$$\theta_1 = 56° 20'.$$

The impedance of branch composed of X_L and R_1 in series is 7.22 ohms.

The current through this branch of the circuit lags 56° 20′ behind the voltage.

To find Impedance Z_2 and phase relations of branch composed of R_2 and X_c draw Fig. 375.

By measurement:

$$Z_2 = 13 \text{ ohms.} \quad \theta_2 = 51°.$$

By computation:

$$Z_2 = \sqrt{R_2^2 + X_c^2}$$
$$= \sqrt{64 + 100}$$
$$= 12.8 \text{ ohms.}$$
$$\tan \theta_2 = \tfrac{10}{8} = 1.25.$$

From table, $\theta_2 = 51° 20'$.

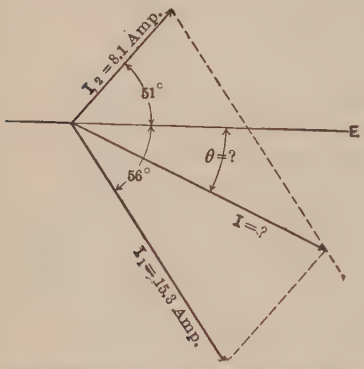


FIG. 376.—Graphical method of finding current through parallel combination in Fig. 373.

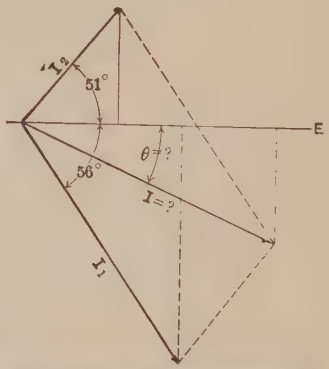


FIG. 377.—Analytical method of finding current through parallel combination in Fig. 373.

Impedance of branch composed of R_2 and $Xc = 12.8$ ohms. The current through this branch leads the voltage by 51° 20′. To find the combined impedance of above branches in parallel draw Fig. 376. By assuming a voltage of 110 volts across AB, the following current must flow:

Through Z_1,

$$I_1 = \frac{E}{Z_1} = \frac{110}{7.22} = 15.3 \text{ amperes}$$

Through Z_2, $$I_2 = \frac{E}{Z_2} = \frac{110}{12.8} = 8.6 \text{ amperes.}$$

In Fig. 376, E represents the phase of the voltage E. Current through Z_1 lags 56° 20′ behind E; then the line, I_1 of 15.3 amperes, at an angle of 56° 20′ lag to E, represents the current through Z_1. Similarly line I_2 represents a current of 8.1 amperes, 51° 20′ ahead of E in phase. The line I, resultant of I_1 and I_2, represents the current through the parallel combination; and the angle θ represents the difference in phase between the current through combination and voltage across combination.

By measurement,

$$I=15 \text{ amperes.} \quad \theta=25°.$$

By computation (referring to Fig. 377),

$$\begin{aligned} I &= I_1 \oplus I_2 \\ &= \sqrt{(I_2 \cos 51° + I_1 \cos 56)^2 + (I_1 \sin 56° - I_2 \sin 51°)^2} \\ &= \sqrt{(5.1+8.54)^2 + (12.7-6.3)^2} \\ &= \sqrt{(13.64)^2 - (6.4)^2} \\ &= \sqrt{186+41} \\ &= \sqrt{227} \\ &= 15.1 \text{ amperes.} \end{aligned}$$

$$\tan\theta = \frac{6.4}{13.64} = .469,$$

$$\theta = 25°.$$

The current through the parallel combination thus equals 15.1 amperes and lags 25° behind the voltage.

The impedance of the parallel combination AB can now be found by the equation:

$$Z = \frac{E}{I},$$

$$= \frac{110}{15.1} = 7.3 \text{ ohms.}$$

The impedance Z of the series combination BC can be found from Fig. 378.

By measurement:

$$Z_3 = 5.8 \text{ ohms.}$$

$$\theta_3 = 31°.$$

By computation:

$$Z_3=\sqrt{R_3^2+X_{L_2}^2}$$
$$=\sqrt{25+9}$$
$$=5.84 \text{ ohms.}$$
$$\tan\theta=\tfrac{3}{5}=.6,$$
$$\theta=31°.$$

The impedance (Z_3) of the series circuit BC is 5.84 ohms, and its effect is to cause the current through it to lag 31° behind the voltage.

But in series with above impedance Z_3, is the impedance Z of the parallel circuit AB, which causes the current to lag

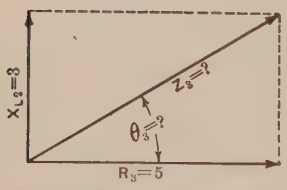


FIG. 378.—Graphical method of finding impedance of series combination R_3 and X_{L_2}.

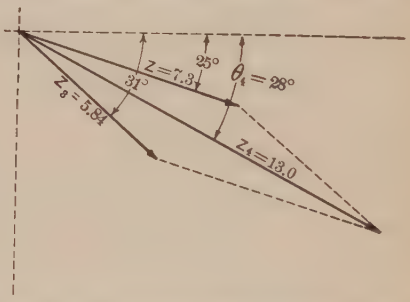


FIG. 379.—Graphical method of finding combined impedance of Z and Z_3.

25° behind the voltage across BC. The combined impedance of these two (Z and Z_3) can be found from Fig. 379.

By measurement:

$$Z_4=13 \text{ ohms},$$
$$\theta_4=28°.$$

By computation:

$$Z_4=\sqrt{(Z\sin 25°+Z_3\sin 31°)^2+(Z\cos 25°+Z_3\cos 31°)^2}$$
$$=\sqrt{(3.08+3)^2+(6.61+5)^2}$$
$$=\sqrt{6.08^2+11.61^2}$$
$$=\sqrt{172}$$
$$=13.1 \text{ ohms.}$$
$$\tan\theta_4=\frac{6.08}{11.61}=.524.$$

From table, $\theta_4 = 27° 40'$.

Thus the impedance of the series-parallel combination is 13.1 ohms, and the phase difference between current and voltage is 27° 40′ lag.

275*a*. Summation of Voltage, Current and Impedance Relations in Series and in Parallel Combinations.

(1) SERIES.

Voltage across combination equals vector sum of separate voltages.

Current is same throughout series combination.

Impedance equals vector sum of separate impedances.

(2) PARALLEL.

Voltage is same across each branch of combination.

Current equals vector sum of separate currents.

Impedance equals (assumed) voltage divided by the current which (assumed) voltage forces through combination.

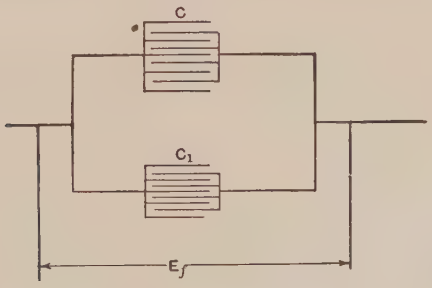


FIG. 380.—Capacities in parallel.

Problem 78–15. What current flows through a circuit connected as in Fig. 380 under following conditions?

C has a resistance of 200 ohms.
C " capacity of 20 mfs.
C_1 " resistance of 400 ohms.
C_1 " capacity of 40 mfs.
$E_f = 110$ volts, 25 cycles.

Problem 79–15. What is difference in phase between E_f and I_f of circuit in Problem 78–15?

Problem 80–15. What current would flow in circuit of Problem 78–15 if the condensers C and C_1 had no resistance?

Problem 81–15.

L has 40 ohms resistance and .4 henry inductance, Fig. 381.

L_1 has 80 ohms resistance and .8 henry inductance.

$E = 110$ volts, 25 cycles.

What current will flow through above circuit?

What will be the phase relations between current I_f and voltage E_f?

Problem 82–15. In Fig. 382,

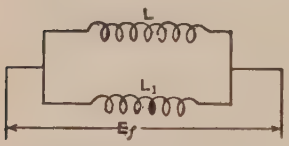


FIG. 381.—Inductances in parallel.

$L_1 = .6$ henry;
$L_2 = .3$ henry;
$C_1 = 40$ mfs.;
$C_2 = 20$ mfs.;
$R_1 = 10$ ohms;
$R_2 = 50$ ohms;
$R_3 = 3$ ohms;
$R_4 = 4$ ohms.
$E_f = 220$ volts, 60 cycles.

Find: Current through AB;
" CD;
" FG.

Phase relation of current through FG to voltage across FG.

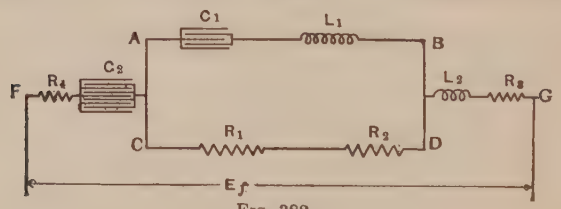


FIG. 382.

276. Power in A.C. Circuits. The power taken at any instant by an alternating current circuit equals the product of the voltage at that instant times the current at that instant. In the form of an equation it may be written:

$p = ie$;

p = instantaneous power in *watts;*

e = " voltage in *volts;*

i = " current in *amperes.*

The power curve can then be drawn, each point on which shall be the product of the value of the current times the value of the voltage at that instant.

The question of power consumed in an alternating current circuit is divided into two parts:

(1) When the current and voltage are IN PHASE.

(2) When the current and voltage are OUT OF PHASE.

(1) Fig. 383 represents such a curve, when the current and voltage are *in phase.*

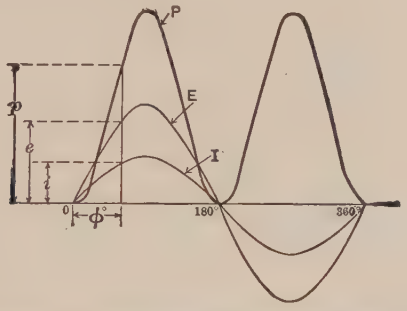


FIG. 383.—Power curve P equals product of sine curves E and I. E and I in phase.

Any value as p, on the power loop P, is the product of the corresponding value of the voltage e, on the voltage loop, E, times the current i, on the current loop I.

Note that all the power loops are positive (above the axis), even when the current and voltage are negative. This is for the reason that the signs of the current and voltage values change at the same instant, and thus the product is always a product of *two positive* values or of *two negative* values; either of which products is always positive.

Therefore, when the current and voltage are in phase the power is all positive. That is, no power is flowing backward and being returned to the generator. The effective power in such a circuit, then, must be the total of the product of the effective voltage times the effective current. The

equation therefore for a non-inductive circuit, or for a circuit containing resistance only, is

$$P = E_f I_f.$$

Another way of considering this case is to note that there being no reactance in the circuit all the power must be expended in overcoming resistance. The power used to overcome resistance at any instant must be i^2R, or;

$$p = i^2R.$$

The average of this power would be $Av\ i^2R$ or,

(1) $$P = Av\ i^2R.$$

But by definition

$$\sqrt{Av\ i^2} = I_f.$$

Therefore,

$$Av\ i^2 = I_f^2;$$

and since

$$P = Av\ i^2R;$$

then

(2) $$P = I_f^2R.$$

But by Ohm's law, the voltage to force a given current through a given resistance is

$$E_f = I_fR.$$

Then, substituting this value for I_fR in equations (2), we have,

$$P = I_fE_f.$$

277. (2) Power in Inductive Circuits. Power Factor. Fig. 384 represents the power curve, when the voltage and current are $\theta°$ OUT OF PHASE. We have seen that such a case exists, when the circuit contains reactance. The same equation for instantaneous power holds true—the instan-

taneous power equals the product of the current value times the voltage value at the same instant, or

$$p = ie.$$

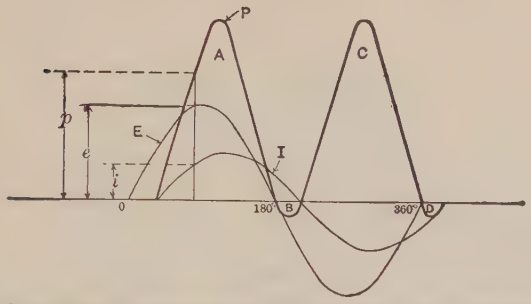


FIG. 384.—Power curve equals product of current curve *I* and voltage curve *E*. *I* lags behind *E*.

Any point in the power curve such as p is the product of the instantaneous values i and e on the voltage and current curves.

But note that the power loops B and D are negative; that is, they are the product of a negative value of voltage times a positive value of current, or vice versa. The power then is not all flowing one way. Part of it, represented by the small negative loops B and D, is being returned to the generator. The average power used by such a circuit is then no longer the entire product of the effective value of voltage times the effective value of the current. If the circuit contained resistance only, such would be the case, as we have seen above. Therefore that part of the power which is used to overcome resistance only in any circuit must be the power consumed in the circuit.

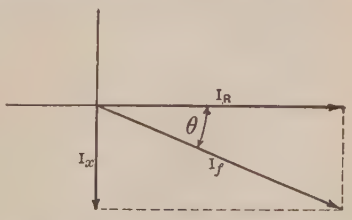


FIG. 385.—Graphical method of resolving a current I_f into two components, I_x and I_R, at right angles to each other.

By means of a vector diagram as Fig. 385, we can see what fraction of the power this must be in each case.

Let I_f represent the effective current lagging $\theta°$ behind the voltage. We may consider this current I_f as composed of two components, I_R and I_x. The component I_R is in phase with the voltage, and the component I_x is at right angles to the voltage.

Now since I_R is the only component of current, to maintain which power is required, we may say that the average power in the circuit equals the effective voltage times the component of the effective current in phase with voltage. The equation would then be

$$P = E_f I_R;$$

but

$$I_R = I_f \cos \theta;$$

therefore

$$P = E_f I_f \cos \theta.$$

This is the general equation for power consumed in any electric circuit.

The term "cos θ" is called the "POWER FACTOR" since it is the factor by which the *apparent power* (amperes times volts) must be multiplied in order to obtain the true power.

When the current is in phase with the voltage, $\theta = 0$ and cos $\theta = 1$. The power factor is then said to be one, or UNITY.

The load which is most likely to produce a lagging current is that which is composed of a number of induction motors. Fig. 385*a* represents a motor of this type. It has the advantage of using no commutator or collecting rings.

278. Wattless Component of Current. Since there is no power consumed in forcing the component of current I_x (Fig. 385), through the circuit, this component is sometimes called the WATTLESS COMPONENT of the current.

The reason why it requires no power to force the wattless component through the circuit can be understood, if we consider that the power taken by this current is used either to charge a condenser, when of course this charge all flows back into the generator, or to set up a counter electromotive force in a coil, which of course helps send

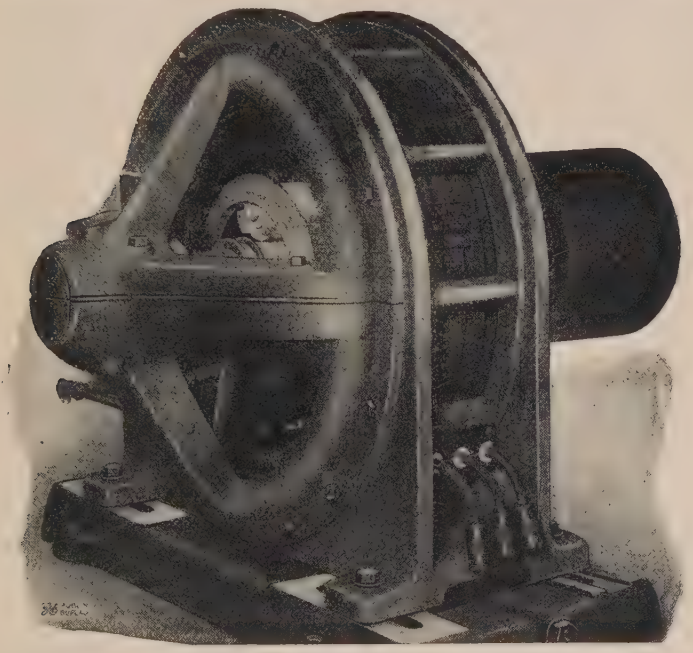


FIG. 385a.—Induction motor. Takes lagging current and reduces power factor of the line.

the current back toward the generator, as the E.M.F. reverses.

279. Use of Ammeter and Voltmeter to Measure Power. Wattmeter. Since A.C. voltmeters and ammeters indicate effective values, when a voltmeter and ammeter are used to measure power, the product of the volts times the amperes or volt-amperes does not always give the true power. If the current "leads" or "lags," the angle of lead (or lag)

must be known, so that the power factor, (cos θ), may be used to reduce the volt-amperes to watts.

A wattmeter, however, is so constructed that it indicates the true power in an A.C circuit. Its readings, therefore, do not need to be multiplied by the power factor (cos θ).

Example 1. In a certain A.C. circuit where the current lags 30°, an ammeter indicates 20 amperes and a voltmeter 115 volts. What would a wattmeter indicate?

A wattmeter would read power directly, or

$$P = 20 \times 115 \times \cos 30°$$
$$= 20 \times 115 \times .866$$
$$= 2000 \text{ watts.}$$

Example 2. There are 5 amperes flowing in an A.C. circuit under a pressure of 110 volts. If the current is in phase with the voltage, what power is consumed by the circuit?

$$P = I_f E_f \cos \theta,$$

Here $\cos \theta = 1.$

Therefore, $P = I_f E_f.$

Fig. 386 shows vector diagram for this case

$$= 5 \times 110$$
$$= 550 \text{ watts.}$$

FIG. 386.—Vector diagram. Current and voltage in phase.

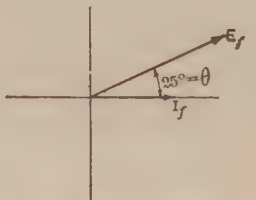


FIG. 387.—Vector diagram. Current lags 25° behind voltage.

If the voltage in the above example leads the current by 25°, what power is being consumed?

$$P = I_f E_f \cos \theta;$$
$$= 5 \times 110 \times \cos 25°$$
$$= 550 \times .905$$
$$= 498 \text{ watts.}$$

Fig. 387 shows vector diagram for a case where voltage leads current by 25°.

Example 3. A 110 volt A.C. circuit contains 40 ohms resistance. What is the average power in the circuit?

$$I_f = \frac{E_f}{R};$$

$$\frac{110}{40} = 2.75 \text{ amperes.}$$

$$P = I_f E_f$$
$$= 2.75 \times 110$$
$$= 303 \text{ watts.}$$

Fig. 388.—Impedance Z, composed of resistance R and reactance X.

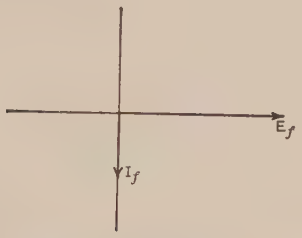


Fig. 389.—Vector diagram. Current lagging 90° behind voltage. No component in phase with voltage.

If the circuit in above example 3 contained 10 ohms resistance and 30 ohms reactance, what would be the average power in the circuit?

Referring to Fig. 388:

$$Z = \sqrt{R^2 + x^2};$$
$$= \sqrt{30^2 + 10^2}$$
$$= \sqrt{1000}$$
$$= 31.6 \text{ ohms;}$$

$$I_f = \frac{E_f}{Z}$$
$$= \frac{110}{31.6} = 3.48 \text{ amperes.}$$

$$\tan \theta = \frac{30}{10} = 3;$$
$$\theta = 71° 65';$$
$$P = I_f E_f \cos \theta$$
$$= 3.48 \times 110 \times .313$$
$$= 120 \text{ watts.}$$

Example 4. Assume that the 40 ohms in above example 3 were reactance, what power would be consumed by the circuit?

Reactance alone in a circuit causes the current and voltage to differ in phase by 90°. There is then no component in phase with the voltage as shown by Fig. 389 and the power should be zero. Fig. 390 indicates this, in that it shows that the negative power loops are as large as the positive power loops, when the phase difference is 90°. Thus all the power is being returned to the generator. The equation shows that no power is being used.

$$P = I_f E_f \cos \theta;$$
$$\theta = 90°;$$
$$\cos 90° = 0;$$

Therefore,
$$P = I_f E_f \times 0$$
$$= 0.$$

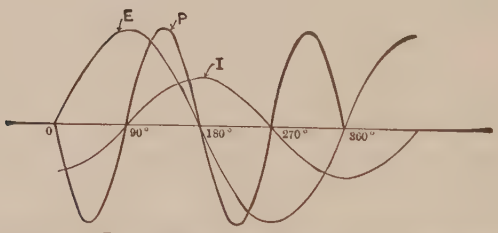


FIG. 390.—Power curve P has as large negative as positive loops when current and voltage differ 90° in phase.

Problem 83–15. What power is consumed in a circuit containing 8 ohms impedance? Voltage = 110 volts. Power factor = .9.

Problem 84–15. What is the power factor in a circuit which contains 20 ohms resistance and 4 ohms reactance?

Problem 85–15. What power is consumed in circuit in Problem 84, if the voltage is 220 volts?

Problem 86–15. How many amperes are there in the wattless current in Problem 85–15?

Problem 87–15. A 110 volt 60 cycle circuit contains 10 ohms resistance. (*a*) What current flows? (*b*) What power is consumed?

Problem 88–15. How much would current in Problem 87 be reduced to, if a coil of negligible resistance and .2 henrys inductance were placed in series with the 10 ohm resistance?

Problem 89–15. What power would be consumed by circuit in Problem 88?

Problem 90–15. A coil of .015 henry inductance and 2 ohms resistance is placed on a 25 cycle 110 volt circuit. (*a*) What is power factor? (*b*) What power is consumed by coil?

Problem 91–15. An ammeter inserted in a 110 volt A.C. circuit indicates 14 amperes. Wattmeter reads 1.4 K.W. What is power factor and phase difference between current and voltage?

280. To Determine the Necessary Brush Potential of A.C. Generators. (1) *When the Power Factor of the Load is Unity.* Assume required voltage V at motor $M = 110$ volts, current of $M = 40$ amperes. Current of M in phase with voltage. Line resistance $= .2$ ohm. No reactance in line.

Solution. Since there is no reactance in the circuit, the current throughout is in phase with the voltage. The voltage across the combination of series resistances then equals the sum of the separate resistances, as in a D.C. circuit.

Voltage across motor $= 110$ volts
Voltage lost in line $= .2 \times 40 = 8$ volts

Volts required at generator $= 118$ volts

Power delivered at generator to line $= 118 \times 40 = 4720$ K.W.

FIG. 391.—A D.C. shunt generator feeding a bank of lamps.

This is exactly the same as though a D.C. generator, Fig. 391, were feeding 40 amperes to a bank of lamps at 110 volts over a line of .2 ohm. The voltage of the generator would have to be 118 volts.

(2) *When the Current in Load Lags.* Assume same power is taken by motor, except that the current through motor lags so as to produce a power factor of .90, i.e., $\cos \theta = .90$.

Power consumed by motor in (1) $=110\times40=4.4$ K.W. Power in motor in this example must equal 4.4 K.W. also. But power $=EI\cos\theta$. If we assume that the voltage is to remain 110 volts, the current through motor can be found by the above equation.

$$4400=110\times I\times.90;$$

$$I=\frac{4440}{99}=44.4 \text{ amperes.}$$

That is, when the current through motor lags, it must be larger, in order that the power remain the same as when the power factor was unity, voltage remaining constant.

Construct vector diagram as per Fig. 392.

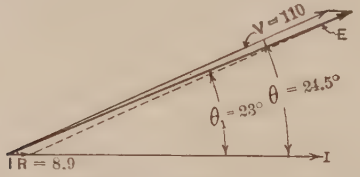


FIG. 392.—Voltage E is vector sum of IR and V.

Draw line I of indefinite length to represent current phase. Voltage to force current through line resistance of .2 ohm equals 8.9 volts and is in phase with current. Therefore, draw IR along I to scale to equal 8.9.

$$\text{P.F.}=.90;$$

Thus

$$\cos\theta=.90;$$

$$\cos 24.5^\circ=.90.$$

Therefore the angle between voltage and current of motor equals 24.5°.

Draw line V to scale to equal 110 at an angle of 24.5° to line I.

The brush potential E of generator, then, equals the vector

sum, or resultant, of IR and V. Construct parallelogram and scale off E.

By this method $E=117$ volts, angle θ_1, between current and generator voltage $=23°$.

Power delivered by generator equals

$$\begin{aligned} P &= 117\times 44.4 \cos 23° \\ &= 117\times 44.4\times .92 \\ &= 4.780 \text{ K.W.} \end{aligned}$$

By computation,

$$\begin{aligned} E &= I \oplus RV; \\ &= \sqrt{(IR+.90\times 110)^2+(V \sin 24.5)^2} \\ &= \sqrt{(8.9+99)^2+(45.7)^2} \\ &= \sqrt{11700+2080} \\ &= 117 \text{ volts.} \end{aligned}$$

The angle θ_1 between voltage and current of generator, can be found by the following equation.

$$\begin{aligned} \tan \theta_1 &= \frac{V \sin 24.5}{IR+.90V} \\ &= \frac{45.7}{108} \\ &= .423 \\ &= 22.9°; \end{aligned}$$

$$\begin{aligned} \text{Power} &= 117\times 44.4\times \cos 22.9° \\ &= 117\times 44.4\times .92 \\ &= 4.780 \text{ K.W.} \end{aligned}$$

The power delivered by the generator, when the current is out of phase with the voltage must be more than when current is in phase, in order to supply motor with given constant load, over line of given resistance.

(3) *When the Load Consists of Parallel Pieces with Different Power Factors.* In Fig. 393, M takes 40 amperes at 110 volts. Power factor = 85%. Each lamp between FB takes 10 amperes. Power factor of lamps, unity.

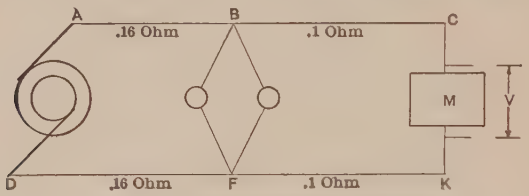


FIG. 393.—A.C. generator feeding parallel pieces.

Find:

(1) Current through each section of line.
(2) Voltage across BF.
(3) Voltage across generator.
(4) P.F. of generator load.
(5) Efficiency of transmission.

Draw Fig. 394 similar to Fig. 392, making $IR = 8$ and the angle between V, voltage across motor, and I, current through motor = 32°, since cos 32° = .85 = power factor.

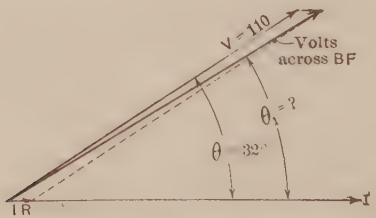


FIG. 394.—Graphical method of finding voltage across BF, Fig. 393.

The resultant of IR, 8 volts and V, 110 volts, by measurement equals 117 volts. Angle θ_1, between current and voltage in line $BCKF$ = 30°.

The current in line AB and FD can now be found by adding vectorially currents of 40 amperes lagging 30° behind the voltage and of 20 amperes in phase with volt-

age, as in Fig. 395. By measurement, current I in AB and $DF = 58$ amperes. Angle between current in line and voltage of line $= 20°$. Voltage to force this current through section of line AB and $DF = .32 \times 58 = 18.6$ volts. Voltage

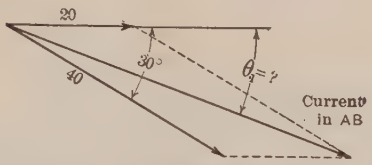


Fig. 395.—Graphical method of finding current in AB, Fig. 393.

across generator can be found by means of vector diagrams in Fig. 396. Voltage E of generator $= 134$ volts $\theta_2 = 17.5°$.

Power factor of load $= \cos 17.5° = .95.$

Power delivered by generator $= 134 \times 58 \times .95 = 7.400$ K.W.

Power used in motor $= 110 \times 40 \times .85 = 3.74$ K.W.

Power used in lamps $= 117 \times 20 \times 1 = 2.34$ K.W.

Total useful power. $= 6.08$ K.W.

Efficiency of transmission $= \dfrac{6.08}{7.40} = 82.2\%.$

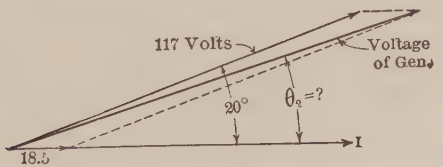


Fig. 396.—Graphical method of finding voltage across generator, Fig. 393.

It is clear from above example that the lamps, having unity power factor, raise the power factor of the load and that if some piece of electrical machinery having capacity reactance were placed in parallel with the motor and lamps, the power factor could be raised still more.

281. Two-phase Distribution. In the distribution of Direct Current, we have seen that two generators may be used in series and the power be distributed by three wires.

This allows a voltage to be obtained across the outside of the two generators which is twice that between either wire and the neutral, as explained in Chapter IX. By means of a special armature, one generator is usually made to supply current to this three-wire system, though the principle is the same as in the two-generator plan.

FIG. 396*a*.—Revolving field of alternator-coils excited from separate source of direct current.

In the same way two A.C. generators can be run in series and made to supply current to a three-wire system. But the voltage across the outside of the generator system, *need not necessarily be twice that between an outside wire and the neutral*, depending on whether or not the voltages of the two generators were in phase. This voltage, however, would *always be the vector sum* of the voltages of the separate generators.

In the case of A.C. generators as in D.C., the armature is wound so that one generator delivers the power to the three wires. Each leg is then called a PHASE, and the system is called a TWO-PHASE SYSTEM. For several reasons, the voltages of the two phases *are not in phase*, but at an angle of 90° to each other.

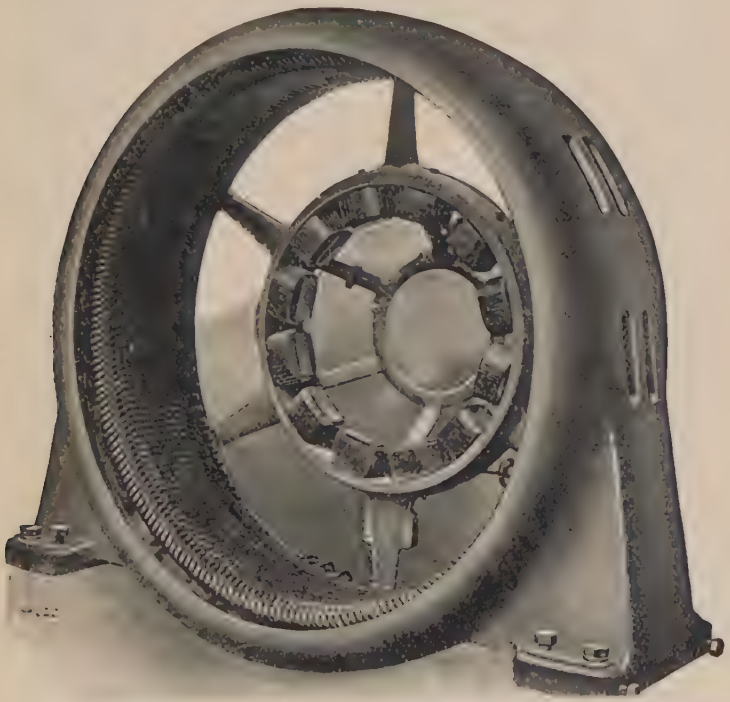


FIG. 396*b*.—Stationary armature of alternator. No moving wires or contact points to insulate for the high voltage generated.

Figs. 396*a* and *b* show the usual construction of alternating current generators. The field revolves while the armature is stationary. In using high voltages better insulation can be obtained by this construction.

The construction and action of such a machine can easily be understood by reference to the following diagrams.

Figs. 310 to 316 inclusive, show the action of a single coil, single-phase generator. Now if another coil be added at right angles to the coil of Figs. 310 to 315, as in Fig. 397, the actions will be the same in each coil, with the exception that they will take place 90° later in one coil than in the other. Thus, when the voltage across coil AB is at a maximum, as in Fig. 397, the voltage across $A'B'$ is zero, and vice versa. The voltages may be represented, then, by two curves differing in phase by 90°, as in Fig. 398. The vector diagram would be that of Fig. 399, in which E equals voltage induced in AB, and E_1 the voltage induced in $A'B'$.

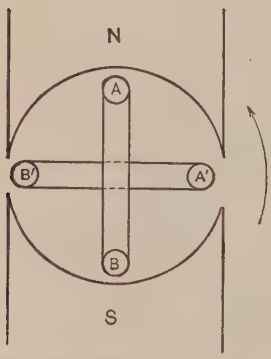


FIG. 397.—Simple 2-phase 2-pole generator. Coils are at 90° to each other.

The internal armature connections and the connections to the receiving circuit determine the amount of the voltage

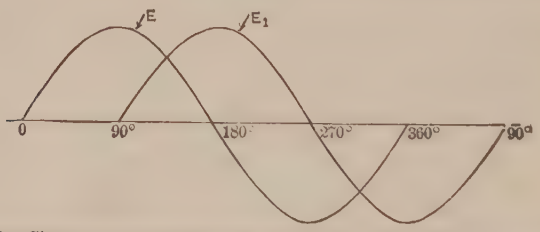


FIG. 398.—Sine curves of the phase relations of the E.M.F.'s in the phases of a 2-phase generator.

delivered to the line. There are three ways of making these connections:

(1) The ends A and B may be brought out to one pair of collecting rings, and A' and B' may be brought out to another pair. In that case, we would have two separate single-phase lines, in which the voltages would be identical

in value but differing in phase by 90°. This might be called a four-wire, two-phase system.

The voltage-current relations in each phase could be treated entirely separately as single-phase relations. Diagrams of these conditions are shown in Figs. 400 and 401, the latter being the conventional representation. Note that the phases are absolutely distinct from each other.

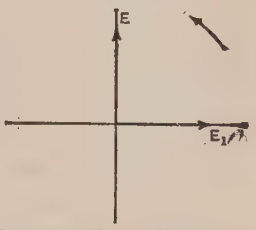


Fig. 399.—Vector diagram of the E.M.F.'s in the phases of a 2-phase generator.

(2) The two coils may be joined in series, as end A' of one coil to end B of the other, and the terminals A and B' brought out to one pair of collecting rings. The induced voltages in the two coils would merely be added vectorially before being brought out, and the machine would deliver a simple single-phase voltage, the curve of which would merely be the sum of the voltage curves of each coil.

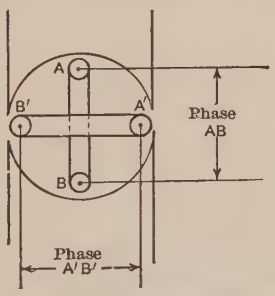


Fig. 400.—Two-phase system using four wires.

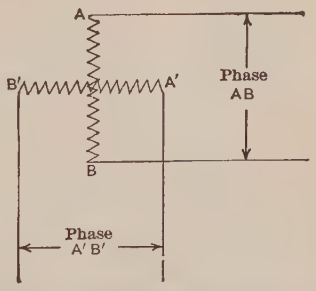


Fig. 401.—Conventional representation of a 2-phase system using four wires.

(3) Another, not very common way, is to join the two coils in series as above and then bring out a lead from the juncture to a third collector ring. This ring then acts as the neutral in a three-wire system, of which each phase forms a leg. The voltages in the two phases differ from each other

by 90°. Fig. 402 represents this case. Assume the voltage across phase AB = voltage across $A'B'$ = 110 volts. Then in the line, the voltage across XN and YN = 110 volts. Across XY, however, it would be not 220 volts, but the vector sum of 110 volts and 110 volts at right angles to each other. Thus in Fig. 403, OX = voltage across NX and OY = voltage across NY; ON = voltage across XY. If $OX = OY = 110$, then $ON = 155$ volts.

Note that line ON is the diagonal of a square, of which OY and OX are sides.

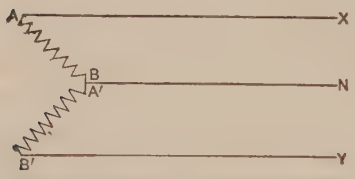


Fig. 402.—Diagram of a 3-wire 2-phase system.

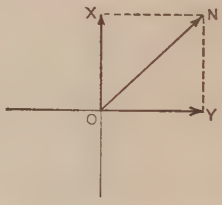


Fig. 403.—Graphical method of finding the voltage across the outer wires of a 3-wire 2-phase system.

The length of the diagonal of a square equals $\sqrt{2}$ times the length of one side. Thus,

$$ON = \sqrt{2}\,OY \text{ or } \sqrt{2}\,OX.$$

This is generally stated:

The voltage across two phases in a two-phase system equals $\sqrt{2}$ times the voltage across one phase (the phases being at 90° to each other).

The current and voltage distribution in such a system is found by the same method as that used in a D.C. three-wire system. *Care* must be exercised, however, *in combining the currents or voltages in the different sections vectorially instead of algebraically.*

282. Three-phase Distribution. Instead of using an armature with two coils at right angles to each other as in Fig. 397, three such coils might be used at an angle of 120°

to one another as in Fig. 404. The voltage induced in each of the three coils would then differ from the other by an angle of 120°. Fig. 405 shows the voltage curves of such an arrangement. Such a combination might also be connected internally and leads brought out to collecting rings in three ways:

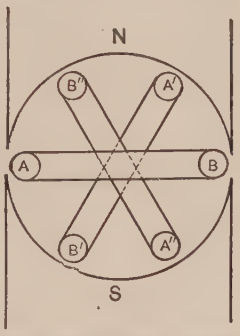


Fig. 404.—Simple 3-phase generator.

(1) The coils may all be connected in series and the two ends brought out to two rings. This would result in a single-phase generator, as the voltages would merely be added vectorially within the armature and the vector sum be supplied to the rings, and distributed to a single pair of wires.

(2) The three corresponding ends (as A, A', A'') may be connected to a common terminal and this be brought out

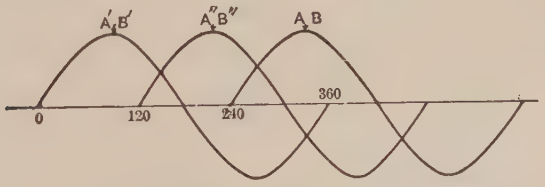


Fig. 405.—Sine curves of voltage in a 3-phase generator.

to a collecting ring. The other ends (B, B', B'') would then be brought out separately to three other rings. We would then have a three-phase system in which all three phases used a common return. This is commonly called a Y connection.

The conventional diagram for such an arrangement is shown in Fig. 406. The voltage from any outside wire (X, Y, or Z) to the neutral N would be the voltage of one phase.

When we desire to find the voltage across any two phases, we must add the voltages in the two phases *vectorially*.

Now, although the coils are said to be at an angle of 120° with one another and the voltages to differ in phase by 120°,

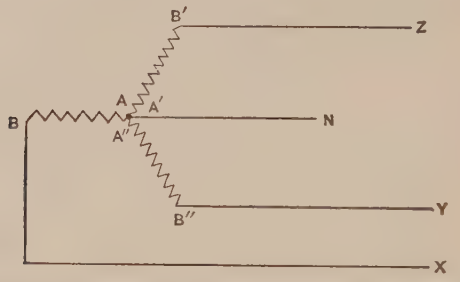


Fig. 406.—Diagram of a 3-phase system; Y-connected.

it will be noted by inspecting Fig. 404 that if we join any two adjacent ends together, that the coils thus joined are really at an angle of 60° and that the phase difference between the voltages is really 60°.

Thus in combining the voltage across phase AB with the voltage across phase $A'B'$, we must construct a vector

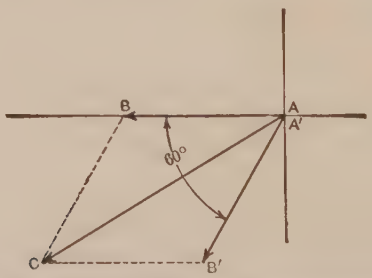


Fig. 407.—Graphical method of finding the voltage across any two phases of a 3-phase system.

diagram as in Fig. 407. Because, when A is joined to A', this diagram will represent the true phase relation between the voltages in the two coils, as is seen by an inspection of Fig. 404. The vector sum, then, of AB and $A'B'$ is the

line AC. If $AB = A'B' = 110$, AC by measurement $= 191$ volts. But the resultant of two equal forces at an angle of 60° to each other, equals $\sqrt{3}$ times the value of either force.

Thus the voltage across any two phases of a Y-connected generator equals $\sqrt{3}$ times the voltage across any single phase. The current and voltage relations of a loaded three-phase system are computed as in a two-phase. *Care must be taken to note correctly the phase relations of voltage and current in the different phases.* When the higher voltage is desired, there is need of but three, instead of four, wires in the line.

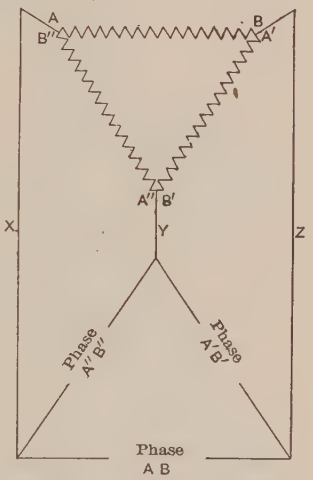


FIG. 408.—Diagram of a 3-phase system Δ-connected.

(3) Instead of joining *like ends* of the three phases together as in (2), we may connect *unlike ends*, as A to B', A' to B'', and A'' to B. This forms a closed loop. The junctures of the three coils are now brought out to three collecting rings. This is commonly called a Δ (Delta) connection. Fig. 408 is the conventional diagram for this arrangement.

It would appear at first sight that we had formed a short circuit of the coils, and that the current would circulate around through the armature coils.

But here, again, particular attention must be paid to the phase relations between the voltages across the coils. By constructing a vector diagram, we see that the vector sum of the voltages in any two coils is equal and opposite to the voltage in the third. The resulting voltage tending to cause current to circulate around through the coils is

then zero. By an inspection of Fig. 404, we see that in joining the coils together in this way, we joined adjacent ends. The coils thus joined are at an angle of 120° to one another, and the voltages induced in the coils are also at an angle of 120°. Thus, in the vector diagram of the voltage relations of the three coils, Fig. 409, the vectors representing the voltages are at an angle of 120°. The resultant or vector sum of any two voltages, such as $A'B'$ and $A''B''$, would be AR, which is equal and opposite to voltage AB. Therefore the voltage tending to cause a current to flow around through the three coils joined in Δ is zero. The

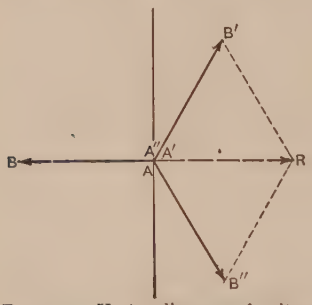


Fig. 409.—Vector diagram of voltage relations across the paths of a 3-phase armature, Δ-connected.

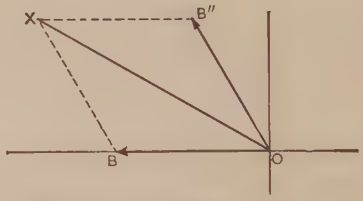


Fig. 410.—Vector diagram of current in armature paths and line wires of 3-phase system, Δ-connected.

voltage induced in each coil is accordingly available to force a current through any outside line which may be joined to the ends of the coil, as the three-phase line, XZ, ZY, and XY of Fig. 408. The voltage across each phase of the line is the voltage across each path of the armature.

By inspection of Fig. 408, it will be seen that each line wire of the three-wire system is fed by two coils of the armature. It will also be seen on close inspection that the currents in any two coils feeding one line wire are at an angle of 60° to each other. The current flowing in the line wire will then be the vector sum of the currents in these two armature coils.

If the load on the three phases is balanced, the current

in each line wire will be the same, and the current in each armature path will be the same.

The current in the line would then be the vector sum of two equal currents in the armature at an angle of 60° to each other. We have seen that the vector sum of two equal quantities making an angle of 60° with each other

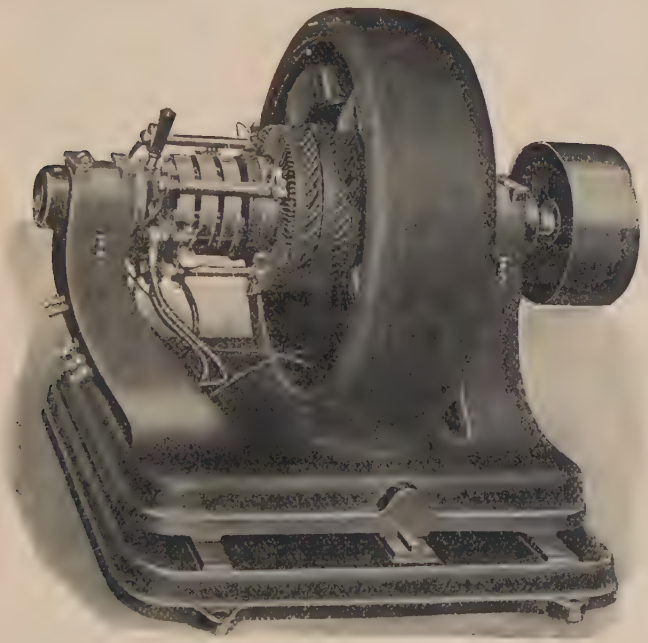


Fig. 411.—Three-phase generator. May be used as a rotary condenser, or a synchronous motor.

is equal to $\sqrt{3}$ times one of the quantities. This current relation is usually stated:

The current flowing in each line wire of a balanced three-phase system equals the $\sqrt{3}$ times the current in each armature path. This will be clear from the vector diagram, Fig. 410, of current in line wire X_1. This wire is fed by coils AB and $A''B''$, as per Fig. 408.

Let OB represent the vector for the current in phase AB. Then vector OB'' of the current in $A''B''$, makes an angle of 60° to AB, since the phase of the current in $A''B''$ is seen by Fig. 404 to be 60° behind the phase of current in AB. OX then represents the current in the line wire X, and is seen to be $\sqrt{3}$ times either OB or OB''.

The current and voltage relations throughout a loaded three-phase Δ system are found in accordance with the foregoing principles. Care must be taken at all times to properly represent the phase relations in the different branches. Fig. 411 is an illustration of a 3-phase generator, using no neutral wire. This machine may also be run as a motor, or as a rotary condenser, to improve the power factor of the line.

SUMMARY OF CHAPTER XV

An ALTERNATING CURRENT flows back and forth through a circuit.

A CYCLE consists of one complete flow back and forth.

The FREQUENCY is the number of cycles completed in one second, symbol, f.

A cycle is divided into 360 degrees; any instant of the cycle being rated as so many degrees and called the PHASE.

An ALTERNATING E.M.F. or an alternating current has four values:

(1) The instantaneous value, symbols e and i.
(2) The maximum value, symbols E and I.
(3) The average value, symbols avE and avI.
(4) The effective value, symbols E_f and I_f.

There are three ways of representing an alternating current:

(1) By curve; generally the sine curve;
(2) Vector diagram;
(3) Algebraic equation.

All alternating currents considered in this chapter are assumed to follow the sine curve.

Any INSTANTANEOUS VALUE depends upon the phase and is found by the equations:

$$i = I \sin \phi,$$
$$e = E \sin \phi,$$

where ϕ = angle of phase.

The AVERAGE VALUE of alternating currents and voltages equals .636 times the maximum value.

The EFFECTIVE VALUE, which is the value always meant when the terms voltage and current are used alone, is that value in amperes of an alternating current which will produce the same heating effect as a direct current of the same number of amperes. The effective value equals the square root of the average of the squares of all the instantaneous values, and is equal to .707 times the maximum value.

The current may either be IN PHASE with the voltage, LAG BEHIND the voltage, or LEAD the voltage. The angle of lead or lag is represented by the letter θ.

POWER. When the current is in PHASE with the voltage,

$$i = I \sin \phi,$$
$$P = I_f E_f$$

When the current LAGS BEHIND the voltage,

$$i = I \sin (\phi - \theta),$$
$$P = I_f E_f \cos \theta,$$

($\cos \theta$ is called the power factor).

When the CURRENT leads the voltage,

$$i = I \sin (\phi + \theta),$$
$$P = I_f E_f \cos \theta.$$

The cause of any lag in an alternating current is the inductive reactance of the circuit.

INDUCTIVE REACTANCE is the opposition offered by self induction to the flow of an alternating current, and is measured in ohms; symbol, X_L. $X_L = 2\pi fL$.

The cause of any lead in an alternating current is the capacity reactance of the circuit.

CAPACITY REACTANCE is the opposition offered by capacity to the flow of an alternating current, and is measured in ohms; symbol, X_c. $X_c = \frac{I}{2\pi fC}$.

REACTANCE is any combination of inductive and capacity reactance symbol, X.

$$X = X_L - X_c.$$

With inductance alone in the circuit the current lags 90 degrees; with capacity alone, the current leads 90 degrees.

$$\text{Current} = \frac{\text{Voltage}}{\text{Reactance}},$$

but the current is always 90 degrees out of phase with the voltage.

IMPEDANCE, symbol Z, is the combination of reactance and resistance, and is found from the following equation:

$$\text{Impedance}^2 = \text{reactance}^2 + \text{resistance}^2,$$

or

$$Z = \sqrt{R^2 + X^2}.$$

Voltage to force a current of I_f amperes through an impedance of Z ohms equals the product of current times impedance, i.e.,

$$E_f = I_f Z,$$

or

$$E = IZ.$$

The angle of lead or lag, in the case of a circuit containing impedance, can be found from the equation,

$$\tan \theta = \frac{X}{R}.$$

Voltage and current values in parallel and series circuits are found by adding vector quantities. See summation on Page (491).

Alternating current is at present distributed either by SINGLE PHASE, TWO PHASE or THREE PHASE SYSTEMS.

In a TWO-PHASE SYSTEM the voltage across one phase is at an angle of 90 degrees to the voltage across the other.

In a THREE-WIRE TWO-PHASE system, the voltage across the two phases equals $\sqrt{2}$ times the voltage across one phase.

In a THREE-PHASE SYSTEM the voltage across each phase is at an angle of 120 degrees to the voltage across the other phases.

When Y CONNECTED, AND BALANCED, the voltage across any phase equals $\sqrt{3}$ times the voltage between one line wire and the neutral. Neutral need not be used.

The current in each path in the armature is the same as the current in one phase.

When Δ CONNECTED and balanced, the voltage across any phase equals the voltage across any one path in the armature.

The current in each line-wire equals $\sqrt{3}$ times current in each armature path.

PROBLEMS ON CHAPTER XV

[*Wave forms of voltage and current in the following problems are assumed to be sine curves.*]

92–15. What is the instantaneous value of an alternating E.M.F. when it has completed 40° of its cycle? Maximum value =135 volts.

93–15. What is the average value of an alternating current which has an instantaneous value of 14 amperes at the 35° phase?

94–15. What would be the effective value of current in Problem 93?

95–15. If current of Problem 93 is caused by voltage of Problem 92, and current phase is 35° when voltage phase is 40°, compute: (*a*) Angle of lead or lag; (*b*) Power factor; (*c*) Instantaneous power; (*d*) Average power.

96–15. The equation for an alternating E.M.F. is $e=152 \sin \phi$. (*a*) What is e, when $\phi=70°$? (*b*) When $\phi=135°$? (*c*) When $\phi=960°$?

97–15. The instantaneous value of a current at 25° phase is 80 amps. What is the equation for the current?

98–15. The equation for an alternating current is, $i=141 \sin (\phi-20°)$. (*a*) What is the instantaneous value of the current when $\phi=70°$? (*b*) When $\phi=240°$?

99–15. What are the instantaneous values of the voltage when the phase values are (*a*) 35°, (*b*) 95°, (*c*) 520°. The equation for the voltage is $e=280 \sin \phi$.

100–15. Two voltages E and E_1 are impressed upon a circuit in series. $E_1=140$ volts and lags 35° behind E, which equals 201 volts. (*a*) What is the instantaneous voltage across the circuit when E is at its 80° phase? (*b*) When E is at 10° phase? (*c*) When E is at its 125° phase?

101–15. Two alternating currents I and I_1 are flowing in parallel branches of a circuit. $I=42$ amperes, $I_1=20$ amperes and lags 35° behind I. (*a*) What is the resultant of these two currents? (*b*) What is the phase relation between the resultant current and I?

102–15. If the voltage across the parallel circuits in Problem 101 is 110 volts and is in phase with the resultant current, find:
(*a*) Power in branch containing I.
(*b*) Power in branch containing I_1.
(*c*) Total power in parallel circuit.

103–15. Find instantaneous power in each branch of circuit of Problem 101, when I is at the 60° phase.

104–15. How many volts are necessary to force 25 amperes alternating current through 8 ohms resistance?

105–15. (*a*) How many watts are consumed in resistance of Problem 104? (*b*) How much direct current would be necessary to cause same heating effect as this alternating current?

106–15. If a coil of 8 ohms inductive reactance and of neglible resistance is used instead of the resistance of Problem 104, how many volts are necessary to force 25 amperes through it?

107–15. What would be the equations for the instantaneous values of voltage and current in Problems 104 and 106?

108–15. (*a*) If the coil in Problem 106 were capacity reactance, what voltage would be needed to force 25 amperes through it? (*b*) Give equations for instantaneous values of current and voltage.

109–15. A coil of .2 henry inductance, negligible resistance, is placed on a 110 volt 60 cycle circuit. What current flows?

110–15. A coil of .2 henry inductance and 15 ohms resistance, is placed in a 110 volt 60 cycle line. (*a*) What current flows? (*b*) What is the angle of phase difference between current and voltage?

111–15. (*a*) Write the equations for the instantaneous values of voltage and current in coil of Problem 110. (*b*) How much power is consumed by the coil?

112–15. Write the equations for the instantaneous values of voltage and current in coil of Problem 109. (*b*) How much power is consumed by the coil?

113–15. If the frequency in Problem 110 were changed to 25 cycles, what would be the answers to Problems 110 and 111?

114–15. If the coil in Problem 110 were put on a 110 volt D.C. line, what would be the answers to Problems 110 and 111?

115–15. When the voltage across coil in Problem 110 is passing through zero (growing), what value will the current have?

116–15. An inductance coil has a resistance of 80 ohms and an inductance of .4 henry. When on a 40 cycle line, the equation for the current through it is $i=5 \sin \phi$. (*a*) What is the equation for the voltage across the coil?

117–15. (*a*) What value will voltage have in Problem 116, when the current is passing through zero (growing)? (*b*) What power is consumed by the coil?

118–15. A choke coil of negligible resistance and .3 henry inductance is placed in series with a 6 ohm resistance across a 25 cycle line. The current which flows through the combination is 12.5 amperes. What is the voltage across each and across the combination?

119–15. The two pieces in Problem 118 were placed in parallel across a circuit of the same frequency and 25 amperes flowed through the choke coil. (*a*) How much current flowed through the resistance and how much current flowed in the main line, assuming these two were the only pieces in the circuit?

120–15. What voltage is required to force .1 ampere through coil in Problem 1–10, if frequency is 25 cycles per sec. and resistance of coil is 240 ohms?

121–15. What current will flow through coil in Problem 5–10 when 220 volts, 60 cycles, are put across the terminals? Resistance equals 400 ohms.

122–15. What would be the line "drop" in transmitting 2 amperes A.C. over line in Problem 10–10? Frequency=60.

123–15. If 50,000 volts were used in Problem 122–15, how much power would be transmitted? Non-inductive load.

124–15. What would be the power factor and efficiency of transmission in Problem 123–15?

125–15. If 20 amperes flow in an A.C. circuit when the pressure is 115 volts, what are the resistance, reactance and impedance? Power factor is .85; current lags.

126–15. A non-inductive resistance of 12 ohms, a capacity of 250 microfarads, and an inductance of .08 henry are connected in series across a 60 cycle, 112 volt line. Find:

(*a*) Voltage across the resistance.
(*b*) Voltage across the inductance.
(*c*) Voltage across the capacity.
(*d*) Current through the combination

127–15. (*a*) Find the power factor of combination in Problem 126. (*b*) Find power consumed by each part of the combination.

128–15. Two inductance coils, having resistances of 12 and 6 ohms respectively, and inductances of .2 and .08 henry respectively, are connected in series across a 25 cycle 110 volt circuit. Find:

(*a*) Current through the coils.

(*b*) Voltage across each coil.

(*c*) Phase relation of voltage across first coil to voltage across second coil.

129–15. If coils in Problem 128 are connected in parallel across the same circuit, what will be the answers to (*a*), (*b*) and (*c*)?

130–15. A generator is to deliver 80 amperes at 110 volts to supply power to incandescent lamps, which are non-inductive. If the line wires have .4 ohm resistance and .2 ohm reactance, what must brush potential of the generator be?

131–15. (*a*) What voltage is lost in the line of Problem 130? (*b*) What is efficiency of transmission?

132–15. An induction coil is connected across a 60 cycle 220 volt line and takes 12 amperes. If it consumes 1400 watts on this circuit what power will it consume on a 25 cycle, 110 volt line?

133–15. A 25 cycle 220 volt generator delivers 50 amperes to run induction motors. The power factor is then .80. What capacity must be put in parallel with the motors to bring the power factor up to .95?

134–15. A choke coil takes 8 amperes and consumes 1200 watts when connected across a 60 cycle 220 volt line. What resistance must be inserted in series with choke coil so that when placed across a 25 cycle, 110 volt line it may take the same current?

135–15. A synchronous motor takes a leading current of 45 amperes when the fields are overexcited. An induction motor takes a lagging current of 85 amperes. Power factor of synchronous motor is .90; of induction motor .80. If the two motors are operated in parallel on a 110 volt line, what current does generator supply?

136–15. What is power factor of load on generator in Problem 135?

137–15. In a three wire, two phase A.C. system, each leg has a pressure of 110 volts across it. If the system is balanced and 40 amperes are flowing in each phase, what current flows in common return wire?

138–15. The load in Problem 137 is non-inductive. What current would flow in each phase if common return wire were broken?

139–15. A three phase delta-connected generator, has equal loads of 50 K.W. Δ-connected in each phase. Voltage across each phase equals 100 volts. Unity power factor. (*a*) What is the current in each phase? (*b*) In each lead wire? (*c*) In each armature circuit.

140–15. If generator of Problem 139 were Y-connected, what would be the voltage across each phase of the line?

APPENDIX

Useful Numbers

$\pi = 3.1416 = \dfrac{\text{circumference}}{\text{diameter}}$. Surface of cyl. $= 2\pi rl + 2\pi r^2$.

$\pi^2 = 9.8696$; $\dfrac{1}{\pi} = .3183$. Volume of cyl. $= \pi r^2 l$.

Area of circle $= \pi r^2 = \dfrac{\pi d^2}{4} = .7854 d^2$. Surface of sphere $= 4\pi r^2$.

Volume of sphere $= \dfrac{\pi d^3}{6} = \dfrac{4\pi r^3}{3}$.

Metric-English Equivalents

1 cm.	= .39 in.	1 in.	= 2.54 cms.
1 m.	= 39.37 ins.	1 ft.	= 30.48 cms.
1 m.	= 3.23 ft.	1 ft.	= .305 m.
1 km.	= .6 mile	1 mile	= 1.60 km.
1 gm.	= .035 oz. (avoir.)	1 oz.	= 28.35 gms.
1 kgm.	= 2.204 lbs. (avoir.)	1 lb.	= 453.6 gms.
1 sq.cm.	= .155 sq.in.	1 sq.in.	= 6.45 sq.cms.
1 cu.cm.	= .061 cu.in.	1 cu.in.	= 16.39 cu.cms.

Units of Force, Work, Power, etc.

1 dyne = .00102 gm.
1 ft.lb. $= 1.356 \times 10^7$ ergs
1 joule $= 10^7$ ergs.
1 horse-power = 33,000 ft.lbs./min.
1 horse-power = 550 ft.lbs./sec.
1 horse-power $= 7.46 \times 10^9$ ergs/sec.
1 horse-power = 746 watts.
1 watt = .00134 horse-power.
1 watt $= 10^7$ ergs./sec. = 1 joule/sec.

Mechanical Equivalents of Heat

1 gm. of water heated 1° C. $= 4.2 \times 10^7$ ergs.
1 lb. of water heated 1° C. = 1400 ft.lbs.
1 lb. of water heated 1° F. = 780 ft.lbs.

The combustion of 1 lb. of coal produces about 14,000 B.T.U.

SIGNIFICANT FIGURES

(From Jameson's "Mechanics," Longmans, Green & Co.)

The results of all experimental work should be so expressed as to indicate as nearly as possible the degree of precision with which the work was performed. It is evident that all numbers that are obtained as the result of measurements are limited in precision by the nature of the apparatus employed, by the care used by the observer, the size of the units, etc. In this respect, then, such quantities are quite different from the pure numbers of absolute value as employed in ordinary arithmetical operations.

The student must observe carefully the following rules in all laboratory work and in reports.

I. Recording Readings

In general, scales, etc., are to be read to tenths of the smallest divisions marked on the instrument. The last figure entered in the record is thus assumed always to be an estimation and *therefore doubtful.*

Example 1.—15.57 cms. means that a distance was measured by a scale subdivided to *millimeters*, and that the observer *estimated* the seven; thus the distance is known to be between 15.5 and 15.6, and *estimated* to be $\frac{7}{10}$ths of the way between these two values. It is misleading, and furnishes only a clue to what we *actually know* about this distance to record it as 15.6 or 15.570 cm.

Example 2.—A distance is being measured with a rule subdivided to *tenths of inches.* The observer finds the distance to be as nearly exactly seven inches *as he can distinguish.* This should be recorded *7.00 in.* (not 7.0 or 7 ins.). Why?

Example 3.—A balance is capable of weighing an object to .01 gm. and .001 gm. can be estimated. Notice the correct records for following:

Eight gms.	8.000 gms.
Eight and $\frac{1}{2}$ gms.	8.500 gms.
Eight and $\frac{7}{100}$ gms.	8.070 gms.
Eight and $\frac{8}{1000}$ gms.	8.008 gms.
Eight-tenths gm.	.800 gms.

In general a series of readings made with the same instrument should all show the same number of places filled in to the right of the decimal point *even if one or all these* places are *zeros*. Why?

It is often convenient to express *in decimal form* readings taken from scales divided into halves of units, quarters, eighths, etc. In all such cases, retain only as many places in the decimal as correspond approximately to the same degree of precision as would be expressed by the fraction, i.e., to the nearest half unit, to the nearest quarter, etc. If the first decimal figure rejected is 5 or greater, call the preceding figure one larger than before.

A study of the following table should make this clear:

$\frac{1}{2}=.5$	$\frac{1}{8}=.1$	$\frac{3}{8}=.4$	$\frac{2}{3}=.7$
$\frac{1}{3}=.3$	$\frac{1}{16}=.06$	$\frac{5}{8}=.6$	$\frac{3}{4}=.8$
$\frac{1}{4}=.3$	$\frac{1}{32}=.03$	$\frac{7}{8}=.9$	$\frac{3}{16}=.19$
$\frac{1}{5}=.2$	$\frac{1}{64}=.02$	Etc.	Etc.
$\frac{1}{6}=.2$			

II. Use of Data in Calculations

Wherever the figure following the doubtful (last retained) figure is 5 or greater than 5, increase the doubtful figure by unity. Thus, if but three figures are to be kept, 15.75, 15.76, 15.77, 15.78, and 15.79 would all be entered 15.8.

Notice especially that the *location of the decimal point has nothing to do with significant figures.* Thus, 275, 27.5,

2.75, .275, .0275, .00275, etc., are all results expressed to some degree of precision, and in each there are *three and only three* significant figures, the 5 being the doubtful figure in each.

Averages.—In averaging a series of determinations, in general, retain in the result the same number of significant figures as in any one item.

But if a *large* number of items closely agreeing with each other are averaged, the result may contain *one more* significant figure than any item.

Multiplication.—After the operation, keep in the result as many figures, *counting from the left*, as there are significant figures in the factor having the *lesser number* of significant figures.

Division.—In dividing one number by another, keep in the quotient as many figures as there are significant figures in the number having the lesser number of significant figures. Continue the divisions only far enough to determine the required figures.

Note on Multiplication and Division.—Ciphers immediately following the decimal point, *when there are no figures to the left of the point, do not count* as significant. Study the following examples:

(*a*) $15.75 \times 3.08 = 48.5.$
(*b*) $.096 \times .096 = .0092.$
(*c*) $.1523 \times .00113 = .000172.$
(*d*) $720 \times 3.1 = 2200.$
(*e*) $900 \times 800 = 720000.$

In (*e*) only the *first cipher* is significant. It is necessary to add the other three to express the number properly.

(*f*) $325.6 \div 72.5 = 4.49.$
(*g*) $.0007859 \div 157 = .00000500.$

Use of Pure Numbers, Constants, etc.—In using pure numbers and constants such as (3.1416), .7854, etc., do not employ more figures than there are significant figures in the values depending on experiment which are used with them in the same calculation. Thus if the diameter of a circle is measured as 4.51, the area is $4.51 \times 4.51 \times .785 = 15.9$. The use of more numbers in the constant lengthens the computation and gives no better result. Why?

CURVES

Coördinate Axes.—The position of a point in space may be fixed by reference to two known straight lines intersecting at right angles in the same plane as the point (OX and OY of the Fig. 412). Such lines are known as *coördinate axes.*

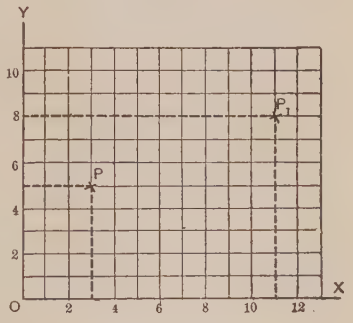


FIG. 412.

The horizontal line (OX) is known as the "*axis of abscissæ*" or "*X axis,*" the vertical line (OY) as the "*axis of ordinates*" or "*Y axis.*" The point of intersection O is called the *origin.* The *abscissa* of a point is its horizontal distance from OY; its *ordinate* is its vertical distance from OX. These given, the position of the point is determined. Thus P is that point which has an abscissa of 3, an ordinate of 5, P_1 the point which has abscissa of 11, ordinate of 8, etc.

For convenience, squared or "cross-section" paper is used for work of this kind.

Curves.—A succession of related points may be connected by a smooth line, thus constituting a "*curve.*" Such curves are frequently the most convenient and the clearest way of representing a physical law, corrections for errors of apparatus, etc. Suppose, for example, that it is desired to show the relation between the stretch of a wire and the stretching loads producing it, data being as follows:

Load.	Increase in Length.
5 lbs.	.010 inch
10 "	.019 "
15 "	.030 "
20 "	.040 "
25 "	.051 "

Taking the stretching loads, expressed in some convenient scale of lengths, as ordinates, and the corresponding elongations similarly expressed, as abscissæ, a series of points may be located as just explained, and through these a smooth line may be drawn. Inspection of the curve thus produced (Fig. 413) will show at a glance what could be obtained from the figures only on more extended analysis. The law, "Elongation is proportional to the load applied," is seen immediately, from the nature of the curve.

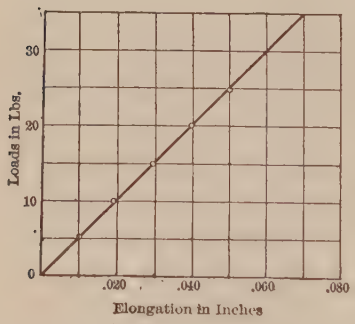


Fig. 413.

Had the curve turned continuously more and more toward either the X or the Y axis, showing in one case a progressive increase, in the other a progressive decrease in

elongation with increase of load, or had, at any time, a sudden change from the conditions which had previously existed occurred, these factors would have been brought to the attention as quickly.

When also, as here, the great majority of points lie along a straight line (or, as in some cases, along a smooth curve), any experimental errors of measurement (as in the elongations for loads of 10 and 25 lbs.), will be shown at once by the fact that these points lie slightly off the line. In all such cases, the curve should be drawn as nearly as may be through all points, and leaving as many points on one side as on the other.

The student must in all cases use his judgment in drawing the curve and consider the conditions of the experiment and the general physical law illustrated.

It is not necessary, and indeed often not advisable, that ordinates and abscissæ be expressed in the same scale. Of course, for the same curve all abscissæ must be in one scale, and all ordinates in one scale. In general, the scale adopted should be that most convenient for the particular values which will at the same time give a curve as large as the paper will permit.

One, two, five, or ten units to a square will be found the best. Avoid the use of three or seven units per square, or other inconvenient subdivisions.

GENERAL DIRECTIONS FOR CURVE SHEETS

(1) The curve must be done neatly in India ink.

(2) Heavy lines one inch in from the margin on the ruled portion are to be taken as axes, except where all the paper is necessary for the curve. The origin, i.e., the intersection of vertical and horizontal axes, should be at the lower left-hand corner. The paper may be used with

either longer or shorter side as vertical axis, according to needs of the curve.

(3) The scale on which the curve is plotted should be so selected as to make the curve as large as possible.

(4) Each axis should be marked with the quantity which it represents, and with the unit in which these quantities are expressed, e.g., "loads in pounds per square inch," "elongations in inches," etc. These titles should be lettered upon the *ruled paper between margin and axes.*

(5) Each *half-inch line* along both vertical and horizontal axes should be marked with the value which it represents. *No other figures* are to be used in locating the curve.

(6) The points fixing the curve are to be located by a small dot around which is drawn a small circle with a pair of dividers.

(7) The curve should usually be a *smooth line* drawn as nearly as possible through all points. It will represent the most probable value of the observations, and any single point lying at a distance on either side of the line will usually be a result of error in observations. Of course judgment must be used in drawing this conclusion, and the conditions of the experiment and the nature of the related quantities of the curve must always be taken into account.

(8) The name of the student and the date should be placed at the bottom of the sheet, at the right, in *small* letters.

(9) The title of the curve should be stated in the lower right-hand portion of the curve sheet unless this interferes with the curve; in which case the lower left-hand or the upper right-hand portion should be used.

(10) If more than one curve is drawn on the same paper for comparison, etc., use the *same origin and scales* for all.

Distinguish the curves by the title printed along the curve, or by lines of different colors.

(11) All titles, explanations, etc., must be in lettering, and no handwriting should appear upon the curve sheet.

THE EQUATION OF A STRAIGHT LINE

It is often desired to find the equation that corresponds to a given line (straight or curved) plotted on squared paper. In this course it will not be necessary to obtain the equation of a curved line. A simple method for the equation of a straight line follows:

Let AB, Fig. 414, be a line plotted as usual on the axes OX and OY, and meeting the axis of Y at the point A. (If the line as first drawn does not cut the axis of Y it must be extended till it does so.)

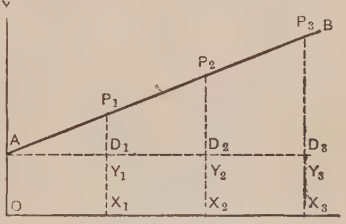


Fig. 414.

At the point A draw a line parallel to the axis of X. Choose any point on the line as P_2, and draw its ordinate y_2. x_2 is the abscissa of this point. We desire to obtain an equation that will give us the relation between the abscissa and the ordinate for this and every other point on this line.

We notice first that the ordinate y_2 equals the intercept OA on the Y axis, plus P_2D_2, or

$$y_2 = OA + P_2D_2.$$

Also,

$$y_1 = OA + P_1D_1,$$

$$y_3 = OA + P_3D_3,$$

and so on for every point on the line.

The value of the *intercept* OA may now be read from the curve. Suppose in the given case $OA=8$. Next read from the curve *values* of the *altitude* and *base* of *any triangle* whose hypothenuse is some part of the line AB. These values are to be expressed in units of the respective scales used in plotting X and Y and *not* as actual lengths in inches. The triangle AP_2D_2 will serve. Suppose $P_2D_2=4$ and $AD_2=10$ in the given case. Then $\frac{P_2D_2}{AD_2}=.4$. But $AD_2=x_2$, therefore $P_2D_2=.4x_2$.

If we had used other triangles we should have obtained the *same ratio* between altitude and base, and thus,

$$P_1D_1=.4x_1,$$

$$P_2D_2=.4x_2,$$

$$P_3D_3=.4x_3.$$

Or, in words, we may now say that any ordinate equals the intercept on the Y axis plus .4 of the abscissa for the same point. Let x and y be the coördinates of *any point on the line* AB; then

$$y=8+.4x,$$

which is the equation desired.

The ratio $\frac{P_2D_2}{AD_2}$ is sometimes called the *slope* of the line.

We may now state the general *rule* as follows:

Rule.—The equation of a straight line is formed by putting y equal to the intercept on the axis of Y plus the slope times x. If intercept $=a$, and slope (ratio) $=m$, we have,

$$y=a+mx.$$

NOTE.—The student will notice that the equation just given is perfectly general. If the line cuts the axis of Y *below* the origin, the intercept will be a *negative* term and the equation will be of the form $y = -a + mx$. If the line slopes so that an increase in the value of the abscissa causes a decrease in the value of the ordinate, then m will be a negative quantity, $y = a - mx$. It is possible, of course, that both a and m may be negative at the same time, as $y = -a - mx$. The student should draw and consider carefully lines to illustrate each case.

EXERCISES

1. Locate following points:

Point.	Abscissa.	Ordinate.
(*a*)	5	3
(*b*)	7	10
(*c*)	5	8
(*d*)	0	12
(*e*)	5	5
(*f*)	9	0

2. Measure carefully the lengths of 9 ins., 7 ins., 5 ins., 3 ins., respectively in metric units. Make each measurement three times, using different parts of the scale. Why? Take the average of the three readings and plot a curve, using inches as ordinates and the corresponding number of centimeters as abscissæ.

The curve will pass through the origin or zero-point of each scale. Why?

From your curve find the value of 1 inch in centimeters. What is the true value? What is your per cent of error?

3. A determination of the relation of bending of beam to load gave following results:

Loads.	Deflections.
10 lbs.	0.05 inch.
20 "	0.10 "
30 "	0.15 "
40 "	0.21 "
50 "	0.25 "
60 "	0.29 "
70 "	0.35 "

Plot curve showing relation of deflection to load, using loads as ordinates.

4. Plot a straight line such that it shall gain 3 units of abscissæ for every unit gained as ordinates.

5. A determination of Boyle's Law gave the following data:

Pressure.	Volume of Gas.
82.1	12.03
88.2	11.20
96.2	10.26
105.5	9.35
118.9	8.31
135.5	7.29
160.1	6.17

Plot the curve of above values, using pressures as ordinates.

6. A determination showing the effect of length of a beam upon its stiffness gave the following data:

Length.	Deflection.
10 inches	.005 inch
20 "	.035 "
30 "	.120 "
40 "	.285 "

Plot curve showing relation of deflections to length, using deflections as abscissæ.

7. The area of a circle varies with square of its diameter. Plot a curve to show relation of area to diameter in circle whose successive diameters are 1, 2, 3, 4, 5, and 6. Use diameters as ordinates.

8. What is the equation of the line which cuts the axis of Y at a point 3.4 above the origin and which rises 5 units for every 8 of horizontal distance?

9. If the intercept of a line is .56 and its slope is 2.58, what is its equation?

10. A line has two points whose coördinates are (8.5) and (3.1). Plot the line and obtain its equation.

11. What is the slope of a line if its intercept is 4 and $x=12$ when $y=6.8$?

12. The slope of a certain line is .532, and it passes through the origin. What is the ordinate of a point on this line whose abscissa is 2.3?

13. Write the equation of the following lines. a=intercept and m=slope.

(*a*)	$a=5$,	$m=$ .13.
(*b*)	$a=-5$,	$m=$ 3.
(*c*)	$a=-2.3$,	$m=$ .70.
(*d*)	$a=3$,	$m=-.68$.
(*e*)	$a=-2.5$,	$m=-.45$.

14. Draw a sketch to show the general character of each of the lines described in Problem 13.

SIMPLE TRIGONOMETRIC FUNCTIONS

Let ABC, Fig. 415, be any given angle, and let P be *any* point on the line BC. From P draw PN perpendicular to the side BA, thus forming the right-angled triangle PNB. Suppose we measure BP and find it to be 10 inches long and PN and find it to be 5.05 inches long. Then

$$\frac{PN}{BP}=\frac{5.05}{10}=.505.$$

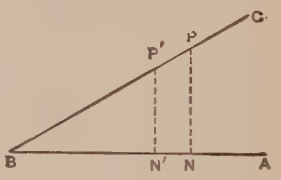


FIG. 415.

Now let us choose another point on BC, as P', draw the perpendicular $P'N'$ and measure lines BP' and $P'N'$. If BP' is $\frac{4}{5}$ as long as BP, or 8 inches, then $P'N'$ will be $\frac{4}{5}$ as long as PN, or $5.05\times4=4.04$ inches. The ratio $\frac{P'N'}{BP'}=\frac{4.04}{8}=.505$ or the same as that of PN to BP. So we might choose any point on BC and always obtain the same ratio of the perpendicular to the hypothenuse so long as we use the same angle ABC.

Similarly, $\frac{BN}{BP}$ gives a second constant ratio for the given angle, and $\frac{PN}{BN}$ a third, all of which are independent of the position of P and dependent only on the angle ABC.

If the angle ABC be changed, then the ratios $\frac{PN}{BP}$, etc., will have new values, but these again will be the same, no matter where P is taken on the line BC. Each angle thus has a certain number of constant ratios among which

are the three here given, and these ratios are given distinguishing names.

The ratio $\frac{PN}{BP}$ is called the *sine* of the angle ABC.

The ratio $\frac{BN}{BP}$ is called the *cosine* of the angle ABC

The ratio $\frac{PN}{BN}$ is called the *tangent* of the angle ABC.

Definitions.—Let B be an angle (not the right angle) of a right-angled triangle. The *sine* of the angle B is the ratio of the side *opposite* the angle to the hypothenuse of the triangle.

The *cosine* of the angle B is the ratio of the side *adjacent* the angle to the hypothenuse of the triangle.

The *tangent* of the angle B is the ratio of the side *opposite* to the side *adjacent.*

In calculations, sine, cosine, and tangent are always written for brevity, sin, cos, tan. Thus, $\sin 30° = .500$; $\cos 45° = .707$; $\tan 50° = 1.19$.

The values of these ratios have been calculated for all angles, and are given in what are called tables of trigonometric functions. Such tables, with the values carried out to three decimals, will be found on the following page.

TRIGONOMETRIC FUNCTIONS

A	Sin.	Cos.	Tan.	A	Sin.	Cos.	Tan.
0	.000	1.000	.000				
1	.017	.999	.017	46	.719	.695	1.04
2	.035	.999	.035	47	.731	.682	1.07
3	.052	.999	.052	48	.743	.669	1.11
4	.070	.998	.070	49	.755	.656	1.15
5	.087	.996	.087	50	.766	.643	1.19
6	.105	.995	.105	51	.777	.629	1.23
7	.122	.993	.123	52	.788	.616	1.28
8	.139	.990	.141	53	.799	.602	1.33
9	.156	.988	.158	54	.809	.588	1.38
10	.174	.985	.176	55	.819	.574	1.43
11	.191	.982	.194	56	.829	.559	1.48
12	.208	.978	.213	57	.839	.545	1.54
13	.225	.974	.231	58	.848	.530	1.60
14	.242	.970	.249	59	.857	.515	1.66
15	.259	.966	.268	60	.866	.500	1.73
16	.276	.961	.287	61	.875	.485	1.80
17	.292	.956	.306	62	.883	.469	1.88
18	.309	.951	.325	63	.891	.454	1.96
19	.326	.946	.344	64	.898	.438	2.05
20	.342	.940	.364	65	.906	.423	2.14
21	.358	.934	.384	66	.914	.407	2.25
22	.375	.927	.404	67	.921	.391	2.36
23	.391	.921	.424	68	.927	.375	2.48
24	.407	.914	.445	69	.934	.358	2.61
25	.423	.906	.466	70	.940	.342	2.75
26	.438	.898	.488	71	.946	.326	2.90
27	.454	.891	.510	72	.951	.309	3.08
28	.469	.883	.532	73	.956	.292	3.27
29	.485	.875	.554	74	.961	.276	3.49
30	.500	.866	.577	75	.966	.259	3.73
31	.515	.857	.601	76	.970	.242	4.01
32	.530	.848	.625	77	.974	.225	4.33
33	.545	.839	.649	78	.978	.208	4.70
34	.559	.829	.675	79	.982	.191	5.14
35	.574	.819	.700	80	.985	.174	5.67
36	.588	.809	.727	81	.988	.156	6.31
37	.602	.799	.754	82	.990	.139	7.12
38	.616	.788	.781	83	.993	.122	8.14
39	.629	.777	.810	84	.995	.105	9.51
40	.643	.766	.839	85	.996	.087	11.43
41	.656	.755	.869	86	.998	.070	14.30
42	.669	.743	.900	87	.999	.052	19.08
43	.682	.731	.933	88	.999	.035	28.64
44	.695	.719	.966	89	.999	.017	57.28
45	.707	.707	1.000	90	1.000	.000	Infinity

TYPICAL VALUES OF *B* AND *H* FOR DIFFERENT IRONS

(From Caldwell's "Electrical Problems")

B in Gausses.	*H* in Gausses.			
	Sheet Steel.	Cast Steel.	Wrought Iron.	Cast Iron.
3,000	1.3	2.8	2.0	5.0
3,500	1.4	3.1	2.2	6.5
4,000	1.6	3.4	2.5	8.5
4,500	1.7	3.7	2.7	11.0
5,000	1.9	3.9	3.0	14.5
5,500	2.1	4.2	3.2	18.5
6,000	2.3	4.5	3.5	24.0
6,500	2.4	4.8	3.7	30.0
7,000	2.6	5.1	4.0	38.5
7,500	2.8	5.4	4.2	49.0
8,000	3.0	5.8	4.5	60.0
8,500	3.2	6.1	4.7	74
9,000	3.5	6.5	5.0	89
9,500	3.7	7.0	5.3	106
10,000	3.9	7.5	5.6	124
10,500	4.1	8.2	6.0	144
11,000	4.4	9.0	6.5	166
11,500	4.7	11.2	7.2	191
12,000	5.0	11.5	7.9	222
12,500	5.5	13.7	8.9	255
13,000	6.0	16.0	10.0	290
13,500	7.0	18.0	13.0	328
14,000	9.0	21.5	15.0	369
14,500	12.0	27.0	18.5	
15,000	15.5	32.0	25.0	
15,500	20.0	40.0	35.0	
16,000	27.0	49.0	49.0	
16,500	37.5	60.0	69.0	
17,000	52.5	74.0	93.0	
17,500	70.0	93.0	120	
18,000	92.0	115	152	
18,500	119	139	186	
19,000	149	175	229	
19,500	189	226	277	
20,000	232	285		
20,500	277			
21,000	327			
21,500	383			
22,000	441			
22,500	498			
23,000	555			
23,500	612			
24,000	669			
24,500	726			
25,000	783			

Material (Commercial).	Resistivity; Ohms per Mil-Foot at 20° C.	Temperature Coefficient of Resistance = Increase per degree C. / Resistance at 0°
Aluminum	18.7	.00435
Copper, annealed	10.4	.0042
Copper, hard drawn	10.65	
Iron, annealed	90	.005
Iron, E. B. B. (Roebling)	64	.0046
German silver	114 to 275	.00025
Manganin	250 to 450	.00001
IAIA (Boker) soft	283	.000005
IAIA (Boker) hard	300	−.00001
Advance (Driver-Harris)	294	.00000

RESISTANCE OF SOFT OR ANNEALED COPPER WIRE

B. & S. Gauge.	Diameter in Mils, d.	Area in Circular Mils. d^2.	Ohms Per 1000 Ft. at 20° C. or 68° F.	B. & S. Gauge.	Diameter in Mils, d.	Area in Circular Mils, d^2.	Ohms Per 1000 Ft. at 20° C. or 68° F.
0000	460.00	211,600	.04893	21	28.462	810.10	12.78
000	409.64	167,810	.06170	22	25.347	642.40	16.12
00	364.80	133,080	.07780	23	22.571	509.45	20.32
0	324.86	105,530	.09811	24	20.100	404.01	25.63
				25	17.900	320.40	32.31
1	289.30	83,694	.1237	26	15.940	254.10	40.75
2	257.63	66,373	.1560	27	14.195	201.50	51.38
3	229.42	52,634	.1967	28	12.641	159.79	64.79
4	204.31	41,742	.2480	29	11.257	126.72	81.70
5	181.94	33,102	.3128	30	10.025	100.50	103.0
6	162.02	26,250	.3944	31	8.928	79.70	129.9
7	144.28	20,816	.4973	32	7.950	63.21	163.8
8	129.49	16,509	.6271	33	7.080	50.13	206.6
9	114.43	13,094	.7908	34	6.305	39.75	260.5
10	101.89	10,381	.9972	35	5.615	31.52	328.4
11	90.742	8,234.0	1.257	36	5.000	25.00	414.2
12	80.808	6,529.9	1.586	37	4.453	19.82	522.2
13	71.961	5,178.4	1.999	38	3.965	15.72	658.5
14	64.084	4,106.8	2.521	39	3.531	12.47	830.4
15	57.068	3,256.7	3.179	40	3.145	9.89	1047
16	50.820	2,582.9	4.009				
17	45.257	2,048.2	5.055				
18	40.303	1,624.3	6.374				
19	35.890	1,288.1	8.038				
20	31.961	1,021.5	10.14				

TABLE OF CARRYING CAPACITY OF WIRES

(National Electric Code)

The following table, showing the allowable carrying capacity of copper wires and cables of 98 per cent conductivity, according to the standard adopted by the American Institute of Electrical Engineers, must be followed in placing interior conductors.

(For insulated aluminum wire, the safe carrying capacity is 84 per cent of that given in the following tables for copper wire with the same kind of insulation.)

B. & S. Gauge.	Table A. Rubber Insulation, Amperes.	Table B. Other Insulations, Amperes.	Circular Mils.	Table A. Rubber Insulation, Amperes	Table B. Other Insulations, Amperes.
18	3	5	200,000	200	300
16	6	8	300,000	270	400
14	12	16	400,000	330	500
12	17	23	500,000	390	590
10	24	32	600,000	450	680
8	33	46	700,000	500	760
6	46	65	800,000	550	840
5	54	77	900,000	600	920
4	65	92	1,000,000	650	1,000
3	76	110	1,100,000	690	1,080
2	90	131	1,200,000	730	1,150
1	107	156	1,300,000	770	1,220
0	127	185	1,400,000	810	1,290
00	150	220	1,500,000	850	1,360
000	177	262	1,600,000	890	1,430
0000	210	312	1,700,000	930	1,490
			1,800,000	970	1,550
			1,900,000	1,010	1,610
			2,000,000	1,050	1,670

The lower limit is specified for rubber-covered wires to prevent gradual deterioration of the high insulations by the heat of the wires, but not from fear of igniting the insulation. The question of drop is not taken into consideration in the above tables.

The carrying capacity for No. 16 and No. 18 B. & S. gauge wire is given, but no smaller than No. 14 is to be used.

EQUIVALENT VALUES OF ELECTRICAL, MECHANICAL, AND HEAT-UNITS

From Kent's "Mechanical Engineers' Pocket-Book"

Work Units.

Unit.	Equivalent Value in Other Units
1 K.W. Hour =	1,000 watt hours. 1.34 H.P. hours. 2,654,200 ft.-lbs. 3,600,000 joules. 3,412 heat-units (B.T.U's.). 367,000 kilogram-meters. .235 lb. carbon oxidized with perfect efficiency. 3.52 lbs. water evap. from and at 212° F. 22.75 lbs. of water raised from 62° to 212° F.
1 H.-P. Hour =	.746 K.W. hour. 1,980,000 ft.-lbs. 2,545 heat-units (B.T.U's.) 273,740 kg.m. .175 lb. carbon oxidized with perfect efficiency. 2.64 lbs. water evap. from and at 212° F. 17.0 lbs. water raised from 62° F. to 212° F.
1 Joule =	1 watt second. .000000278 K.W. hour. .102 kg.m. .0009477 heat-units (B.T.U's.). .7373 ft.-lb.
1 Ft.-lb. =	1.356 joules. .1383 kg.m. .000000377 K.W. hour. .001285 heat-unit (B.T.U.). .0000005 H.-P. hour.
1 Kilogram meter =	7.233 ft.-lbs. .00000365 H.-P. hour. .00000272 K.W. hour. .0093 heat-unit (B.T.U.)

Power Units.

Unit.	Equivalent Value in Other Units
1 Kilo-Watt =	1,000 watts. 1.34 horse-power. 2,654,200 ft.-lbs. per hour. 44,240 ft.-lbs. per minute. 737.3 ft.-lbs. per second. 3,412 heat-units per hour. 56.9 heat-units per minute. .948 heat-unit per second. .235 lb. carbon oxidized per hr. 3.52 lbs. water evap. per hour from and at 212° F.
1 H.-P. =	746 watts. .746 K.W. 33,000 ft.-lbs. per minute. 550 ft.-lbs. per second. 2,545 heat-units per hour. 42.4 heat-units per minute. .707 heat-unit per second. .175 lb. carbon oxidized per hr. 2.64 lbs. of water evap. per hour from and at 212° F.
1 Watt =	1 joule per second. .00134 H.-P. 3.412 heat-units per hour. .7373 ft.-lb. per second. .0035 lb. water evap. per hour from and at 212° F. 44.24 ft.-lbs. per minute. 1 lb. pull at half a mile per hour (approx.).
1 Watt per sq. in. =	8.19 heat-units per sq.ft. per minute. 6,371 ft.-lbs. per sq.ft. per minute. .193 H.-P. per sq.ft.

Heat-Units, Work.

Unit.	Equivalent Value in Other Units.
1 Heat-Unit =	1 British Thermal Unit. Symbol, B.T.U. 253 calories.
1 Heat-Unit. B.T.U. =	1,055 watt seconds. 780 ft.-lbs. 107.6 kilogram-meters. .000293 K.W. hour. .000393 H.-P. hour. .0000688 lb. carbon oxidized. .001036 lb. water evap. from and at 212° F.
1 lb. carbon oxidized with perfect efficiency =	14,544 heat-units. 1.11 lb. anthracite coal oxidized. 2.5 lbs. dry wood oxidized. 21 cu.ft. illuminating gas. 4.26 K.W. hours. 5.71 H.-P. hours. 11,315,000 ft.-lbs. 15 lbs. of water evap. from and at 212° F.
1 lb. water evap. from and at 212° F. =	.283 K.W. hour. .379 H.-P. hour. 970 heat-units. 103,900 kg.m. 1,019,000 joules. 751,300 ft.-lbs. .0664 lb. of carbon oxidized.

Heat Units, Power.

Unit.	Equivalent Value in Other Units.
1 Heat-Unit per sq.ft. per min. =	.122 watt per sq.in. .0176 K.W. per sq.ft. .0236 H.-P. per sq.ft.

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